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**ELEMENTARY ENGINEERING  
THERMODYNAMICS**





# ELEMENTARY ENGINEERING THERMODYNAMICS

BY

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## PREFACE TO THE THIRD EDITION

The chief reasons which prompt the publication of the third edition of this text are the changes in emphasis on some applications of thermodynamics during the war years and the desire to present what the author believes to be improved methods of approach toward certain topics such as Joule's law, entropy, etc.

The author wishes to acknowledge gratefully the helpful suggestions of Prof. Linn Helander of Kansas State College and of Prof. Walter A. Downing, Jr., of the United States Military Academy, and to express his appreciation for the help given by Prof. Walter B. King of the Oklahoma Agricultural and Mechanical College in the preparation of material.

VINCENT W. YOUNG

STILLWATER, OKLA.

*August, 1947*



## PREFACE TO THE FIRST EDITION

In engineering instruction it has become the usual practice to introduce the theory and the practice of heat-work apparatus in two parallel courses, each about three semester hours in length. For the purpose of flexibility, it is desirable that either of these courses may precede the other or they may be offered concurrently, without handicap to the student. For students of mechanical engineering, these introductory courses are followed by later more detailed and more specialized instruction in the various related subfields such as internal-combustion engines, power-plant design and testing, refrigeration, and air conditioning.

This text has been designed to meet the requirements of the first named of these introductory courses and, it is hoped, will prove an acceptable treatment of the fundamental theoretical basis of apparatus of this type for the purposes of a beginning course for students of mechanical engineering or a survey course for students interested in other engineering fields.

The book has been made as brief and uncomplicated as a fairly comprehensive treatment will allow and many illustrative examples are included. Effort has been made by the authors to develop the subject in logical order and to give especial attention to features which, in their experience, have offered particular difficulty to the average student. Among the latter are the distinction between stored energy and energy in transition (internal energy and heat, stored mechanical energy and work), the unvarying character of the change of internal energy for a perfect gas, the two types of process (nonflow and steady-flow) under which all thermodynamic transactions may be classified, the character and properties of entropy, the subcooled (super-pressure) liquid and its properties, and the effect of friction on the interpretation of the cycle as presented on temperature-entropy coordinates.

A somewhat unusual feature in a treatise of this scope is the inclusion of a chapter on atmospheric air and the psychrometric chart. This has been added to give a thermodynamic foundation for the study of air-conditioning processes and apparatus.

The authors wish to express their appreciation of the criticism and assistance of Professors G. A. Munro, H. L. Solberg, and D. P. Craig of Purdue University and Professors V. L. Maleev, C. M. Leonard, and G. A. Hale of the Oklahoma Agricultural and Mechanical college.

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## SYMBOLS

$A$	Thermal equivalent of work
$a$	Area, square inches or square feet
$\beta$	Coefficient of performance (refrigeration)
$C$	Specific heat, B.t.u. per pound, per degree; also a constant
$C_p$	Specific heat at constant pressure, B.t.u. per pound per degree
$C_v$	Specific heat at constant volume, B.t.u. per pound per degree
$E$	Sum of available and unavailable energy, B.t.u.
$E_a$	Available energy, B.t.u.
$E_b$	Unavailable energy, B.t.u.
$e$	Efficiency
$F$	Force, pounds
$f$	Number of degrees of molecular freedom
$g$	Acceleration due to gravity = 32.2 ft. per second per second
$H$	Total enthalpy, B.t.u.
$h$	Specific enthalpy, B.t.u. per pound
$h_f$	Specific enthalpy of the saturated liquid, B.t.u. per pound
$h_{fg}$	Increase of specific enthalpy during vaporization, B.t.u. per pound
$h_g$	Specific enthalpy of the dry saturated vapor, B.t.u. per pound
$J$	Mechanical equivalent of heat
$K$	Kinetic thermal energy, B.t.u.; also a constant
$k$	Ratio of the specific heats = $C_p/C_v$
$L$	Distance, feet
$M$	Weight, pounds
$m$	Mass, slugs; also, molecular weight
$N$	Revolutions per minute
$n$	Exponent of polytropic
$P$	Pressure, pounds per square foot; also potential thermal energy, B.t.u.
$p$	Pressure, pounds per square inch
$Q$	Thermal energy in transition, heat, B.t.u.
$R$	Gas constant
$S$	Total entropy
$s$	Specific entropy
$s_f$	Specific entropy of the saturated liquid
$s_{fg}$	Increase of specific entropy during vaporization
$s_g$	Specific entropy of the dry saturated vapor
$T$	Absolute temperature, degrees Fahrenheit or degrees centigrade
$t$	Scale temperature, degrees Fahrenheit or degrees centigrade
$U$	Total internal energy, B.t.u.
$u$	Specific internal energy, B.t.u. per pound
$V$	Total volume, cubic feet
$v$	Specific volume, cubic feet per pound
$v_f$	Specific volume of the saturated liquid, cubic feet per pound

$v_{fg}$	Increase of specific volume during vaporization, cubic feet per pound
$v_g$	Specific volume of the dry saturated vapor, cubic feet per pound
$W$	Mechanical energy in transition, work, foot-pounds
$w$	Velocity, feet per second
$X$	Number of stages, air compression
$x$	Quality of the saturated vapor
$y$	Reheat fraction
$Z$	Elevation, feet

## ABBREVIATIONS

Absolute	abs.
Absolute temperature in Fahrenheit degrees	°F. abs.
British thermal unit	B.t.u.
Cubic foot	cu. ft.
Degree centigrade	°C.
Degree Fahrenheit	°F.
Foot	ft.
Foot-pound	ft.-lb.
Grain	gr.
Horsepower	hp.
Inch	in.
Inch of mercury	in. Hg
Logarithm (natural)	log.
Mean effective pressure	m.e.p.
Pound	lb.
Pounds per square inch	p.s.i.
Pounds per square foot	p.s.f.
Revolutions per minute	r.p.m.
Square foot	sq. ft.
Square inch	sq. in.

## CHAPTER I

### INTRODUCTION

**1:1. Thermodynamics.** The study of thermodynamics is the study of energy. Just as energy is found in many different forms (mechanical, thermal, chemical, etc.), so the broad field of thermodynamics is subdivided into many narrower fields of study which attempt to define and explain the energy changes that take place under given conditions between energy in two or more of its various forms.

One of the most important of these phases of thermodynamics is engineering thermodynamics, which deals with interchanges of energy between the mechanical and thermal forms and offers a theoretical basis for the interpretation of many engineering processes and for the design of engineering apparatus such as the internal-combustion engine, the steam engine and turbine, and the mechanical refrigerator.

It will be the purpose of this text to introduce the student to the ideas of engineering thermodynamics. The study of thermodynamics is essentially a precise study. That is, it consists of a mathematical development of and from certain natural laws which have been time-tried and tested. Basically, these laws have been developed through observation and experiment. The mathematical statements which define them are then analyzed and used as a basis for the extension of our knowledge of the subject, and this additional knowledge is then again tested experimentally. Our knowledge of thermodynamics is constantly expanding in this way and will continue to expand as long as the knowledge of the human race continues to grow.

Only the basic facts of engineering thermodynamics will be within the scope of this book, the purpose of which is to provide a theoretical foundation for the study of heat and power apparatus to the engineering student. The simpler, more essential phases will be discussed, and an effort will be made to eliminate the more complicated mathematical processes as far as possible.

**1:2. A thermodynamic medium** is the name given a substance taking part in a thermodynamic process through its ability to receive, store, and emit energy as required by the nature of the process. The more important mediums included under this head in the study of engineering

thermodynamics are classified as gases and vapors, and the distinction between them will be more clearly defined in a later chapter. Liquids and solids may also act as mediums in certain thermodynamic processes although they are of less importance to the student. The gaseous medium which is especially important from a practical standpoint is either air or a similar mixture of gases such as is produced through the combustion of fuel in an atmosphere of air. The most important vapor is undoubtedly steam, although others such as mercury and ammonia have properties which make them useful in certain processes and under certain conditions.

**1:3. A thermodynamic system** is composed of that portion of a thermodynamic medium which is essential to the thermodynamic change under consideration. Such a system has the function of absorbing or emitting heat, expanding or contracting as may be required by the conditions of the process. Thus a specified quantity of air, water, steam, or any combination of substances may form a thermodynamic system. It should be understood that the thermodynamic system consists of the medium only and does not include the apparatus necessary for the practical demonstration of the change. As has been stated above, in the study of engineering thermodynamics, interest is largely confined to vapors, to the liquids which result from their condensation, and to gases. Thus in a thermodynamic analysis of the performance of a steam engine or turbine, the *system* is the water and steam which undergo the changes and does not include the apparatus, such as boiler, prime mover, and condenser, necessary to bring about those changes.

**1:4. The nature of energy** is usually clearly understood by the student but is incapable of being satisfactorily defined. The conventional definition states that energy is the capacity for doing work. Energy can be better understood by reviewing its properties and characteristics.

1. Energy may be *stored* in a body in the form of thermal, mechanical, electrical, or chemical energy. Such a storage of energy is always evidenced by some change of temperature, phase (as from solid to liquid), or composition. A pan of water when heated (*i.e.*, raised in temperature) increases its store of thermal energy.

2. Energy may also *flow* from one body to another without any movement of the particles which make up the bodies themselves. Examples of *energy in transition* are *work* and *heat*. The first is mechanical energy and the second is thermal energy in transition. When these terms are used hereafter, it will be understood always that the energy is in process of transition. Work and heat, since they can exist only instantaneously, must immediately find a medium to inhabit as stored energy. However,

either work or heat, after their instantaneous passage from one medium to another, do not necessarily reappear as the equivalent form of stored energy (*i.e.*, mechanical energy or thermal energy). For instance, when heat is applied to a thermal couple there are two effects. First, a part of the thermal energy absorbed is utilized in increasing the temperature of the metals forming the couple; and, second, the remainder is converted into electrical energy. This property of energy in transition is, as we shall see, of the utmost importance.

3. The many forms in which energy appears are mutually convertible. This is merely a restatement of what has been implied in the preceding paragraph. Its application is not restricted to changes involving mechanical, thermal, and electrical energy but extends to energy in all its forms. For instance, the process of charging a storage battery is essentially a conversion of electrical into chemical energy.

4. Energy is measured not in absolute but in relative terms. A moving object possesses energy dependent upon its mass and the velocity at which it moves. While its mass may be measured in absolute units, its velocity must be expressed relative to some other object such as the surface of the earth. In that case, its energy as calculated will be expressed relative to the earth's surface.

5. Energy can be neither created nor destroyed. As soon as energy leaves one system it must reappear in another or a number of others and in equivalent amount. The various forms of energy are measured in different units, all of which must evidently bear a definite numerical relationship to one another.

6. All forms of energy are two-dimensional in character, *i.e.*, the product of two quantities such as force and distance, pressure and volume, temperature and entropy, is required to express the magnitude of the energy received by a thermodynamic system. Heat and work are, therefore, commonly represented diagrammatically by an area. Energy is a scalar rather than a vector quantity.

7. The various forms of energy are associated with different types of systems. Thus, mechanical energy is associated with tangible objects, or *molar bodies* (bodies composed of collections of molecules), thermal energy with systems of molecules, chemical energy with the atom, etc.

8. The equivalence of energy and mass has long been known and has of late taken on increased significance. Development in this field lies within the scope of the broad field of thermodynamics, but progress in this direction will not destroy the importance and usefulness of specialized study in thermodynamics such as is the purpose of this text.



In the study of engineering thermodynamics, interest is centered on energy in the mechanical and thermal forms. These special forms of energy have certain common characteristics, a review of which will aid the student in a proper understanding of their nature. In the first place, both may properly be divided into two main classifications, *viz.*, stored energy and energy in transition. Second, stored energy in both these forms may be divided into two subclasses, kinetic energy and potential energy. The basis of this subclassification will be more fully discussed in later paragraphs. The following outline will be helpful to the student:

$$\begin{array}{l} \text{Mechanical} \\ \text{or} \\ \text{Thermal} \end{array} \quad \begin{array}{l} \text{energy} \\ \\ \end{array} \quad \left\{ \begin{array}{l} \text{stored energy} \left\{ \begin{array}{l} \text{kinetic energy—energy due to motion} \\ \text{potential energy—energy due to position} \end{array} \right. \\ \text{energy in transition—work or heat} \end{array} \right.$$

**1:5. Stored mechanical energy**, as has been seen, may appear in either of two forms, kinetic or potential. Kinetic mechanical energy is possessed by a body in motion, and its amount is measured by the product

$$\text{Kinetic mechanical energy} = \frac{1}{2}mw^2 = \frac{Mw^2}{2g} \quad (1:1)$$

in which  $m$  is the mass of the body,

$w$  is its velocity,

$M$  is its weight,

and  $g$  is the acceleration due to gravity.

If  $M$  is measured in pounds,  $w$  in feet per second, and  $g$  in feet per second per second, the kinetic energy will be expressed in foot-pounds. If the body is a fluid, all its parts may not have the same velocity owing to a progressive change of form during its movement. In this case  $w$  must be defined as the rate of movement of its center of gravity. In any case, since its velocity must be stated relative to the velocity of some other object (see Art. 1:4), the kinetic energy of a body is a purely relative quantity.

Potential mechanical energy is due to the position or internal stress of a body; it is possessed by water in an elevated tank or by a compressed spring. Its amount is measured by the work which the body is capable of performing in returning to a standard reference position or condition and is relative to the base which is selected.

**1:6. Stored Thermal Energy.** In order to explain the way in which thermal energy may be stored in a body, it is first necessary to evolve some theory on the basis of which the explanation may be understood.

For this purpose, we shall make use of the following theory as to the internal composition of the body.

We shall assume that the body is composed of an extremely large number of extremely small, perfectly elastic particles which we shall call molecules. These molecules are in constant motion within the confining surfaces of the body as a whole. Not all of them, of course, are moving in the same direction at the same time, or a movement of the whole body would result. This movement, or *molecular vibration*, is entirely an internal effect; and, because of the large number of these molecules, the law of averages makes it certain that at any given moment their individual kinetic energies will be so balanced as to leave unchanged the position of the center of gravity of the entire body. Since it is evident that the motion of these numerous molecules within a confined space must bring about many collisions and since these collisions, because of the perfect elasticity of the individual molecules, must result in an unreduced velocity in new directions opposite to each other as regards the individual molecules, the balanced position of the center of gravity of the body as a whole will be assured.

The energy possessed by these small molecules due to their random motion is classified as kinetic thermal energy. It is distinguished from energy in the kinetic mechanical form since the latter implies a movement of the body as a whole. The rate of movement of these molecules depends upon the intensity of the thermal-energy store which the body possesses. The intensity of thermal energy is evidenced by *temperature*, a more complete discussion of which will be attempted in a later paragraph. For the present, it is sufficient if we understand that, as thermal energy is added to the body, the resulting rise of temperature, *if it occurs*, is evidence of an increased rate of molecular vibration.

But we all are familiar with processes which result in adding to the store of thermal energy which a body possesses without a corresponding increase of its temperature. For example, the melting of ice represents an addition of thermal energy which leaves the temperature unchanged. The molecular theory explains this by the assumption that all these tiny particles have a mutual attraction for each other and that it will require energy either to increase the distance between them or to change the character or amplitude of their vibration. For instance, in the case of a solid, the molecular motion is interpreted as being confined to a certain mean or average position, in other words as being restricted within extremely narrow limits—such a motion as might better, perhaps, be termed an oscillation. As the temperature (and therefore the rate of this oscillation) gradually increases, the point is finally reached where it is

sufficient to break down the boundary walls of cohesion which previously had limited the motion of each molecule. When this temperature has been reached, frequently no further increase of temperature will occur until all these walls have been destroyed and the molecules are free to move throughout the entire body, which is now a liquid. The destruction of these walls has required the absorption of thermal energy, however, even though no increase of temperature results and this is given the name of *latent heat of fusion*. The word "latent" denotes that no increase of temperature has occurred during its addition.

The molecules of the liquid are now free to move about at will within the confines of the whole body of liquid but are restrained from leaving the body by a property known as *surface tension*. This property is again a phase of the mutual attraction of the molecules. As further thermal energy is added to the liquid, the rate of molecular vibration (as evidenced by the temperature) will increase. This has the effect of gradually breaking down the surface tension until the individual molecule is free to leave the confines of the liquid and assume a new position and a greater range of activity in space. The thermal energy required, at constant temperature, to change the form of the body from liquid to gaseous is called the *latent heat of vaporization*. The resulting vapor now has no surface tension and, to be restrained at all from free movement in space, must be confined within limiting walls of some liquid or solid substances.

Evidently stored thermal energy is analogous to stored mechanical energy in that it may be divided into the kinetic and potential forms. The internal motion of the molecules, a change in the rate of which, as we have seen, is always accompanied by a change of temperature, is clearly to be classified as internal kinetic or *thermal kinetic energy*, while the thermal energy which is added to break down the boundaries which limit molecular motion and to increase the distance between molecules against the force of their mutual attraction is just as certainly to be considered *thermal potential energy*.

To summarize, an increase in the store of thermal energy possessed by a system is attested by one or more of the following effects:

1. An increase of temperature.
2. A change in the state of aggregation of the system, as from liquid to vapor.
3. A change of volume.

The first effect represents an increase in the kinetic energy of the molecules of which the system is composed and, unlike the kinetic energy of the system as a whole, should be classified as kinetic thermal energy.

The second and third effects are evidences of a change of potential thermal energy analogous to the change of potential mechanical energy which takes place in a spring when it is elongated.

Stored thermal energy, whether kinetic or potential, is usually given the name of *internal energy* and will be so designated in this text. Also, the term *internal energy* will, for our purposes, include only stored thermal energy. The symbol used for total internal energy will be  $U$ , the lower-case letter  $u$  being used for specific internal energy. Internal energy is always expressed relative to some arbitrarily established reference condition, and interest is usually centered in the *changes* of internal energy that take place during a thermodynamic process. The total *change* of internal energy is the sum of the kinetic thermal and potential thermal effects, or

$$dU = dK + dP \quad (1:2)$$

in which  $dU$  is the *change* of internal energy,

$dK$  is the *change* of kinetic thermal energy,

and  $dP$  is the *change* of potential thermal energy.

In conclusion, it should be stated that, although the molecular theory has been used as the basis of the foregoing discussion, the validity of that theory is not essential to our study of engineering thermodynamics.

**1:7. The Properties of a Thermodynamic System.** The state or condition of a system is usually defined by a statement of its properties such as mass, velocity, pressure, volume, temperature, entropy, enthalpy. In this article certain of these properties will be defined and discussed, while others will be left for such time as will make their introduction into our ideas of engineering thermodynamics logical. For the present, it will be sufficient to say that the store of mechanical or internal (thermal) energy possessed by a system is a function of these properties.

The condition of a thermodynamic system is sometimes expressed in terms of its *pressure* and *volume*. *Pressure* may be defined as the force exerted by the system on a unit area. It may be expressed in various units; for the purposes of this text, pressures will be denoted by  $P$ , pounds per square foot, or  $p$ , pounds per square inch. The *volume* of a thermodynamic system is denoted by  $V$  and is ordinarily measured in cubic feet. The symbol for *specific volume*, or volume per pound, is  $v$ . Therefore, if  $M$  is the total weight of the system in pounds,  $V = Mv$ .

*Atmospheric pressure* is the force exerted on a unit area of the earth's surface due to the weight of the atmosphere. Atmospheric pressure decreases in general with the altitude and varies from day to day at the same location with atmospheric conditions. This variation is within

comparatively narrow limits; and, for convenience, *standard atmospheric pressure* is based on average sea-level conditions and is equivalent to the pressure sufficient to support a column of mercury 29.92 in. high. Since a pressure of 0.491 p.s.i. is required to support 1 in. of mercury, the standard atmospheric pressure is 14.7 p.s.i.

The construction of most pressure-measuring devices is such that they measure differences between the pressure of the confined substance and that of the atmosphere. If the pressure of the substance is greater than that of the atmosphere, the difference so recorded is called the *gage pressure*. The result obtained by adding the gage pressure to the atmos-

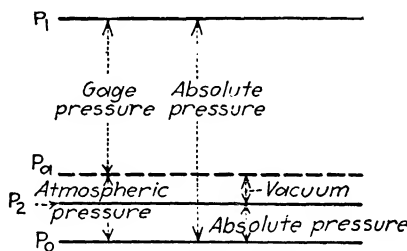


FIG. 1:1.—Pressure measurement.

pheric pressure is the *absolute pressure* and is the difference between the actual pressure of the substance and the true zero of pressure.

Pressures below atmospheric are sometimes expressed directly as absolute pressures and sometimes in terms of *inches of mercury vacuum*; if the latter designation is used, the equivalent absolute pressure may be found by subtracting the indicated vacuum from the atmospheric pressure as recorded by a barometer. When the actual atmospheric pressure is not known, it will be assumed as standard.

The relationships which exist between gage, absolute, atmospheric, and vacuum pressures are illustrated in Fig. 1:1.

In this figure, two pressures are used for purposes of illustration,  $P_1$  being any pressure above and  $P_2$  any pressure below the pressure of the atmosphere. The horizontal line  $P_a$  represents the pressure of the atmosphere and is dashed to indicate that it varies somewhat with atmospheric conditions. The full line  $P_o$  represents the absolute zero of pressure.

*Temperature* is one of the two dimensions of thermal energy and may be regarded as a measure of the intensity of thermal energy in the same sense that voltage measures the intensity of electrical energy; as we have seen, temperature is actually a measure of the internal *kinetic* energy of the system. Temperature determines the direction of flow of

heat from one body to another; without external aid, it can flow only from the warmer body to the colder, just as water, unless it is acted upon by some external mechanism such as a pump, cannot flow from a lower to a higher elevation.

A rise in temperature may be considered as being caused by an increase in the rate of vibration of the molecules which make up the body and is therefore an evidence of an increase in the amount of thermal energy stored in it.

In the United States, temperature is measured on the Fahrenheit scale, on which the melting point of ice is represented by a temperature of  $32^{\circ}$  and the boiling point of water at standard atmospheric pressure by a temperature of  $212^{\circ}$ . The centigrade scale, from which conversion is sometimes necessary, places these points at 0 and  $100^{\circ}$ , respectively; the degree centigrade is therefore equivalent to  $1.8^{\circ}\text{F}$ . Temperatures on either of the two scales may be changed to their equivalents on the other by the use of the following expressions:

$$\text{Degrees centigrade} = \frac{5}{9} (\text{degrees Fahrenheit} - 32) \quad (1:3)$$

$$\text{Degrees Fahrenheit} = \frac{9}{5} \text{degrees centigrade} + 32 \quad (1:4)$$

The zero point on neither of these scales, Fahrenheit

	Fahrenheit	Centigrade
	+932	+500
	+500	+260
	+212	Water boils +100
	+32	Ice melts 0
	-459.7	Absolute zero -273

of course, corresponds to an absolute zero of molecular kinetic energy. That such an absolute zero of temperature does exist has been established by research in physics. This absolute zero is located at  $-459.7^{\circ}$  on the Fahrenheit scale, equivalent to  $-273^{\circ}\text{C}$ . Temperature as measured by a thermometer will be denoted by  $t$ , and the symbol  $T$  will be reserved for absolute temperatures; therefore, on the Fahrenheit scale,

$$T = t + 459.7$$

A graphic comparison of the Fahrenheit and centigrade scales is made in Fig. 1:2. In this figure no attempt has been made to carry the comparison into the higher range of temperatures. Other scales of temperature are in use but are of minor importance to the engineering student.

FIG. 1:2.—Temperature scale.

Temperature is measured ordinarily on the basis of the increase in volume of some liquid, such as mercury, confined in a glass tube. The liquid used must have a boiling point well above the upper range and a

freezing point below the lower range of temperatures to be measured. For extremely high temperatures, the measurements are based either upon the amount of radiant energy emitted (the principle of the optical pyrometer) or upon the thermoelectric effects set up in an electrical circuit (the thermal-couple method).

**1:8. Mechanical energy in transition** is called *work*. It is produced by a force  $F$  acting through a distance  $L$ . If the force is measured in pounds and the distance in feet, the product  $FL$  is the work and is measured in foot-pounds. The foot-pound is the unit of work in the English system; and since work is a form of energy and all forms of energy are mutually convertible, it may also be used to designate the amount of energy either stored in a system or passing from one system to another in any form whatever.

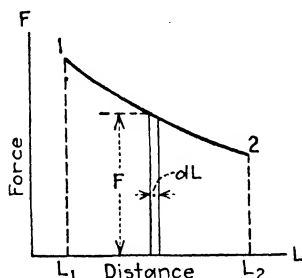


FIG. 1.3.—Work in terms of force and distance.

It should be clearly understood that work is neither kinetic nor potential mechanical energy, although it may be produced by the dissipation of either or may reappear in either form; rather, work represents a *transfer* of mechanical energy from one system to another as along a shaft due to its rotary or transverse movement; this is a most common example in engineering and is given the special designation of *shaft work*. Work cannot be *stored* in a system except in the sense that, if work is done upon it, its stored energy content may be increased.

In Fig. 1.3, the force  $F$ , instead of remaining constant, is represented as varying with the distance traversed. In this case, the work performed is represented graphically by the area under the force curve 1-2, and, in order to calculate its amount, the areas of all the little segments  $F dL$  must be integrated. The mathematical expression for this process and therefore the amount of work performed as the force travels from 1 to 2 is

$${}_1W_2 = \int_{L_1}^{L_2} F dL \quad (1:5)$$

If the force is expressed in pounds and the distance in feet, the resulting work will be in foot-pounds.

In the study of thermodynamics, it is often convenient to represent work on coordinates of pressure and volume rather than in terms of force

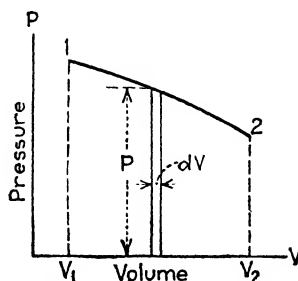


FIG. 1.4.—Work in terms of pressure and volume.

and distance. Pressure has been defined as force per unit area, and volume may be interpreted as the product of area and length (distance). Therefore the process, as represented in Fig. 1:4, involves the integration of the elements of area  $P dV$ , and the expression for work becomes

$${}_1W_2 = \int_{V_1}^{V_2} P dV \quad (1:6)$$

in which  $P$  is an expression for the pressure in terms of the volume. If the pressure is measured in pounds per square foot absolute and the volume in cubic feet, the work will be expressed in foot-pounds as before. It is evident that the work performed is positive if the volume of the system is increasing and negative if decreasing, the positive sign indicating that energy in the form of work is leaving the system, the negative sign that it is entering.

*Example.* A thermodynamic system has a pressure of 120 p.s.i. gage and a volume of 2 cu. ft. During an expansion to a volume of 5 cu. ft., the gage pressure changes according to the equation

$$p = 150 - 15V$$

How much work was performed by the system during the expansion?

*Solution.*

$$W = \int_2^5 P dV = 144 \int_2^5 p_a dV$$

in which  $p_a$  is the absolute pressure in pounds per square inch. Assuming standard atmospheric pressure, the variation of absolute pressure becomes

$$p_a = 164.7 - 15V$$

Substituting this value for  $p_a$  in the expression for work,

$$\begin{aligned} W &= 144 \int_2^5 (164.7 - 15V) dV = 144 \left[ 164.7V - 7.5V^2 \right]_2^5 \\ &= 144(823.5 - 329.4 - 187.5 + 30) = 48,470.4 \text{ ft.-lb.} \end{aligned}$$

In practical engineering apparatus, the expansion (and compression) of a system is often carried on within a cylinder through which a piston travels. An instrument called an indicator, by means of which a pressure-volume record of the condition of the fluid within the cylinder may be obtained, is often attached to this cylinder. Thus, in Fig. 1:5, as the

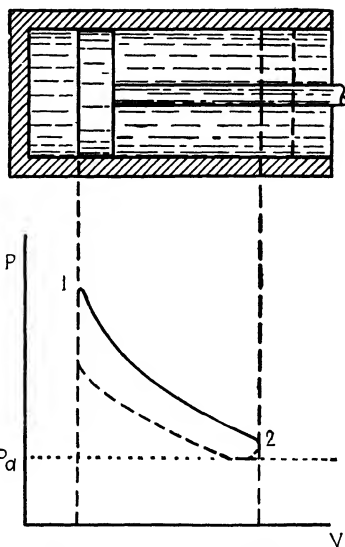


FIG. 1:5.—Indicator diagram.



piston moves from its position at the beginning of the stroke to the opposite end of the cylinder, the indicator traces the curve 1-2. If the engine piston is frictionless, the area beneath this curve represents energy in the form of work that has left the system. A part of this work travels out the piston rod to be employed for useful purposes; the balance is expended in pushing against the pressure of the atmosphere on the outer side of the piston.

The practical process is always a cyclic process, and the pressure-volume record during the return stroke is shown as a dashed line in the illustration. The complete pressure-volume record of the cycle is called the *indicator diagram*. During this return stroke, energy in the form of work reverses its direction of flow and travels back into the system, *i.e.*, negative area accumulates under the curve as the volume is reduced to its initial value. Again this energy return comes from two sources, a part of the energy stored in the flywheel (not shown) on the outward stroke is returned, and the pressure of the atmosphere working against the outer side of the piston accounts for the balance. Since the pressure of the atmosphere is constant during the double stroke, the amount of work returned by the atmosphere exactly balances the amount expended in displacing it on the power stroke. This amount of work is represented by the area under the dotted line, the height of which represents atmospheric pressure.

Again referring to the illustration, it is evident that the area between the full line and the dotted line representing atmospheric pressure is that part of the work delivered during the forward stroke which moves out along the piston rod and results in work delivery to the engine shaft (shaft work). The area below the dotted line evaluates the work expended in displacing the atmosphere on this stroke. In the same manner, the area between the dashed and dotted lines represents work returned by the shaft during the backstroke, and the area under the dotted line is that amount returned by the atmosphere. The *net* amount of shaft work delivered during the cycle is equivalent therefore to the area enclosed within the indicator diagram.

The analysis above has been developed by considering that the piston is frictionless. Friction is always present in some degree in the actual machine and always has the effect of changing work into heat. If friction is taken into account in the analysis, it will be noted that a corresponding deduction from the net shaft work will result. The heat developed through friction may reappear as stored thermal energy in the system enclosed within the cylinder or may flow directly into the atmosphere. Often a combination of these two effects is present; but, in any

case, its eventual destination is as stored thermal energy in the atmosphere.

It is important to note that Eq. (1:6) may be used for the calculation of the work delivered externally *only when no friction is present*. This imposes an important restriction on the use of this equation. However, in the development of thermodynamic theory, frictionless operation may usually be assumed as a limiting condition, and Eq. (1:6) assumes an important place in engineering thermodynamics.

Since most practical thermodynamic processes have as their purpose the *continuous* production of work, the idea of power, or work per unit time, is often encountered in engineering thermodynamics. The unit of power in the English system is the horsepower (hp.), which is equivalent to 550 ft.-lb. of work performed per second. The kilowatt, equivalent to 1.34 hp., is also frequently used.

**1:9. Thermal Energy in Transition.** When two bodies at different temperatures are placed in contact, the temperature of the colder will be observed to rise and that of the warmer to fall. This represents a *transfer* of thermal energy. To explain it, we say that *heat* has passed from the warmer to the colder body. The term *heat* is indiscriminately used in many references to refer to thermal energy in both the stored and the transitional states; but in this text, to avoid confusion, it will be reserved for thermal energy in transition, and the term *internal energy* will be used to denote stored thermal energy.

The unit of heat (and of all forms of thermal energy) in the English system is the British thermal unit (B.t.u.). The B.t.u. may be described as the amount of heat which is required to raise the temperature of 1 lb. of water from 63 to 64°F. This amount of heat is one one-hundred-eightieth of the amount necessary to raise the temperature of a pound of water from the freezing to the boiling point at standard atmospheric pressure or from 32 to 212°F.

**1:10. Mechanical Equivalent of Heat.** It is well known that thermal energy may be produced by the expenditure of work; this is illustrated in the processes involving friction which are so common in nature. Also, work may be produced from thermal energy as in the internal-combustion engine or the steam turbine. In fact, all forms of energy are mutually convertible; and, from the law of conservation of energy, it may be inferred that a definite ratio exists between the units of the various forms of energy. Engineering thermodynamics is more directly concerned with the relationship which exists between energy in the thermal and mechanical forms. This ratio has been shown to be

$$1 \text{ B.t.u.} = 778 \text{ ft.-lb. (approximately)}$$

so that, if the amount of heat converted into work is denoted by  $Q$  and the resulting work by the symbol  $W$ , the following statements may be written:

$$W = 778Q$$

$$Q = \frac{1}{778}W$$

in which  $Q$  is expressed in B.t.u. and  $W$  in foot-pounds. These are often abbreviated to the forms

$$W = JQ \quad (1:7)$$

$$Q = AW \quad (1:8)$$

in which  $J = 778$  and  $A = \frac{1}{778}$ .

It will be noted that no *definition* of the B.t.u. has as yet been attempted in this text; it has merely been *described* (see Art. 1:9). The relationship between the B.t.u. and the foot-pound outlined above is the basis of the accepted definition of the B.t.u. as the thermal equivalent of 778 ft.-lb. of mechanical energy.

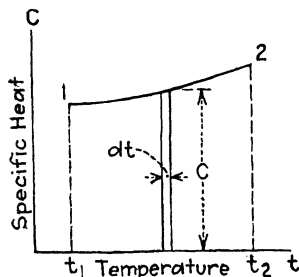


FIG. 1:6.—Effect of varying specific heat.

**1:11. Specific Heat.** The addition of the same quantity of heat to equal weights of different substances will, in general, produce different temperature changes. The amount of heat in B.t.u. required to raise the temperature of 1 lb. of a substance  $1^{\circ}\text{F.}$  is known

as the *specific heat* of that substance. Thus the specific heat of water at  $63^{\circ}\text{F.}$  is unity.

If the specific heat ( $C$ ) is constant or may be considered as remaining practically constant during a specified change of temperature, the amount of heat required to raise  $M$  lb. of the substance from  $t_1$  to  $t_2$  may be calculated by the equation

$${}_1Q_2 = MC(t_2 - t_1)$$

If the specific heat is a variable, it becomes necessary to integrate as shown below and illustrated in Fig. 1:6:

$${}_1Q_2 = M \int_{t_1}^{t_2} C dt \quad (1:9)$$

in which  $C$  is an expression for the specific heat in terms of  $t$ .

The specific heat of gases and vapors depends also upon the conditions under which heat is added. The specific heat at constant volume, for which the symbol is  $C_v$ , is the number of B.t.u. required to increase the temperature of 1 lb.  $1^{\circ}\text{F.}$  when the gas or vapor is confined in such a

manner that the volume does not change during the process. The specific heat at constant pressure ( $C_p$ ) is the heat required to produce the same effect when the pressure is constant.

*Example A.* The specific heat of a substance is expressed by the equation

$$C = 0.48 + 0.0008t$$

How many B.t.u. will be required to raise the temperature of 5 lb. from 20 to 40°F.

*Solution.*

$$\begin{aligned} {}_1Q_2 &= M \int_{t_1}^{t_2} C \, dt = 5 \int_{20}^{40} (0.48 + 0.0008t) \, dt = 5 \int_{20}^{40} [0.48t + 0.0004t^2] \\ &= 5[(0.48)(40) + (0.0004)(1,600)] - 5[(0.48)(20) + (0.0004)(400)] \\ &= 50.4 \text{ B.t.u.} \end{aligned}$$

*Example B.* The heat absorbed by 1 lb. of a substance as it increases in temperature is expressed as

$$Q = 0.612t + 0.00034t^2$$

for the range of temperature from 0 to 200°F. What is the specific heat of the substance at a temperature of 100°F.?

*Solution.* From Eq. (1:9), the following relationships may be derived for unit weight of a substance:

$$dQ = C \, dt; \quad C = \frac{dQ}{dt}$$

From the latter,

$$C = \frac{dQ}{dt} = \frac{d}{dt} (0.612t + 0.00034t^2) = 0.612 + 0.00068t$$

and, substituting the value of  $t$  at which the specific heat is desired,

$$C(\text{at } 100^\circ\text{F.}) = 0.612 + 0.00068(100) = 0.680$$

When the addition of heat is accompanied by a continuous increase of temperature, the heat so added is called *sensible* heat. On the other hand, when a change of aggregation is taking place as during the melting of ice or the vaporization of water, no increase of temperature accompanies the process and the specific heat is infinite. Heat absorbed under these conditions is called *latent* heat. If the substance is changing from the state of a solid to that of a liquid, the term *latent heat of fusion* is used; if from the liquid to the vapor state, *latent heat of vaporization*.

### Problems

1. Give examples of transformation of energy as follows:

- |                           |                              |
|---------------------------|------------------------------|
| (a) Thermal to mechanical | (b) Mechanical to electrical |
| (c) Electrical to thermal | (d) Chemical to electrical   |
| (e) Chemical to thermal   | (f) Electrical to chemical   |

2. What is the kinetic energy of a 2,000-lb. automobile traveling at 60 miles per hour (m.p.h.)? This amount of kinetic energy is relative to what base or standard?

3. What is the kinetic energy of a truck weighing 8,000 lb. and traveling at 30 m.p.h.?

4. If the automobile of Prob. 2 is traveling along the same road as the truck of Prob. 3, in the same direction as that of the truck, what is its kinetic energy relative to the truck? What is the kinetic energy of the truck relative to the automobile?

5. In Prob. 4, if the automobile is traveling in a direction opposite to that of the truck, what is its kinetic energy relative to the truck? What is the kinetic energy of the truck relative to the automobile?

6. In Prob. 5, if the truck and the automobile were to collide and come to a stop, how much mechanical energy would be released? What would become of this energy?

7. If the automobile of Prob. 2 is traveling on a road at an elevation of 5,000 ft. above sea level, what is its potential mechanical energy referred to sea level? Referred to the top of a mountain with an elevation of 14,000 ft. above sea level?

8. In Prob. 7, what is the total stored mechanical energy of the automobile referred to sea level? To the top of a 14,000-ft. mountain?

9. Assume a theoretical substance the specific internal energy of which is in direct proportion to its absolute temperature. If the temperature of a system composed of this substance is increased from 32 to 212°F. without changing its state of aggregation or its volume, in what proportion is its internal energy increased? In what proportion does the velocity of its molecules increase?

10. Change the following pressures to pounds per square inch absolute. The barometric pressure is 29 in. Hg.

(a) 60 p.s.i. gage

(b) 25 in. Hg vacuum

(c) 2,000 p.s.f. abs.

(d) 6 in. Hg abs.

11. Change the following pressures to pounds per square foot absolute. The barometric pressure is standard.

(a) 30 p.s.i. gage

(b) 27 in. Hg abs.

(c) 27 in. Hg vacuum

(d) 60 p.s.i. abs.

12. Convert the following centigrade temperatures to degrees Fahrenheit:

(a)  $-20^{\circ}\text{C}$ .

(b)  $50^{\circ}\text{C}$ .

(c)  $-70^{\circ}\text{C}$ .

(d)  $400^{\circ}\text{C}$ .

13. Convert the following Fahrenheit temperatures to degrees centigrade:

(a)  $120^{\circ}\text{F}$ .

(b)  $10^{\circ}\text{F}$ .

(c)  $-70^{\circ}\text{F}$ .

(d)  $600^{\circ}\text{F}$ .

14. At what temperature will the readings on a Fahrenheit and on a centigrade thermometer coincide?

15. Two bodies are placed in contact with each other. The first has a temperature of  $110^{\circ}\text{F}$ ., the second a temperature of  $50^{\circ}\text{C}$ .. (a) In what direction will heat flow between them? (b) On the basis of the given data, is it possible to determine which has the larger original store of internal energy?

16. Assume a scale of temperature on which the melting point of ice is designated as  $-40^{\circ}$  and the boiling point of water at standard atmospheric pressure as  $+110^{\circ}$ . (a) At what temperature is absolute zero located on this scale? (b) Derive an equation for changing temperatures on this scale to degrees Fahrenheit.

17. A thermodynamic system is compressed without friction from a volume of 12 cu. ft. to a volume of 5 cu. ft. at a constant pressure of 80 p.s.i. abs. What amount of work accompanies the process? Is this work positive or negative in sign?

18. A thermodynamic system expands frictionlessly at a constant pressure of 25 p.s.i. gage. The original volume is 20 cu. ft.; the final is 25 cu. ft. What amount of work is involved?

19. During a process which takes place at a constant volume of 8 cu. ft., the pressure of a system increases from 100 p.s.i. abs. to 120 p.s.i. abs. Is work done by or on the system, and how much?

20. The absolute pressure of a system changes according to the expression  $p = 120 - 0.4V + 0.03V^2$  ( $p$  in pounds per square inch absolute,  $V$  in cubic feet) during a frictionless process which increases the volume from 12 to 20 cu. ft. What amount of work accompanies the process?

21. Derive an expression for the amount of work developed as a system changes in volume from  $V_1$  to  $V_2$  when  $PV = P_1V_1 = C$  (a constant). Assume the process to be frictionless.

22. During a frictionless process, the absolute pressure of a system varies according to the expression  $p = 400/V$ . Is work done by or on the system as the pressure decreases from 80 to 20 p.s.i. abs., and how much?

23. During a frictionless process, the product of the pressure (pounds per square inch absolute) and the volume (cubic feet) remains constant at 500. The initial volume is 5 cu. ft.; the final is 10 cu. ft. How much work is performed?

24. Derive an expression for the amount of work developed during the frictionless expansion of a system from a volume  $V_1$  to a volume  $V_2$  when

$$PV^n = P_1V_1^n = P_2V_2^n = C \quad (n \text{ and } C \text{ are both constants})$$

25. During the frictionless compression of a system, the absolute pressure varies according to the equation  $PV^{1.4} = 40,000$ . What amount of work is involved in the compression of the system from 12 to 5 cu. ft.?

26. When a certain frictionless process is plotted on a  $PV$  diagram, the area between the state path and the  $V$  axis is 2.5 sq. in. If the scale of pressures is 100 p.s.i. abs. per inch and the scale of volumes is 2 cu. ft. per inch, how much work is performed during the process?

27. One horsepower delivered over a period of 1 hr. is called a *horsepower-hour*. Is this a unit of power or energy? Calculate its magnitude in terms of the foot-pound and the B.t.u.

28. Calculate the thermal equivalent of the kilowatt-hour.

29. Convert 1,000,000 ft.-lb. per hour into horsepower; into kilowatts.

30. Convert 100,000 ft.-lb. per minute into horsepower; into kilowatts.

31. An electrical heater is rated at 750 watts. During a period of 30 min., how much heat will it generate?

32. If electrical energy costs 3 cents per kilowatt-hour, how much will it cost per day to heat a house electrically if an average of 40,000 B.t.u. per hour is required for that purpose?

33. A rock weighing 800 lb. falls from a cliff to the surface of a glacier 200 ft. below. Assuming the only thermal effect is to melt the ice, how much ice will be melted? (The latent heat of fusion of ice is 143 B.t.u. per pound.)

34. A jet of water is directed horizontally at a velocity of 120 m.p.h. against a vertical wall. Assuming that all the kinetic energy of the jet is converted into heat at the point of impact and is used to heat the water, what increase in temperature results? The specific heat of water is unity.

35. A waterfall 150 ft. in height has a horizontal current velocity of 30 m.p.h. at

the top of the fall. Assuming that all the mechanical energy is converted into heat at the bottom of the fall and is used to heat the water, how much will the temperature of the water be raised?

- ✓ 36. A gas engine converts 22 per cent of the energy in its fuel into useful work and uses 1,500 cu. ft. of natural gas per hour. If the heat value of the gas is 950 B.t.u. per cubic foot, what horsepower does the engine develop?

37. How many miles could gasoline (heat value 18,000 B.t.u. per pound) be raised if *all* its energy content could be utilized for that purpose? If 30 per cent is available for the purpose?

- ✓ 38. An airplane engine consumes 0.5 lb. of gasoline (heat value 18,500 B.t.u. per pound) per horsepower-hour developed. What per cent of the energy in the fuel is delivered as useful work?

39. If a steam power plant is capable of converting 22 per cent of the energy in coal into useful work, how many pounds of 12,000 B.t.u. coal will be required per horsepower-hour delivered? Per kilowatt-hour?

- ✓ 40. How many pounds of gasoline (heat value 18,500 B.t.u. per pound) will be required to raise 100,000 lb. of water 200 ft. if the engine and pump combined deliver as useful work 12 per cent of the energy in the fuel?

41. A Diesel engine converts 38 per cent of the energy in the fuel into useful work. It burns 18 lb. of fuel oil (18,600 B.t.u. per pound) per hour at full rated load. At what horsepower is it rated?

42. During a 24-hr. test a steam power plant delivers an average of 3,900 kw. to the switchboard and burns 50 tons of coal (heat value 13,000 B.t.u. per pound). What per cent of the energy in the fuel is delivered to the switchboard?

- ✓ 43. The constant specific heat of a substance is 0.12. How much heat will be required to raise the temperature of 9 lb. from 60 to 180°F.?

44. Fifty B.t.u. of heat is added to 4 lb. of aluminum. As a result, its temperature is increased from 50 to 110°F. What is the specific heat of aluminum?

45. When 30 lb. of lead at an original temperature of 180°F. is immersed in 50 lb. of water at 60°F., what is the final temperature of the water and the lead? Specific heat of water is unity, of lead is 0.0302.

- ✓ 46. When 50 lb. of zinc at 200°F. is immersed in 80 lb. of water at 60.4°F., the final temperature of the water and zinc is observed to be 68.0°F. If the specific heat of water is unity, what is the specific heat of zinc? How much heat is transferred from the zinc to the water?

47. For alcohol  $Q = 0.5476t + 0.001122t^2$  ( $t$  in degrees centigrade,  $Q$  in calories per gram). (a) Find the specific heat at 30°C. in calories per gram per degree centigrade and in B.t.u. per pound per degree Fahrenheit. (b) Find the specific heat at 59°F. (c) How much heat is required to raise 5 lb. of alcohol from 10 to 50°C.? (d) What is the mean, or average, specific heat over this range?

48. The specific heat at constant pressure of carbon dioxide is

$$C_p = 0.1625 + 0.089(10)^{-3}T - 0.0136(10)^{-6}T^2$$

in which  $T$  is measured in degrees Fahrenheit absolute. (a) What is its specific heat at 40°F.? (b) At 540°F.? (c) How much heat is required to raise the temperature of 5 lb. of carbon dioxide from 40 to 540°F.? (d) What is the mean specific heat over this range of temperature?

## CHAPTER II

### GENERAL LAWS AND EQUATIONS

**2:1. Subdivisions of Engineering Thermodynamics.** There are two phases into which the study of engineering thermodynamics naturally divides itself. These are the thermodynamics of gases and the thermodynamics of vapors. Although the dividing line between a gas and a vapor is not sharp and distinct, a *gas* may be defined as a fluid which cannot be liquefied except by large changes of temperature and pressure. In the strict sense, all gases, since it is possible at sufficiently low temperature and high pressure to condense them, are really highly superheated vapors. Whether a fluid is considered a gas or a vapor for the purposes of thermodynamics depends on how closely it obeys the laws of perfect gases as stated by Boyle and Charles. No gases obey them exactly; fluids which show considerable variation from the behavior predictable by these laws are known as vapors and must be considered on a somewhat different basis in the study of thermodynamics. Air and hydrogen are examples of substances which, under all usual conditions, may be considered as perfect gases; steam and ammonia represent the vapor classification. Some substances, as, for example, carbon dioxide, may ordinarily be treated as gases but, under the influence of high pressures, must be handled as vapors.

In addition to the particular methods and equations that apply to only one or the other of these two groups of thermodynamic mediums, there are certain laws and expressions which are general in their application, and this chapter will be devoted to their consideration.

**2:2. The first law of thermodynamics** is merely a restatement of the law of conservation of energy with special application to the idea of thermodynamics. It may be stated as follows:

*When conversions take place between energy in the thermal and energy in the mechanical forms, the amount of mechanical energy developed is proportional to the amount of thermal energy which disappears and, if the change takes place in the opposite direction, the amount of thermal energy generated is equivalent to the amount of mechanical energy expended.*

When this law is applied to the thermodynamic process, an operation similar to that of bookkeeping is implied, in which the energy content of



the thermodynamic system at two successive conditions is totaled, the external energy flow to or from external systems which occurs during the interval being debited or credited to the system in arriving at a balance. The items which enter into this balance are *heat*, *work*, *internal energy*, *potential (mechanical) energy*, *kinetic (mechanical) energy* and *flow energy*.

Heat and work are the forms in which energy must enter or leave the system. In the practical thermodynamic process, the heat entering the system is often derived from chemical energy, as in the combustion of fuel, but it is usual to represent this by the equivalent amount of heat generated; this has the advantage of simplifying the calculation. Work that leaves or enters the system continuously to or from some external system is called *shaft work*; this designation arises from the nature of the practical thermodynamic apparatus.

Internal energy, as we have seen, is possessed by the system at any instant in an amount dependent upon its state (pressure, volume, and temperature).

Potential mechanical energy is dependent upon the elevation of the system above some accepted base. It is the product of the weight and the elevation; the elevation will be represented by the symbol  $Z$ . This type of energy is of minor importance in engineering thermodynamics owing to the relatively low density of the gases and vapors which are the mediums employed and, for simplicity, is often omitted altogether from the energy balance.

Kinetic mechanical energy is a function of the mass of the system and of its velocity. If the system is not in a state of flow, the velocity is zero and this term will not appear in the energy balance.

When a fluid is in motion, it is customary to set up an energy balance between two successive sections across its channel. At the entry section, work is supplied to an element of the fluid by following elements to force it past that boundary; at the exit section, the element itself must, in turn, do work on preceding elements to pass this second boundary. This is work in a special sense since, instead of being an energy transfer between two distinct systems, it represents an interchange of energy between successive elements of the same flowing fluid. It is given the special designation of *flow work*, or *flow energy*, to differentiate it from external, or *shaft*, work such as might be performed by the flowing fluid in turning a turbine wheel.

Flow energy is usually expressed with respect to unit weight of the fluid and may be evaluated on this basis by calculating the work required to force entry past the section under consideration. For con-

venience, let the flow be assumed as taking place in a channel having a uniform cross-sectional area  $a$ . Let us first consider this unit weight of fluid as located entirely to the left of section  $A$ , which represents the boundary to be passed and with respect to which its flow energy is to be measured. This is illustrated in Fig. 2:1. The pound of fluid will fill the channel through the distance  $L$ , a distance such that  $aL = v$ , the specific volume of the fluid. Now let the fluid be slowly forced past section  $A$  as by the piston shown in the figure, which represents the following elements of fluid. The minimum work done by the piston in forcing the entry of this unit weight of fluid may be measured as the product of the force which it impresses on the column of fluid ( $Pa$ ) and the distance through which it travels,  $L$  ( $= v/a$ ), or

$$\text{Flow energy per pound of fluid} = F \times L = Pa \times \frac{v}{a} = Pv \quad (2:1)$$

It will be noted that, according to the sign convention adopted in the first chapter and *with respect to the fluid which lies between the two sections*,

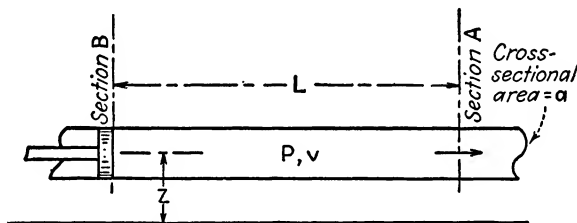


FIG. 2:1.—Flow energy.

flow energy at the entry section is negative while at the exit section it is positive in sign.

A review of the principles of hydraulics will remind the student that, when no important change of temperature occurs during flow and no friction is present, flow energy and kinetic mechanical energy are interchangeable. Thus, when the fluid must increase its velocity to traverse a narrow section of the channel, the corresponding *increase* in kinetic mechanical energy is accomplished by an equal *reduction* of its flow energy; and when a larger channel section is reached, the energy exchange takes place in the opposite direction.

It should be noted also that, unless the fluid is in flow, neither flow energy nor kinetic mechanical energy should be included in the energy balance. It therefore follows that the product  $PV$ , though it may be *expressed* for a fluid at rest and is measured in foot-pound units, is not to be considered as energy except when flow conditions exist. This may be better understood if the reader remembers that work and moment are

both represented as the product of a force times a distance but that only the first can be classified as energy.

**2:3. The State Function.** It is evident that some of these items which are contained in the energy balance are functions of the state of the system, *i.e.*, of the pressure, volume, temperature, and elevation. Whenever the system returns to its original state, the internal energy, the potential mechanical energy, the kinetic mechanical energy, and the flow energy will again have their original value. Quantities of this kind which depend only upon the state of the system will be called *state functions*. If a system changes from an initial condition represented by the state values  $P_1, V_1, T_1, Z_1, w_1$  to some succeeding condition where these values have become  $P_2, V_2, T_2, Z_2, w_2$ , the initial internal energy may be designated as  $U_1$  and the internal energy in the second state may be called  $U_2$ . The *change* of internal energy between the two conditions is the difference between  $U_2$  and  $U_1$  or  $U_2 - U_1$ . If this difference is positive in sign, it denotes an increase of internal energy during the process, and vice versa. Similarly, the change of potential mechanical energy is  $MZ_2 - MZ_1$ , the change of kinetic mechanical energy is  $(Mw_2^2/2g) - (Mw_1^2/2g)$ , and the net flow energy is  $P_2V_2 - P_1V_1$ .

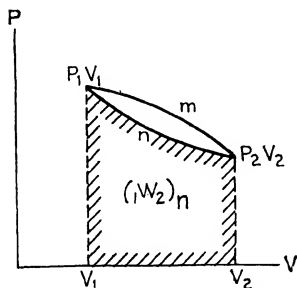


FIG. 2:2.—Variation of work with path.

On the other hand, work and heat, in the sense in which we shall use these terms, are not the inherent possessions of the system at any given state but functions of the nature of the process, or *change*, which has taken place. For instance, with reference to Fig. 2:2 the change of state which has occurred in the system from 1 to 2 is carried out by two paths  $m$  and  $n$ , the areas under the respective curves indicating the amounts of work developed in each case. The *change* of internal energy (or any state function) which takes place as the system passes from condition 1 to condition 2 must always be the same, regardless of the path followed. Evidently the amount of work developed depends rather upon the variation of pressure and volume of the system *during* the process. A similar conclusion may be drawn in regard to the flow of heat, as will be shown in the succeeding article.

It will evidently be impossible to denote the work or heat developed or absorbed by a notation similar to that used to denote the change of internal energy. We can, however, represent these amounts by the notations  ${}_1W_2$  and  ${}_1Q_2$ , indicating the amount of each form of energy which has entered or left the system *during* the change in the condition

of the system from 1 to 2. Therefore, for the calculation of the amount of heat or work developed or absorbed by a thermodynamic system during a thermodynamic process, not only the end points of the process must be known but also the character of the change which takes place between them.

**2:4. The Nonflow Process.** The investigation of the nonflow process involves the consideration of the state of the system at successive intervals of *time*. A practical example of this type of process is the type of expansion which takes place within the walls of a cylinder as the piston moves from its initial position to some subsequent location, increasing or decreasing the volume. The nonflow process, however, does not necessarily involve any change in volume, as will be seen later; the time element is the important factor. The process to be analyzed is represented in Fig. 2:3, the initial condition in this case being represented by the subscript 1 and the final condition by the subscript 2. The energy balance may now be written as follows:

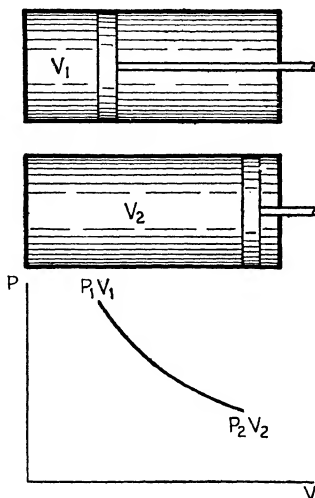


FIG. 2:3.—The nonflow process.

$$U_1 + {}_1Q_2 = U_2 + A({}_1W_2) \quad (2:2)$$

But since, from Art. 1:8, for a frictionless process

$${}_1W_2 = \int_{V_1}^{V_2} P \, dV$$

Eq. (2:2) may be restated as

$$U_1 + {}_1Q_2 = U_2 + A \int_{V_1}^{V_2} P \, dV \quad (2:3)$$

as it applies to a process involving no friction. Since no flow is involved, the terms representing flow energy and kinetic mechanical energy do not appear in this energy balance. Equation (2:2) and the equation immediately following it are based on the assumption that no vertical displacement of the center of gravity occurs. If vertical displacement does take place, a corrective term,  $AM \, \Delta Z$ , must be added to the right-hand side of Eq. (2:2) and to the left side of the second equation; this takes into account the fact that the average pressure acting against the

piston differs from the average value for the system as a whole. For the types of systems and processes in which our interest is centered, this term would be negligibly small and is therefore omitted. Equation (2:3) is, on the other hand, complete as stated. In both these equations, the term  ${}_1Q_2$  represents the heat *entering* the system during the process and the term  $A({}_1W_2)$  the heat equivalent of the work *leaving* the system during the same period. Since internal energy and heat are both conventionally measured in B.t.u. and work in foot-pounds, the conversion factor  $A$  is necessary. If heat *leaves* the system during the process, the term  ${}_1Q_2$  will be negative; similarly, if work is performed *on* the system as, for example, in compressing it, the term  $A({}_1W_2)$  will have the negative sign.

Transposing, these equations become

$${}_1Q_2 = U_2 - U_1 + A({}_1W_2) = U_2 - U_1 + A \int_{V_1}^{V_2} P \, dV \quad (2:4)$$

the second form being the typical form of the nonflow energy equation as it applies to a frictionless process. This means that the heat added during this type of process is equal to the change of internal energy between the initial and final conditions plus the heat equivalent of the external, or shaft, work delivered by the system during the same period. Since the amount of work depends upon the character of the process and the change of internal energy depends only upon the initial and final states of the system, it now becomes evident that heat, like work, is a function of the nature of the change.

*Example.* During a nonflow process 100 B.t.u. of heat is added to a system, and the internal energy of the system is increased 70 B.t.u. How many foot-pounds of work is performed?

*Solution.*

$${}_1Q_2 = +100 \text{ B.t.u.}; \quad U_2 - U_1 = +70 \text{ B.t.u.}$$

Substituting in Eq. (2:4),

$$\begin{aligned} 100 &= 70 + A({}_1W_2) \\ A({}_1W_2) &= 100 - 70 = 30 \text{ B. t. u.} \\ {}_1W_2 &= 778(30) = 23,340 \text{ ft.-lb.} \end{aligned}$$

Since the sign is positive, this work was done *by* the system and is available for performing some external task.

**2:5. The Steady-flow Process.** This type of process is illustrated in Fig. 2:4, the heavy lines showing the limiting walls through which the fluid is passing and the designated sections indicating the positions in the

channel at which the condition of the fluid is to be investigated. The term "steady flow" may be interpreted to mean that the flow has reached a condition of equilibrium such that the *weight* passing any section in unit time is equal to the *weight* passing any other section in the same time and no energy is being stored in or withdrawn from the system as a whole. The investigation of the steady-flow process may therefore be said to involve the consideration of the condition of the fluid at successive intervals of *position*. Since the flow is steady and continuous, no definite bounds may be set as the limits of the system at these two positions and it is usual to base the energy balance upon a *unit weight* of the fluid.

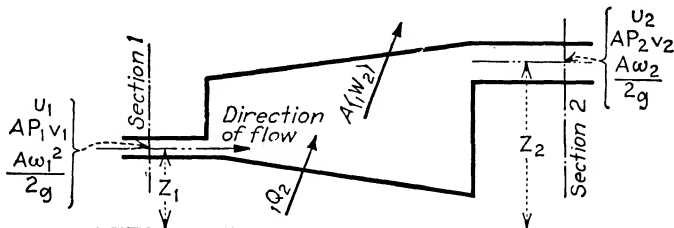


FIG. 2:4.—The steady-flow process.

The application of the definition of steady flow to the flow between these two sections across the channel may be summarized as follows:

1. The *state* of the fluid is uniform over the entrance section and does not change with time.
2. The same is true of the state of the fluid at the exit section.
3. The weight passing the entrance section in unit time is constant and is equal to the weight passing the exit section in the same time.
4. The rate at which energy enters or leaves (in the form of heat or work) between the two sections is constant.

The flow which takes place through a steam turbine is a practical example of the steady flow process.

In fact, nearly all practical engineering apparatus operates on a continuous basis, and steady-flow conditions may be assumed, permitting the application of the equation derived below to analyze its action. Thus, the principle of steady flow can be applied even to a device such as the steam engine, which at first thought would seem to require an analysis on the basis of the nonflow energy equation, by the consideration of the condition of the fluid at sections in the steam supply line just before entrance to the engine and in the exhaust line just after leaving it, as well as heat and work energy amounts entering or leaving between the two sections. It is often helpful to employ both methods of attack, where this can be done.

Based upon a unit weight of fluid, the energy balance may be written as follows:

$$u_1 + AP_1v_1 + \frac{Aw_1^2}{2g} + AZ_1 + {}_1Q_2 = u_2 + AP_2v_2 + \frac{Aw_2^2}{2g} + AZ_2 + A({}_1W_2) \quad (2:5)$$

It will be noted that all the forms of energy listed in Art. 2:2 are represented in this balance; this is necessary since the fluid is in motion and possesses kinetic mechanical energy, since the flow energy terms will apply, and since in flow a significant change in vertical position may take place, altering the store of potential mechanical energy. However, the latter effect is so small in practice in comparison with the other energy changes that it is usually neglected. If this is done, the steady-flow energy equation becomes

$$u_1 + AP_1v_1 + \frac{Aw_1^2}{2g} + {}_1Q_2 = u_2 + AP_2v_2 + \frac{Aw_2^2}{2g} + A({}_1W_2) \quad (2:6)$$

in which  ${}_1Q_2$  is the heat added *per pound of fluid* and  $A({}_1W_2)$  is the shaft work delivered *per pound of fluid* as the fluid passes from the first to the second position.

*Example.* A gas flows through a turbine. At entrance, its pressure is 100 p.s.i. abs., its specific volume is 3 cu. ft. per pound, and its velocity is 500 ft. per second. At exit, the corresponding values are 50 p.s.i. abs., 4 cu. ft. per pound, and 1,000 ft. per second. Between entrance and exit, the internal energy decreases 40 B.t.u. per pound, and 5 B.t.u. of heat per pound is lost through radiation. What amount of shaft work is developed per pound of flow?

*Solution.*

$$u_2 - u_1 = -40 \text{ B.t.u.}; \quad {}_1Q_2 = -5 \text{ B.t.u.}$$

Transposing, Eq. (2:6) becomes

$$A({}_1W_2) = u_1 - u_2 + AP_1v_1 - AP_2v_2 + \frac{Aw_1^2}{2g} - \frac{Aw_2^2}{2g} + {}_1Q_2$$

Substituting assigned values,

$$\begin{aligned} A({}_1W_2) &= 40 + \frac{(144)(100)(3)}{778} - \frac{(144)(50)(4)}{778} + \frac{(500)^2}{(778)(64.4)} - \frac{(1,000)^2}{(778)(64.4)} - 5 \\ &= 40 + 55.5 - 37.0 + 5.0 - 20.0 - 5 \\ &= 38.5 \text{ B.t.u. or } 29,950 \text{ ft.-lb.} \end{aligned}$$

Although  ${}_1Q_2$  and  ${}_1W_2$  are, individually, dependent on the character of the process, a study of both the nonflow and the steady-flow energy

equations will indicate that their difference,  ${}_1Q_2 - A_1W_2$ , can be expressed entirely in terms of properties and therefore has a constant value for a given change in the condition of the system, a value which is independent of the nature of the process which connects the initial and final states. This *difference* can thus be classified as a *state function*.

**2:6. Enthalpy** is the name given a composite property of the thermodynamic system which is of great convenience, especially in the study of the thermodynamics of vapors. The expression that defines enthalpy, for which the symbol is  $H$  ( $h$  for enthalpy per pound), is

$$H = U + APV \quad (2:7)$$

If this expression is differentiated, considering all terms except  $A$  as variables, the following is obtained:

$$dH = dU + AP \, dV + AV \, dP \quad (2:8)$$

Further, if the latter expression for the *change* of enthalpy is integrated between definite limits,

$$\int_{H_1}^{H_2} dH = H_2 - H_1, \quad \int_{U_1}^{U_2} dU = U_2 - U_1$$

and the following statement results:

$$H_2 - H_1 = U_2 - U_1 + A \int_{V_1}^{V_2} P \, dV + A \int_{P_1}^{P_2} V \, dP \quad (2:9)$$

If the pressure is constant, it is evident that the last term will become zero since  $dP$  is zero. A comparison of the resulting form of the equation with Eq. (2:4) indicates that, if heat is added to a system *under a condition of constant pressure*, the enthalpy of the system will be increased by exactly the amount of heat added if the process is frictionless.

Enthalpy, since it is expressed entirely in terms of the properties of the thermodynamic system, is in itself a function of the state of the system. Thus, the variation of a state property may be used to measure heat flow, which depends upon the path which has been traced during the change of state. However, reference to the preceding paragraph will remind the reader that the nature of the state path has been restricted to one following a condition of constant pressure and warn him that this convenient method of evaluating heat flow is available only under this strictly limited condition.

Enthalpy, although it has in the past borne such names as "heat content," "total heat," and "thermal potential," is not to be classified as an energy quantity unless the system is in flow. During the addition of heat at constant pressure to a system at rest, an expansion has taken



place which has resulted in a retransfer of a portion of this energy into surrounding systems in the form of work. Thus the enthalpies found in tables of the properties of vapors such as steam are not to be considered as listings of the energy content of the steam.

*Example.* The original pressure of a system is 50 p.s.i. gage, and its volume is 5 cu. ft. During a change the internal energy of the system increases 20 B.t.u. The final pressure is 120 p.s.i. gage, and the final volume is 3 cu. ft. The barometric pressure is 29 in. Hg. What is the change of enthalpy?

*Solution.* The barometric pressure is equivalent to

$$29 \times 0.491 = 14.25 \text{ p.s.i.}$$

The initial and final pressures of the system are therefore

$$\begin{aligned} p_1 &= 64.25 \text{ p.s.i. abs.} \\ p_2 &= 134.25 \text{ p.s.i. abs.} \end{aligned}$$

From Eq. (2:7),  $H_2 = U_2 + AP_2V_2$ ,  $H_1 = U_1 + AP_1V_1$ , and, subtracting,

$$\begin{aligned} H_2 - H_1 &= U_2 - U_1 + AP_2V_2 - AP_1V_1 \\ &= +20 + \frac{(144)(134.25)(3)}{778} - \frac{(144)(64.25)(5)}{778} \\ &= +35.1 \text{ B.t.u.} \end{aligned}$$

As the sign is positive, the enthalpy increases during the change.

If the enthalpy is substituted in Eq. (2:6) for the sum of the internal and flow energies and if the further assumptions are made that the entrance and exit velocities are equal and that no external work is done between the two sections (assumptions very nearly accurate for most heat-exchange apparatus), it will be observed that the solution of the steady-flow energy equation indicates that the heat flow is equal to the change of enthalpy of the flowing fluid, even though the pressures at the two sections may not be the same.

**2:7. The second law of thermodynamics** may be called the law of degradation of energy. Although all forms of energy are mutually convertible, some changes take place much more readily than others. For example, all of a given quantity of work may be easily converted into heat; but, if the attempt is made to reconvert it to the form of work, the process is much more difficult and only a part of the original energy can be reconverted. Work or any form of energy which is theoretically capable of complete conversion into other forms is called *high-grade* energy; heat, not being capable of complete conversion, is classed as *low-grade* energy. A general tendency exists for all high-grade forms of energy to become low-grade, and it is with the principles that govern this

tendency that the second law deals. Two statements which interpret the meaning of this law, as it applies to engineering thermodynamics, will suffice for the present:

1. *When heat is continuously changed into work, a source must exist at a higher temperature from which heat must flow to a medium (such as a body of gas or vapor) whereby work is done. From the medium, heat must flow to a refrigerator at some lower temperature. Only a portion of the heat which is taken from the source can be changed into work; the balance will reappear as stored thermal energy in the refrigerator.*

2. *Heat cannot flow from a body at a lower temperature to another at a higher temperature; thermal energy can be continuously "lifted" to a body at higher temperature only if energy is continuously supplied by some external system to bring about the change in location.*

The first statement establishes definite conditions and limits for the performance of heat engines and indicates that no heat engine can convert into work all the heat which is supplied to it. A portion, and usually a large portion, must be rejected in the form of unused heat. The *efficiency* of a heat engine may be defined as the proportion of the heat supplied the engine which is converted into work, or

$$e = \frac{AW}{Q_S} \quad (2:10)$$

in which  $e$  is the efficiency, expressed as a decimal fraction,

$W$  is the work developed by the engine during a definite time interval, in foot-pounds,

and  $Q_S$  is the heat supplied the engine from the high-temperature source during the same interval, in B.t.u.

Since  $AW$  is equal to the difference between  $Q_S$  and  $Q_R$  (the unused heat rejected to the refrigerator by the engine), Eq. (2:10) may be written

$$e = \frac{Q_S - Q_R}{Q_S} \quad (2:11)$$

The second statement of the second law defines the general requirements for the operation of a refrigerating machine which will continuously remove thermal energy from a cold body and store it in a body at higher temperature.

The reader should make a careful distinction between the term  ${}_1Q_2$  as used heretofore and the terms  $Q_S$  and  $Q_R$ , as used above. The first was a definite quantity of heat flow which took place as the system underwent a definite change of state. The second are *rates* of heat flow (per engine cycle or per second), and the subscript has no reference to a state of the

system but refers only to the temperature level from or to which the heat flow proceeds.

### Problems

1. For the following nonflow processes, determine whether the energy transfers in the form of heat and work are, in the thermodynamic sense, positive, negative, or zero:

(a) The system consists of air confined in a nonconducting cylinder. The air expands, moving a closely fitted piston and causing it to turn a flywheel.

(b) Same as (a) except that the piston moves in a direction such that the air in the cylinder is compressed.

(c) Same as (b) except that the cylinder is no longer nonconducting but is surrounded by a water jacket containing water at atmospheric temperature. At the beginning of the process, the air is at atmospheric pressure and temperature. As a result of the compression, its pressure and temperature rise.

(d) The system comprises the water and water vapor contained in a closed, rigid boiler. At the beginning of the process the system is at atmospheric temperature. A fire is built under the boiler, and the pressure and temperature of the system rise.

(e) In (d), the boiler explodes after a high pressure and temperature have been reached, and the system escapes into the surrounding atmosphere.

(f) A mixture of hydrogen and oxygen in the proper proportions for complete combustion and confined in a rigid, nonconducting pressure vessel constitutes the system. A spark, which may be considered negligible, ignites the mixture.

(g) Same as (f) except that the system is originally confined in a balloon which is surrounded by the atmosphere. The original pressure and temperature of the mixture are atmospheric. As a result of the explosion, the balloon bursts.

(h) Same as (g) except that the balloon is submerged in a large tank of water. The process ends when a state of thermal equilibrium has been reached.

2. A gas system expands within a cylinder, delivering 50,000 ft.-lb. of work and radiating 5 B.t.u. of heat to the surrounding atmosphere during the process. What is the change of internal energy?

3. During a compression within a cylinder, 30,000 ft.-lb. of work is expended, and the internal energy of the system increases by 35 B.t.u. What is the heat flow?

4. During a process which involves no heat flow, the internal energy of a system increases by 50 B.t.u. How much work is performed? Is this process an expansion or a compression?

5. A system receives 60 B.t.u. of heat as its internal energy increases by 43 B.t.u. How much work is performed? Is this an expansion or a compression?

6. The internal energy of a system remains constant as it receives 20 B.t.u. of heat. How much work is performed? Is this an expansion or a compression?

7. Steam expands behind a piston doing 75,000 ft.-lb. of work. If 10 B.t.u. of heat is radiated to the surrounding atmosphere during the expansion, what is the change of internal energy?

8. A gas system expands at a constant pressure of 80 p.s.i. abs., increasing in volume from 5 to 7 cu. ft. During the process, the internal energy increases by 74 B.t.u. What is the heat flow?

9. During an expansion which takes place at constant pressure, the internal energy of a gas system increases by 50 B.t.u. If the heat flow is 3.5 times the heat equivalent of the work during the process, find the heat flow (B.t.u.) and the work performed (foot-pounds).

10. Twenty B.t.u. is radiated from a pound of air which is confined in a rigid tank. What is the change of internal energy?

11. A gas system undergoes a double process. It first expands at constant pressure, doing 24,000 ft.-lb. of work and receiving 108 B.t.u. of heat. During the second process it undergoes a further expansion, performing an additional 30,000 ft.-lb. of work but without heat flow. A third process returns it to its original state. What is the change of internal energy during the third process?

12. A system rejects 70 B.t.u. of heat at constant volume. It then receives 98 B.t.u. of heat during a constant pressure expansion which is accompanied by 21,800 ft.-lb. of work. What is the change of internal energy during a third process which restores it to its original state?

13. Liquid water flows through a horizontal converging tube at constant internal energy and without heat loss or gain. At a certain section its pressure is 100 p.s.i. abs., its density is 62.4 lb. per cubic foot, and its velocity is 20 ft. per second. At a later section the velocity has increased to 50 ft. per second. Assuming that water is incompressible, what is the pressure at the second section? Neglect frictional losses.

14. Same as Prob. 13 except the tube is not horizontal and the second section is 100 ft. below the first section.

15. Liquid water flows through a horizontal Venturi tube without change of internal energy. The density is constant at 62.5 cu. ft. per pound. At entrance the pressure is 50 p.s.i. abs., and the velocity is 50 ft. per second. At the throat the pressure is 24.6 p.s.i. abs. Considering the flow to have been frictionless, calculate the ratio of the cross-sectional area of the tube at the throat to that at the entrance.

16. Compressed air flows in an open line. At a section near the entrance, the pressure of the air is 90 p.s.i. abs., its specific volume is 2.5 cu. ft. per pound, and its velocity is 100 ft. per second. At a section near the exit, the pressure is 85 p.s.i. abs., and the volume 2.3 cu. ft. per pound. The line is horizontal and of constant diameter. The specific internal energy decreases by 13.3 B.t.u. between the sections. What heat flow took place and in what direction?

17. The specific internal energy (relative to a standard reference condition) of steam flowing in a pipe is 1115.8 B.t.u., its pressure is 250 p.s.i. abs., its specific volume is 1.884 cu. ft. per pound, and its velocity is 100 ft. per second. The elevation of the pipe above datum is 10 ft. (a) What is its total stored mechanical and thermal energy (referred to same reference conditions) per pound of steam? (b) If the steam had been at rest in the pipe but the internal energy, pressure, and volume had been as stated above, what would its stored mechanical and thermal energy have totaled? (c) What is the specific enthalpy of the steam in both cases?

18. At a downstream section of the pipe of Prob. 17, the enthalpy of the steam is 1190 B.t.u. per pound, and the velocity is 100 ft. per second. The pipe is horizontal between the two sections. What heat flow takes place between the sections and in what direction?

19. By the use of suitable receivers the flow to and from an engine may be smoothed out to the extent that it approximates steady-flow conditions. A certain engine receives 3,000 lb. of steam per hour at an enthalpy of 1195 B.t.u. per pound and exhausts steam having an enthalpy of 1080 B.t.u. per pound. The heat loss from the engine cylinder to the atmosphere is at the rate of 6000 B.t.u. per hour. Assume that the velocities at entrance and exit are equal, and neglect differences in elevation. What indicated horsepower is developed by the engine?

20. Apply the steady-flow energy equation to the flow through an engine as in Prob. 19, but assuming no radiation losses as well as no change in elevation between

the two sections. Obtain an expression for the work delivered per pound of steam expanded in terms of (a) the internal energies, pressures, and specific volumes and (b) the enthalpies at the two sections.

21. Assume that steady-flow conditions exist in the lines which lead exhaust steam to and remove condensate from a surface condenser. The enthalpy of the steam entering the condenser is 1080 B.t.u. per pound; that of the condensate at exit is 150 B.t.u. per pound. Three thousand pounds of steam is condensed per hour. The velocity of the steam is 300 ft. per second; of the condensate, 30 ft. per second. If the cooling water increases 50°F. in flowing through the condenser and 50,000 B.t.u. per hour is radiated directly to the atmosphere from the shell of the condenser, how many pounds of cooling water must be supplied per hour? Neglect differences of elevation; specific heat of water is 1.

22. Applying the steady-flow energy equation to the flow through a condenser and neglecting differences of velocity and of elevation, obtain an expression for the heat rejected per pound of steam condensed in terms of (a) the internal energies, pressures, and specific volumes at entrance and exit and (b) the original and final enthalpies.

23. A boiler evaporates 50,000 lb. of water per hour. At a section of the feed-water line entering the boiler the enthalpy of the water is 68 B.t.u. per pound, the velocity is 30 ft. per second, and the elevation is 5 ft. above the floor. At a section of the steam line leaving the boiler the enthalpy is 1200 B.t.u. per pound, the velocity is 150 ft. per second, and the elevation is 30 ft. above the floor. If 75 per cent of the energy content of the coal (heat value 12,000 B.t.u. per pound) is used in heating the water and steam, how many pounds of coal are burned per hour?

24. Apply the steady-flow equation to sections at entrance to and at exit from a steam boiler, neglecting differences in velocity and elevation, and derive an expression for the amount of heat received by 1 lb. of water as it changes into steam in the boiler.

25. Derive an expression for the work required to operate a feed-water pump per pound of water flowing by applying the steady-flow energy equation. Assume inlet and exit velocities and elevations equal, no heat transfer, no friction, and that the specific volume of the water does not change.

26. A steam turbine receives steam having a specific enthalpy of 1250 B.t.u. per pound. The rate of steam flow is 12,000 lb. per hour, and the turbine delivers 800 kw. The heat loss may be considered negligible. Find the final enthalpy of the steam (a) if the entrance and exit velocities are equal and the difference in elevation is neglected; and (b) if the initial velocity is 100 ft. per second, the final 1,000 ft. per second, and the exit section is 5 ft. below the entrance.

27. The specific enthalpy of the air as it leaves an air compressor is 47 B.t.u. per pound greater than as it entered. One thousand pounds of air is compressed per hour, and 27 hp. is required to drive the compressor. Neglecting differences of velocity and elevation at the entrance and exit sections, calculate the heat rejected to the cooling water jacket and the atmosphere per pound of air compressed.

28. Air flows through a small opening from a region of higher pressure to a large chamber in which a lower pressure is maintained. If sections are chosen at suitable distances on the two sides of the opening so that the velocities at these points may be considered negligible and if no heat enters or leaves the air between the sections, show that no change of enthalpy takes place.

29. Air enters a horizontal nozzle at negligible velocity and specific enthalpy  $h_1$ . At a section near the nozzle exit the velocity has become  $w_2$  and the enthalpy  $h_2$ .

Neglecting heat losses or gains between the sections, express  $w_2$  in terms of the initial and final enthalpies.

30. The enthalpy of a pound of steam at a pressure of 200 p.s.i. abs. and temperature of 500°F. is listed in the tables of the properties of steam as 1268.9 B.t.u., and its specific volume is given as 2.726 cu. ft. per pound. What is its internal energy?

31. A gas system expands at a constant pressure of 60 p.s.i. abs., increasing in volume from 12 to 18 cu. ft. During the process the internal energy of the system increases by 166 B.t.u. What is the heat flow? What is the change of enthalpy?

32. At a constant volume of 12 cu. ft., the pressure of a system increases from 60 to 80 p.s.i. abs. as it receives 155 B.t.u. of heat. What is the change of internal energy during the process? The change of enthalpy?

33. During an expansion at constant pressure, the enthalpy of a system increases by 33 B.t.u. as it performs 7,000 ft.-lb. of work. What is the change of internal energy?

34. The enthalpy of a system is the same at the end as at the beginning of a process during which its pressure decreased from 400 to 14.7 p.s.i. abs. and its volume increased from 1.82 to 30.53 cu. ft. What is the change of internal energy during the process?

35. A heat engine receives 180,000 B.t.u. per hour from its source and converts 40,000 B.t.u. per hour into useful work. What is its efficiency?

36. A heat engine develops 100 hp. and receives heat from its source at the rate of 1,000,000 B.t.u. per hour. What is its efficiency?

37. The efficiency of an aircraft engine is 29 per cent at rated power. If it is rated at 400 hp., how many pounds of 18,000-B.t.u. gasoline will it require per hour at rated power?

38. A heat engine receives 55,000 B.t.u. per hour from its source and rejects 47,000 B.t.u. per hour to its refrigerator. What is its efficiency?

39. An engine consumes 400 lb. of gasoline (18,000 B.t.u. per pound) per hour. The heat rejected in the exhaust is 60,000 B.t.u. per minute, 25,000 B.t.u. per minute is carried away in the cooling water, and 4,500,000 ft.-lb. of work per minute is expended in overcoming engine friction. Assuming there are no other losses, what is the efficiency of the engine? What horsepower does it deliver?

## CHAPTER III

### THERMODYNAMIC CHARACTERISTICS OF GASES

**3:1. Boyle's Law.** Gases have earlier been defined as highly super-heated vapors which obey with considerable exactitude the laws of a perfect gas as stated by Boyle and Charles. Boyle's law, which holds very closely at moderate pressures, may be stated as follows:

*As a gas system is compressed or expanded at a constant temperature, the pressure of the system will vary inversely with its volume.*

Or, in mathematical terms,

$$P_1 V_1 = P_2 V_2 \quad \text{or} \quad PV = C \quad (3:1)$$

in which  $P_1$  is the initial pressure, absolute,

$P_2$  is the final pressure, absolute,

$V_1$  is the initial volume,

$V_2$  is the final volume,

and  $C$  is a constant.

**3:2. Charles's law** may be stated in two ways.

1. *If the pressure is constant during the expansion or compression of a gas system, the volume will vary directly as the absolute temperature.*

2. *If the volume is constant, the pressure of a gas system will vary directly with the absolute temperature.*

1. At constant pressure,

$$\frac{V_1}{V_2} = \frac{T_1}{T_2} \quad \text{or} \quad \frac{V}{T} = C \quad (3:2)$$

2. At constant volume,

$$\frac{P_1}{P_2} = \frac{T_1}{T_2} \quad \text{or} \quad \frac{P}{T} = C \quad (3:3)$$

in which  $T_1$  and  $T_2$  are the initial and final absolute temperatures, respectively.

It is by an application of Charles's law that the point on the thermometric scale which represents the absolute zero of temperature has been established.

**3:3. The characteristic equation** of a perfect gas results from a combination of Boyle's and Charles's laws. If no one of the basic properties

$P$ ,  $V$ , or  $T$  is constant, the relationship which exists may be developed as follows:

Let the initial pressure, volume, and temperature of a gas system be represented by  $P_1$ ,  $V_1$ , and  $T_1$  and the final values of these quantities at the end of a specified change by  $P_2$ ,  $V_2$ , and  $T_2$ . The change will be accomplished in two steps, during the first of which the pressure will be constant at  $P_1$  as the temperature changes to its final value  $T_2$ ; during the second step the temperature will be constant at  $T_2$  as the pressure attains its final value  $P_2$ . The volume at the end of the first step will be represented by  $V_3$ .

Then, for the first step, which takes place at constant pressure,

$$\frac{V_1}{V_3} = \frac{T_1}{T_2} \quad \text{or} \quad V_3 = \frac{V_1 T_2}{T_1}$$

and, for the second step, taking place at constant temperature,

$$P_1 V_3 = P_2 V_2 \quad \text{or} \quad V_3 = \frac{P_2 V_2}{P_1}$$

Equating the two values of  $V_3$ ,

$$\frac{V_1 T_2}{T_1} = \frac{P_2 V_2}{P_1}$$

which may be stated as

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \quad \text{or} \quad \frac{PV}{T} = C \quad (3:4)$$

in which  $C$  is a constant.

The constant  $C$  may be replaced by the product of the weight of gas contained in the system, which will be denoted by  $M$ , and a new constant, which will be represented by the symbol  $R$ . The equation then becomes

$$\frac{PV}{T} = MR \quad \text{or} \quad PV = MRT \quad (3:5)$$

If both sides of the equation are divided by  $M$ ,

$$\frac{PV}{M} = RT$$

or, since  $V = Mv$ ,

$$Pv = RT \quad (3:6)$$

in which  $v$  is the *specific volume* and  $R$  is called the *gas constant* for the gas under consideration. For air, the value of the gas constant is 53.3 if the pressure is measured in pounds per square foot, the volume in cubic feet, and the absolute temperature in degrees Fahrenheit.



*Example.* Calculate the volume of 3 lb. of air at a pressure of 25 p.s.i. gage and a temperature of 120°F. Atmospheric pressure is 14.5 p.s.i.

*Solution.* From Eq. (3:5),

$$V = \frac{MRT}{P} = \frac{3 \times 53.3 \times (120 + 459.7)}{144 \times (25 + 14.5)} = 16.3 \text{ cu. ft.}$$

Note that absolute pressures and temperatures are used and that the pressure is changed to pounds per square foot.

**3:4. The Gas Constant.** From a consideration of Eq. (3:6) it will be observed that the values of the gas constant for various gases will vary directly as the specific volumes or indirectly as the densities of the gases at equal pressures and temperatures. In Table 3:1 will be found the

TABLE 3:1.—PROPERTIES OF GASES

Gas	Density at 14.7 p.s.i. abs. and 32°F., lb. per cu. ft.	Gas con- stant $R$	$C_p$	$C_v$	$C_p/C_v$ ( $k$ )	Approximate molecular weight $m$
Air.....	0.08071	53.3	0.2375	0.169	1.405	29
Carbon dioxide.....	0.12323	34.9	0.207	0.162	1.28	44
Hydrogen.....	0.005621	769.0	3.41	2.42	1.41	2
Nitrogen.....	0.07847	54.9	0.244	0.173	1.41	28
Oxygen.....	0.08927	48.2	0.218	0.156	1.40	32

densities at standard pressure and temperature and the corresponding values of the gas constant for some of the more common gases.

*Example.* At a pressure of 50 p.s.i. abs. and a temperature of 100°F., 10 cu. ft. of a certain gas has a weight of 2 lb. What is the gas constant?

*Solution.* The specific volume of the gas at the stated condition is 5 cu. ft. per pound. Substituting in Eq. (3:6),

$$R = \frac{Pv}{T} = \frac{144 \times 50 \times 5}{100 + 459.7} = 64.3$$

**3:5 Joule's law** is based on the laws of Boyle and Charles and may be stated as follows:

*The internal energy of a perfect gas is independent of the pressure or volume and depends only on its temperature.*

By purely mathematical processes, which are, however, beyond the scope of this text, it is possible to demonstrate the truth of this statement

for a substance for which the equation of state is as expressed in Eq. (3:6)<sup>1</sup>

Conversely, if a perfect-gas system does not change its temperature, its internal energy will also remain unchanged. In Fig. 3:1, lines of constant temperature  $T_1$  and  $T_2$  are represented on  $PV$  coordinates. According to Joule's law, these are also lines of constant internal energy if the system is a perfect gas; thus they may be labeled  $U_1$  and  $U_2$ , respectively. On the diagram, three paths connect line 1 with line 2. Path  $a$  represents a constant-volume process, path  $b$  a constant pressure, and path  $c$  is any general path connecting the two lines. Since internal energy is a property of the system and this property is constant at  $U_1$  along line 1 and at  $U_2$  along line 2, it is evident that the change of internal energy along *any* path connecting line 1 with line 2 is the same and is equal to  $U_2 - U_1$ . Then, if an expression for  $U_2 - U_1$  can be developed which

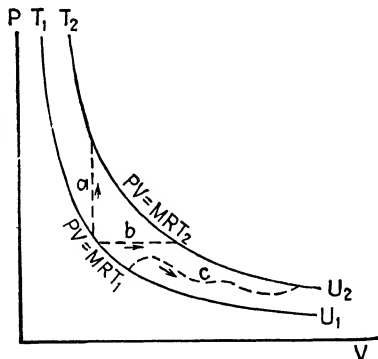


FIG. 3:1.—Joule's law.

is valid for any one of these paths, it will be valid for all of them. The constant volume path  $a$  is convenient to our purpose since, in applying the nonflow energy equation, the  $A_1W_2$  term becomes zero and  ${}_1Q_2 = U_2 - U_1$ . But, for a constant-volume process, the heat flow is, by definition of  $C_v$ , equal to  $MC_v(T_2 - T_1)$ , and we may write, for the perfect-gas system and constant  $C_v$ ,

$$U_2 - U_1 = MC_v(T_2 - T_1) \quad (3:7)$$

or, for variable specific heat,

$$U_2 - U_1 = M \int_{T_1}^{T_2} C_v dT \quad (3:8)$$

It should be noted that these equations are valid for the calculation of the change of internal energy of the perfect-gas system during *any* change of state of the system and are not limited to constant-volume processes in their application.

*Example.* Five pounds of air is cooled from 160 to 80°F. What is the change of internal energy? Assume constant specific heat.

*Solution.*

$$U_2 - U_1 = M \int_{T_1}^{T_2} C_v dT = 5 \times 0.169 \times (80 - 160) = -67.6 \text{ B.t.u.}$$

<sup>1</sup> KIEFFER and STUART, "Principles of Engineering Thermodynamics," p. 529, John Wiley & Sons, Inc., 1930.

Therefore the *decrease* of internal energy is 67.6 B.t.u. Note that the nature of the change was not specified and was not necessary for the solution of the problem and that Fahrenheit-scale temperatures may be used when the specific heat is constant or when it is expressed in terms of scale temperature.

It is evident from a study of the energy balance that if a perfect gas is allowed to expand but without absorbing or emitting heat or performing *external* work, so that  ${}_1Q_2$  and  ${}_1W_2$  are both zero, and under a further condition that the system is at rest before and after the change, so that  $w_1 = w_2 = 0$ , then  $U_2 - U_1$  must also be zero and there should be no change of temperature. Joule and Lord Kelvin carried out this process by allowing a quantity of gas to expand under conditions such that heat was prevented from entering or leaving the system during the process and the work performed as the volume increased was not transferred to any external system but was entirely utilized in increasing the internal energy of the expanded gas. All the gases on which this experiment was performed showed slight changes of temperature, indicating that none behaved exactly as a perfect gas. This deviation, known as the *Joule-Thomson effect*, is small for such gases as air and nitrogen, and they may be treated thermodynamically as perfect gases.

**3:6. The application of the nonflow energy equation** to a gas system leads to a better understanding of certain important relationships. The change of internal energy, for *any* type of expansion or compression, due to an infinitesimal change of temperature  $dT$  is, from Eq. (3:8),

$$dU = MC_v dT$$

Similarly, for a frictionless process which takes place at constant pressure, the amount of heat entering or leaving the system during the same infinitesimal change of temperature is

$$dQ = MC_p dT$$

Let us now apply the nonflow energy equation to a frictionless constant-pressure process over this infinitesimal temperature range, as follows:

$$MC_p dT = MC_v dT + AP dV \quad (3:9)$$

or, since  $dV = M dv$ ,

$$MC_p dT = MC_v dT + AMP dv \quad (3:10)$$

But from the characteristic equation of a perfect gas [Eq. (3:6)] may be obtained, by differentiation with  $P$  constant,

$$P dv = R dT \quad (3:11)$$

Substituting this value of  $P dv$  in Eq. (3:10),

$$MC_p dT = MC_v dT + AMR dT \quad (3:12)$$

or, dividing by  $M dT$ ,

$$C_p = C_v + AR \quad (3:13)$$

The symbol  $k$  will be employed to represent the ratio of the specific heats, *i.e.*,

$$C_p = kC_v \quad (3:14)$$

From Eqs. (3:13) and (3:14) may be obtained the following relationships:

$$C_p - C_v = AR \quad (3:15)$$

$$C_v = \frac{AR}{k - 1} \quad (3:16)$$

and

$$C_p = \frac{k}{k - 1} AR \quad (3:17)$$

A study of Eqs. (3:14) and (3:15) indicates that, since  $R$  has a constant value for a given perfect gas, the ratio of the specific heats,  $k$ , can be constant only if  $C_p$  and  $C_v$  are also constants (are invariable not only with pressure and volume but also with temperature). Although, for all known gases,  $C_p$  and  $C_v$  increase somewhat with increasing temperature, the special type of perfect gas with which we shall deal will have constant specific heats and consequently their ratio will also be constant. Equation (3:7) will therefore apply for the calculation of the change of internal energy, and other mathematical simplifications will be possible. In the case of the actual gas for which  $C_p$  and  $C_v$  change somewhat with the temperature, it has been found that the difference between them,  $C_p - C_v$ , remains constant while  $k$  varies slightly.

It may be shown, by methods which are founded on the thermodynamics of chemistry, that the ratio of the specific heats is dependent on the number of degrees of freedom of the molecule. This relationship may be expressed as

$$k = \frac{f + 2}{f} \quad (3:18)$$

in which  $f$  is the number of degrees of molecular freedom. A molecule which consists of a single atom, as, for instance, helium, would be free to move to either side, forward and backward, and up and down. The number of degrees of freedom would therefore be three, and the value of  $k$  should be 1.67; this is in good agreement with experimental results.

A diatomic molecule (as  $H_2$ ,  $O_2$ , etc.) has two additional degrees of freedom since the molecule can rotate about two axes perpendicular to

each other, the total number of degrees of freedom is five, and the value of  $k$  is therefore  $\frac{7}{5}$ .

Depending on the number of atoms which make up the molecule and their arrangement, gases which have three or more atoms per molecule will have six or more degrees of freedom, and the value of  $k$  will be  $\frac{4}{3}$  or less.

*Example.* Assume a certain diatomic perfect gas with a gas constant of 20. Predict its specific heats at constant volume and constant pressure.

*Solution.* A diatomic molecule has five degrees of freedom, and  $k = 1.4$ .

$$C_v = \frac{AR}{k - 1} = \frac{20}{778(1.4 - 1)} = 0.0642$$

$$C_p = kC_v = 1.4(0.0642) = 0.0898$$

**3:7. Interpretation of the Gas Constant.** An important insight into the true significance of the gas constant  $R$  is given by an examination of Eqs. (3:11) to (3:13). It will be noted that the gas constant is not a dimensionless quantity but is measured in foot-pounds per pound per degree and is, specifically, the amount of external work performed when a unit weight of gas expands at constant pressure owing to a unit increase of temperature.

*Example.* Calculate the external work done when 1 lb. of air at 20 p.s.i. abs. and 60°F. increases in temperature to 61°F. at constant pressure.

*Solution.* The original volume of the air is

$$v = \frac{RT}{P} = \frac{53.3 \times (60 + 459.7)}{144 \times 20} = 9.61 \text{ cu. ft. per pound}$$

The change of volume as the temperature increases to 61°F. is

$$dv = v_2 - v_1 = v_1 \left( \frac{T_2}{T_1} - 1 \right) = v_1 \left( \frac{T_2 - T_1}{T_1} \right) = 9.61 \left( \frac{1}{519.7} \right) = 0.0185 \text{ cu. ft.}$$

The external work done is

$$P dv = 144 \times 20 \times 0.0185 = 53.3 \text{ ft.-lb.}$$

**3:8. Internal Energy of a Gas.** The internal energy of a gas is a purely relative quantity. In the study of gases, the actual internal energy content at a given condition is of little importance. Instead, interest is centered in the *changes* of internal energy that take place. In the integration of Eq. (3:8) between two successive conditions of the system, the constants of integration will cancel each other. *If the specific heat is considered as constant,*

$$\text{Change of internal energy} = U_2 - U_1 = MC_v(T_2 - T_1) \quad (3:7)$$

and, since  $C_v = AR/k - 1$ ,  $R = Pv/T$ , and  $Mv = V$ ,

$$U_2 - U_1 = \frac{A(P_2V_2 - P_1V_1)}{k - 1} \quad (3:19)$$

in which the change of internal energy is measured in B.t.u. This expression will apply in the calculation of the change of internal energy of a perfect-gas system no matter what type of change the system undergoes.

*Example.* A body of air changes from a pressure of 90 p.s.i. gage and volume of 10 cu. ft. to a pressure of 30 p.s.i. gage and a volume of 20 cu. ft. Calculate the change of internal energy. Barometer 28 in. Hg.

*Solution.* The atmospheric pressure is

$$28 \times 0.491 = 13.75 \text{ p.s.i.}$$

The change of internal energy is

$$\begin{aligned} U_2 - U_1 &= \frac{A(P_2V_2 - P_1V_1)}{k - 1} = \frac{144[(43.75)(20) - (103.75)(10)]}{778 \times 0.405} \\ &= -74.3 \text{ B.t.u.} \end{aligned}$$

The negative sign indicates a decrease of internal energy.

**3:9. Gas Mixtures.** Air is a mixture of a number of gases, the principal constituents being oxygen and nitrogen. The thermodynamic treatment of such mixtures is based upon Avogadro's law, which states:

*At identical temperature and pressure, equal volumes of all gases contain the same number of molecules.*

This law indicates that the densities of all gases at the same pressure and temperature are proportional to their molecular weights.

A *mole* is the weight of a standard number of the molecules of any gas and is numerically equal in pounds to the molecular weight of the gas, *i.e.*, 1 mole of nitrogen weighs 28 lb., 1 mole of oxygen, 32 lb., etc. By applying Eq. (3:5), the volume of 1 mole under specified conditions of pressure and temperature can be found. At a pressure of 14.7 p.s.i. abs. and temperature of 32°F., so-called "standard" conditions, this volume is found to be 358.7 cu. ft. This, by Avogadro's law, applies to all gases, *i.e.*, the volume of 1 mole of any gas under the specified conditions is 358.7 cu. ft. Conversely, the molecular weight of any gas can be calculated by multiplying the density under standard conditions by the volume of 1 mole, or

$$m = 358.7 \times \frac{1}{v_s} \quad (3:20)$$

in which  $v_s$  is the specific volume at standard conditions. If the gas is a mixture such as air, the molecular weight so obtained is, of course, a weighted average of the molecular weights of the constituent gases and is called the *apparent* molecular weight.

If Eq. (3:6) is written

$$Pmv = mRT \quad (3:21)$$

in which  $mv$  is the volume of 1 mole, the value  $mR$  is called the *universal gas constant* and has a constant value of 1,544 for all gases. The following relationships may now be written:

$$R = \frac{1,544}{m} \quad (3:22)$$

$$m = \frac{1,544}{R} \quad (3:23)$$

**3:10. Dalton's Law.** *If two or more gases exist as a mixture in a closed vessel, the total pressure exerted by the mixture on the walls of the vessel will be equal to the sum of the individual pressures exerted by the gases making up the mixture.*

This statement is known as Dalton's law.

Its meaning may be illustrated by considering a number of gases all confined within a single large vessel but held in separate compartments by suitable partitions. The pressures and temperatures of all these bodies of gas are equal. The pressure exerted by each gas upon the walls of the compartment that confines it is, according to the molecular theory to which reference has been made in Chap. I, the effect produced by the bombardment of these walls by the molecules of which the gas is composed. The magnitude of the pressure created will vary directly with the number of molecules striking a unit area of wall surface in unit time, the mass of the individual molecule, and its velocity, the last being a function of the temperature. If the gas in one of these compartments were released to occupy the entire vessel (at unchanged temperature), the distance between its molecules would be increased to an extent such that fewer molecules would strike a unit area of wall surface in unit time and the pressure would decrease in inverse proportion to the increase in volume. When all the gas constituents are released to occupy simultaneously the total volume of the vessel, they form a mixture; and each accounts, by the bombardment effect of its molecules, for a part of the total pressure of this mixture. Furthermore, if the temperature has not changed, it is evident by simple calculation that the total pressure which this mixture exerts against the walls of the containing vessel will be equal to the original (equal) pressures of the constituents as confined within their individual compartments.

The *percentage by volume* expresses the volume that each constituent alone would occupy, at the same temperature and pressure that are associated with the mixture, as a proportion of the volume of the mixture. Its usefulness develops since many gas analyses (*e.g.*, the Orsat) are made at constant pressure and thus give volumetric percentages directly. It

may be noted that the partial pressure of each constituent of a mixture may therefore be calculated as the product of its volumetric proportion in the mixture and the total pressure of the mixture.

A further method of attack in developing the properties of the gas mixture is found in applying Eq. (3:5). By using the subscripts  $a$ ,  $b$ ,  $c$ , etc., to denote the various gases contained in the mixture, the pressure that each, considered separately, will exert on the walls of the containing vessel may be calculated as

$$P_a = \frac{M_a R_a T_a}{V_a}, \quad P_b = \frac{M_b R_b T_b}{V_b}, \text{ etc.}$$

It will be noted that the temperatures  $T$ ,  $T_b$ ,  $T_c$ , etc., are all equal to the temperature of the mixture and  $V_a$ ,  $V_b$ ,  $V_c$ , etc., are all equal to the total volume of the mixture. The pressures  $P_a$ ,  $P_b$ ,  $P_c$ , etc., are called the partial pressures of the corresponding constituent gases, and, by Dalton's law, the sum of all of these partial pressures is the total pressure of the mixture, or

$$P_m = P_a + P_b + P_c + \cdots + P_n \quad (3:24)$$

For such mixtures an *apparent* molecular weight and an *apparent* gas constant can be found by applying Eqs. (3:5), (3:20), and (3:22). The specific heat at constant pressure for such mixtures can be calculated as follows:

$$C_m = \frac{M_a C_{pa} + M_b C_{pb} + M_c C_{pc} + \cdots + M_n C_{pn}}{M_m} \quad (3:25)$$

$C_{vm}$  can be calculated by an equivalent process.

*Example.* Calculate the apparent molecular weight, gas constant, and specific heats of a mixture of oxygen, 21 per cent, and nitrogen, 79 per cent, by volume. If the total pressure of the mixture is 14.7 lb. per square inch absolute, what are the partial pressures of the oxygen and nitrogen?

*Solution.* One mole of the mixture contains 0.21 mole of oxygen and 0.79 mole of nitrogen. The weight of oxygen in 1 mole of the mixture is equal to 0.21 times the molecular weight of oxygen. The weight of nitrogen can be found in a similar manner.† The following table can now be prepared:

Constituent		Number of moles	Molecular weight	Weight, pounds	Per cent by weight of mixture
$a$	O <sub>2</sub>	0.21	32	6.72	23.25
$b$	N <sub>2</sub>	0.79	28	22.12	76.75
Total . . . . .	..	1.00	..	28.84	100.00



The total weight of 1 mole of the mixture is 28.84 lb., and this is also the apparent molecular weight.

The gas constant of the mixture is

$$R = \frac{1,544}{m} = \frac{1,544}{28.84} = 53.5$$

The specific heats of the mixture are

$$C_{p_m} = \frac{M_a C_{p_a} + M_b C_{p_b}}{M_a + M_b} = \frac{(0.2325)(0.218) + (0.7675)(0.244)}{0.2325 + 0.7675} = 0.238$$

$$C_{v_m} = \frac{M_a C_{v_a} + M_b C_{v_b}}{M_a + M_b} = \frac{(0.2325)(0.156) + (0.7675)(0.173)}{0.2325 + 0.7675} = 0.169$$

Since the temperature is the same for both the oxygen and the nitrogen and since the volume of each is the total volume of the mixture, the partial pressures of the two constituents are proportional to the product of their respective weights and gas constants.

$$\frac{p_a}{p_b} = \frac{M_a R_a}{M_b R_b} = \frac{(0.2325)(48.2)}{(0.7675)(54.9)} = 0.266 \quad \text{or} \quad p_a = 0.266 p_b$$

The total pressure of the mixture

$$p_m = p_a + p_b = 1.266 p_b = 14.7 \text{ p.s.i. abs.}$$

Therefore,

$$p_b = \frac{14.7}{1.266} = 11.62 \text{ p.s.i. abs.}$$

and

$$p_a = 14.7 - 11.62 = 3.08 \text{ p.s.i. abs.}$$

the partial pressures of the nitrogen and the oxygen, respectively. These partial pressures might have been calculated more simply as the product of the volumetric percentages of each constituent and the total pressure of the mixture. Thus

$$p_a = (0.21)(14.7) = 3.08 \text{ p.s.i. abs.}$$

$$p_b = (0.79)(14.7) = 11.62 \text{ p.s.i. abs.}$$

### Problems

1. A perfect-gas system doubles in volume as it undergoes an expansion at a constant temperature of 100°F. The original pressure of the system was 60 p.s.i. abs. What is its final pressure?

2. Two pounds of air at an initial pressure of 80 p.s.i. gage and volume of 6 cu. ft. expands isothermally (at constant temperature) to a volume of 10 cu. ft. What is the final gage pressure?

3. Ten cubic feet of gas is compressed at constant temperature from standard atmospheric pressure to a pressure of 60 p.s.i. gage. What is its final volume?

4. Five cubic feet of a gas having an initial pressure of 45 p.s.i. abs. undergoes a constant temperature change until the pressure becomes 20 p.s.i. abs. What is the final volume?

5. Using absolute temperature as ordinates and volumes as abscissas, illustrate the variation of volume with absolute temperature when the pressure of a gas system is constant. What is the volume of the system at absolute zero? Interpret in terms

of molecular activity. For all actual gases, what portion of this diagram does not hold and why?

6. Eight cubic feet of air at 70°F. is heated to 300°F. at constant atmospheric pressure. What is the final volume?

7. Fifteen cubic feet of a perfect gas at 250°F. is cooled at constant pressure until its volume becomes 12 cu. ft. What is the final temperature?

8. A new temperature scale is devised. The volume of a perfect-gas system at 0° on this scale and a pressure of 20 p.s.i. abs. is 11 cu. ft. At the same pressure but a temperature of 100° on this temperature scale, the volume of the same system is 14 cu. ft. Locate absolute zero on this temperature scale.

9. Using absolute temperature as ordinates and absolute pressure as abscissas, prepare a chart showing the variation of pressure and temperature of a perfect gas system during changes of condition which take place at constant volume. What is the pressure of the system at a temperature of absolute zero? Interpret in terms of molecular activity.

10. A gas system having an original pressure of 40 p.s.i. abs. and temperature of 60°F. is heated at constant volume until its temperature becomes 200° F. What is its final pressure?

11. A gas system is confined in a rigid tank under a pressure of 25 p.s.i. gage at a temperature of 400°F. Its pressure gradually decreases until it becomes 20 p.s.i. gage. What is the final temperature?

12. A perfect-gas system is confined in a rigid tank under a pressure of 50 p.s.i. abs. Its temperature as measured on a certain temperature scale is 170°. The pressure is allowed to decrease until it becomes 30 p.s.i. abs; the corresponding temperature reading is -20°. At what temperature is absolute zero located on this temperature scale?

13. Describe the surface represented by the equation  $Pv = RT$ . What is the trace of this surface on the plane  $T = 0$ ? On the plane  $T = 100$ ?  $T = 500$ ? What effect on this surface would result from increasing the value of the gas constant  $R$ ?

14. What is the volume of 2 lb. of air at a pressure of 50 p.s.i. abs. and a temperature of 90°F.?

15. What is the volume of 3 lb. of oxygen at a vacuum of 5 in. Hg and a temperature of 60°F.? Barometric pressure is 29.92 in. Hg.

16. A tank contains 3 lb. of air at a temperature of 100° F. The internal volume of the tank is 8 cu. ft. What pressure would be recorded on a pressure gage attached to this tank if the barometric pressure is standard?

17. At a pressure of 75 p.s.i. abs., 3 lb. of nitrogen has a volume of 7 cu. ft. What is the temperature of the system?

18. A certain gas has a density of 0.068 lb. per cubic foot at standard conditions. What is its gas constant?

19. A tank contains oxygen. The internal volume of the tank is 5 cu. ft. The pressure recorded on a gage attached to the tank is 30 p.s.i. The temperature of the oxygen is 70°F. What weight of oxygen is confined in the tank?

20. Five pounds of air is heated at constant pressure, the temperature meanwhile increasing from 60 to 140°F. How much heat is added to the air? What is its change of internal energy?

21. Eight pounds of oxygen is contained in a closed tank at a pressure of 20 p.s.i.

abs. and a temperature of 60°F. How much heat is required to raise its temperature to 100°F? What is the change of internal energy during this process?

22. Three pounds of carbon dioxide decreases in temperature from 120 to 80°F. During the process both the pressure and the volume undergo changes. What is the change of internal energy?

23. Two pounds of air at a pressure of 50 p.s.i. abs. is confined in a tank with an internal volume of 8 cu. ft. Heat is added to the air until the pressure rises to 70 p.s.i. abs. How much heat is added? What is the change of internal energy of the air?

24. A closed tank contains 10 cu. ft. of air at a pressure of 90 p.s.i. gage and a temperature of 50°F. If 80 B.t.u. of heat is added to the air within the tank, what will be its final pressure as recorded by a pressure gage attached to the tank? Barometric pressure is standard. What is the change of internal energy of the air?

25. Seven pounds of air increases in temperature by 40°F. at constant pressure. How much work is performed?

26. As a certain gas system weighing 3 lb. increases from 100 to 210°F. at constant pressure, 15,000 ft.-lb. of work is performed. What is its gas constant?

27. A certain gas has a gas constant of 62. The ratio of the specific heats is 1.33. What are the specific heats at constant pressure and constant volume?

28. If  $C_p$  for a certain gas is 0.41 and its gas constant is 92, what is the specific heat at constant volume?

29. A certain perfect gas has a molecule with six degrees of freedom. If its gas constant is 86, what are its specific heats?

30. If  $C_p$  is 0.22 and  $C_v$  is 0.17, what is the value of the gas constant?

31. A tank having a volume of 6 cu. ft. contains 1.67 lb. of a certain gas at a temperature of 80°F. and a pressure of 40 p.s.i. abs. It requires 3.72 B.t.u. to heat the contents of the tank to a temperature of 110°F. What is the gas constant? Calculate the specific heats. How many atoms comprise a molecule of this gas?

32. Six cubic feet of air at a pressure of 15 p.s.i. abs. is compressed to a final volume of 2 cu. ft. at a pressure of 60 p.s.i. abs. What is the change of internal energy?

33. Five cubic feet of carbon dioxide at a pressure of 80 p.s.i. gage expands to occupy a final volume of 12 cu. ft. at a pressure of 35 p.s.i. gage. Does its internal energy increase or decrease and by how much?

34. The gas methyl chloride ( $\text{CH}_3\text{Cl}$ ) has a molecular weight of 50.5. What is its gas constant?

35. The monatomic gas helium has an atomic (and molecular) weight of 4. Assuming it to follow exactly the laws of a perfect gas, calculate its gas constant and its specific heats at constant pressure and at constant volume.

36. The monatomic gas argon has an atomic weight of 39.9. Calculate its specific heats at constant pressure and at constant volume.

37. Calculate the specific heats of the diatomic gas carbon monoxide (CO) assuming it to follow exactly the laws of a perfect gas.

38. Nitrogen has an atomic weight of 14; the atomic weight of oxygen is 16. Calculate the specific heats of the diatomic gas nitric oxide (NO).

39. A mixture is composed of two-thirds hydrogen, one-third oxygen by volume. Its total pressure is 15 p.s.i. abs. Calculate its apparent molecular weight, gas constant, specific heats at constant pressure and at constant volume, and the partial pressures of its constituents.

40. A mixture is composed of one-ninth hydrogen, eight-ninths oxygen by weight.

Its total pressure is 15 p.s.i. abs. Calculate its apparent molecular weight, gas constant, specific heats at constant pressure and at constant volume, and the partial pressures of its constituents.

41. A gas mixture produced through the complete combustion of pure carbon with no excess air consists of 79 per cent of nitrogen, 21 per cent of carbon dioxide by volume. Calculate the apparent molecular weight, gas constant, and specific heats of this mixture. If its total pressure is 14.7 p.s.i. abs., what are the partial pressures of its constituents?

42. A typical gas sample taken from a boiler flue when dried shows the following percentages by volume:  $\text{CO}_2$ , 10.5 per cent;  $\text{O}_2$ , 9.0 per cent. The balance may be assumed to be nitrogen. Calculate the apparent molecular weight, the gas constant, and the specific heats of the dry flue gas.

43. A mixture consists of  $\text{CO}_2$ , 17 per cent;  $\text{O}_2$ , 9 per cent; and  $\text{N}_2$ , 74 per cent by weight. Its total pressure is 15 p.s.i. abs. Calculate its composition by volume, the apparent molecular weight, gas constant, specific heats, and the partial pressure of its constituents.

## CHAPTER IV

### FRICTIONLESS NONFLOW PROCESSES—GAS SYSTEMS

**4:1. Thermodynamic Processes.** Whenever a change takes place in the state or condition of a thermodynamic system which is composed of a gas or a mixture of gases, at least two of the three variables  $P$ ,  $V$ , and  $T$  must change their value during the process. The relationship which exists between these three quantities has been expressed in the characteristic equation [Eq. (3:6)], which holds no matter what the nature of the process. In this chapter, the infinite variety of nonflow processes which may take place will be classified and the various types analyzed individually with the object of determining what special

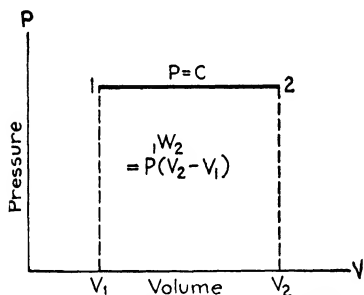


FIG. 4:1—Constant-pressure change.

relationships exist between the variables in each type and what thermal and mechanical effects result. For simplicity, constant specific heats will be assumed and the processes will be assumed to take place without friction.

**4:2. The Constant-pressure Process.** In Fig. 4:1 is represented a constant-pressure change (from point 1 to point 2) on  $PV$  coordinates.

The external work performed is represented by the area of the rectangle under the line 1-2, and its thermal equivalent may be expressed in heat units (see Art. 3:6) as

$$A(1W_2) = AP(V_2 - V_1) = AMR(T_2 - T_1) \quad (4:1)$$

The change of internal energy of a perfect-gas system may always be calculated by applying Eq. (3:7),

$$U_2 - U_1 = MC_v(T_2 - T_1)$$

The heat added during the change is, by definition of  $C_p$ ,

$${}_1Q_2 = MC_p(T_2 - T_1) \quad (4:2)$$

Substituting these values in the nonflow energy equation [Eq. (2:4)],

$$MC_p(T_2 - T_1) = MC_v(T_2 - T_1) + AP(V_2 - V_1) \quad (4:3)$$

The change may, of course, take place in either direction; *i.e.*, it may involve either an increase or a decrease in volume and temperature.

Since  $AMR(T_2 - T_1)$  may be substituted for the external-work term in Eq. (4:3), an inspection of this equation will indicate that, during a given constant-pressure process, all terms must bear the same sign, *i.e.*, if the heat flow is positive, there will be an increase of internal energy, and external work will be done by the system as it expands. A further inspection will show that the magnitudes of these three terms will be fixed with respect to each other in the proportion

$$\frac{\text{Heat flow}}{k} = \frac{\text{change of internal energy}}{1} = \frac{\text{external work (B.t.u.)}}{k - 1}$$

*Example.* Two pounds of air expands from a volume of 10 cu. ft. to a volume of 14 cu. ft. at a constant pressure of 25 p.s.i. gage. Calculate the heat added, the change of internal energy, and the work done. Barometer 30 in. Hg.

*Solution.* The absolute pressure during the expansion is

$$25 + (30 \times 0.491) = 39.73 \text{ p.s.i. abs.}$$

$$T_1 = \frac{P_1 V_1}{MR} = \frac{144 \times 39.73 \times 10}{2 \times 53.3} = 537^\circ \text{F. abs.}$$

$$T_2 = \frac{P_2 V_2}{MR} = \frac{144 \times 39.73 \times 14}{2 \times 53.3} = 752^\circ \text{F. abs.}$$

The heat added during the change is

$${}_1Q_2 = MC_p(T_2 - T_1) = 2 \times 0.2375 \times (752 - 537) = 102.1 \text{ B.t.u.}$$

The increase of internal energy is

$$U_2 - U_1 = MC_v(T_2 - T_1) = 2 \times 0.169 \times (752 - 537) = 72.7 \text{ B.t.u.}$$

The heat equivalent of the external work done is

$$A({}_1W_2) = AP(V_2 - V_1) = \frac{144 \times 39.73 \times (14 - 10)}{778} = 29.4 \text{ B.t.u.}$$

If these values are substituted for the terms of the nonflow energy equation, the equation will balance.

**4:3. Constant Volume.** Figure 4:2 illustrates a change which takes place under a condition of constant volume. Since the volume is constant, the line is vertical and the area under it and consequently the external work are zero. By substitution in the nonflow energy equation,

$${}_1Q_2 = U_2 - U_1 + 0 \quad (4:4)$$

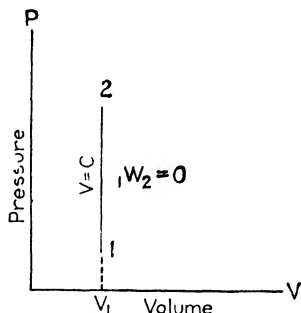


FIG. 4:2.—Constant-volume change.

and, since  $U_2 - U_1 = MC_v(T_2 - T_1)$  for all gas changes,

$${}_1Q_2 = MC_v(T_2 - T_1) \quad (4:5)$$

*Example.* A closed tank having a volume of 10 cu. ft. is filled with air at a pressure of 40 lb. per square inch absolute and a temperature of 100°F. Heat is added until the pressure of the air in the tank becomes 60 lb. per square inch absolute. How much heat is added and what is the change of internal energy?

*Solution.*

$$T_2 = T_1 \left( \frac{P_2}{P_1} \right) = (100 + 459.7) \left( \frac{60}{40} \right) = 839.4^\circ\text{F. abs.}$$

$$M = \frac{P_1 V_1}{R T_1} = \frac{144 \times 40 \times 10}{53.3 \times 559.7} = 1.93 \text{ lb.}$$

The increase of internal energy is

$$U_2 - U_1 = MC_v(T_2 - T_1) = 1.93 \times 0.169 \times (839.4 - 559.7) = 91.4 \text{ B.t.u.}$$

Since no external work is performed, the heat added is also 91.4 B.t.u.

The change of internal energy might have been calculated by Eq. (3:19), which would have made the preliminary calculations for the weight and final temperature unnecessary.

$$U_2 - U_1 = \frac{A(P_2 V_2 - P_1 V_1)}{k - 1} = \frac{144(60 \times 10 - 40 \times 10)}{778(1.405 - 1)} = 91.4 \text{ B.t.u.}$$

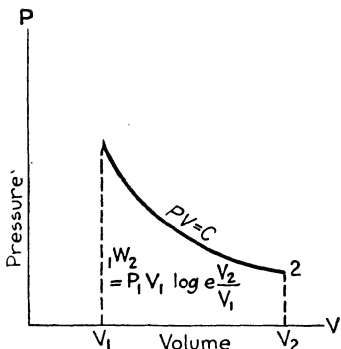


FIG. 4:3.—Isothermal change for a gas.

**4:4. Isothermal changes** are those which take place at constant temperature. By considering the temperature constant in the characteristic equation, it will be observed that, since  $M$  and  $R$  are also constants for any gas system, the product of the two variables  $P$  and  $V$  is equal to a constant for this condition, *i. e.*,

$$PV = MRT = C \quad (4:6)$$

The curve represented by this equation is a rectangular hyperbola and is illustrated in Fig. 4:3. The value of the constant  $C$  is evidently equal to the product of the coordinates  $P_1$  and  $V_1$  of any point on the curve, or

$$C = P_1 V_1 = MRT \quad (4:7)$$

The amount of external work performed during an isothermal change

can be found by integration to find the area under the curve 1-2 in Fig. 4:3, as follows:

$${}_1W_2 = \int_{V_1}^{V_2} P \, dV$$

but

$$P = \frac{C}{V} = \frac{P_1 V_1}{V}$$

and therefore the external work

$$\begin{aligned} {}_1W_2 &= P_1 V_1 \int_{V_1}^{V_2} \frac{dV}{V} \\ &= P_1 V_1 \log_e \frac{V_2}{V_1} \end{aligned} \quad (4:8)$$

$$= P_1 V_1 \log_e \frac{P_1}{P_2} \quad (4:9)$$

Note that  $MRT$  may be substituted for  $P_1 V_1$  in Eqs. (4:8) and (4:9).

The change of internal energy is zero since there is no change of temperature (see Joule's law); therefore, substituting in the nonflow energy equation,

$${}_1Q_2 = 0 + AP_1 V_1 \log_e \frac{V_2}{V_1} = AP_1 V_1 \log_e \frac{V_2}{V_1} \quad (4:10)$$

*Example.* Ten cubic feet of air at a pressure of 60 p.s.i. abs. expands isothermally until the pressure becomes 40 p.s.i. abs. Barometer 29 in. Hg. Is heat added or abstracted, and how much?

*Solution.* The heat equivalent of the external work is

$$\begin{aligned} A({}_1W_2) &= AP_1 V_1 \log_e \frac{P_1}{P_2} = \frac{144 \times 60 \times 10 \times \log_e \frac{60}{40}}{778} \\ &= 111 \times 0.405 = 45 \text{ B.t.u.} \end{aligned}$$

Since there is no change of internal energy, the amount of heat added during the process is also 45 B.t.u.

**4:5. Adiabatic processes** take place without the absorption or emission of heat, *i.e.*, with  ${}_1Q_2 = 0$ . *Ideal* adiabatic processes are those which take place without friction, a condition which can be approached but never attained in practical operation. The effect of friction is to change a part of the work, which otherwise would be available during the expansion or expended on the compression of a system, into thermal agitation, which, in turn, increases the thermal energy of the system, having the effect of increasing the final temperature above that which would have resulted at the end of a frictionless process. Such a process must be considered an adiabatic in the general sense since the *external*



energy transferred was entirely in the form of work; heat transfers alone cannot take place during an adiabatic.

If the process is frictionless, the nature of the curve which represents it on  $PV$  coordinates may be determined by an application of the principles of differential equations. The general relationships which apply are based upon the characteristic and the nonflow energy equations, as follows:

$$PV = MRT \quad (3:5)$$

$$Q_2 = 0 = U_2 - U_1 + A({}_1W_2) \quad (2:4)$$

Differentiating Eq. (3:5), keeping in mind the fact that  $P$ ,  $V$ , and  $T$  are all variables for this condition,

$$P dV + V dP = MR dT$$

from which

$$dT = \frac{P dV + V dP}{MR} \quad (4:11)$$

Since, for an infinitesimal change  $U_2 - U_1 = dU = MC_v dT$  and  $A({}_1W_2) = AP dV$ , Eq. (2:4) may be written

$$0 = MC_v dT + AP dV \quad (4:12)$$

It should be noted that the development has here been limited to a *frictionless* adiabatic process, since only in the absence of friction can external work be measured as  $\int P dV$ .

Substituting the value of  $dT$  from Eq. (4:11) in Eq. (4:12) and transposing,

$$\frac{MC_v}{MR} (P dV + V dP) + AP dV = 0 \quad (4:13)$$

$$\left( \frac{C_v}{R} + A \right) P dV + \frac{C_v}{R} V dP = 0 \quad (4:14)$$

and, since  $C_v + AR = C_p$ ,

$$C_p P dV + C_v V dP = 0 \quad (4:15)$$

Dividing both terms by  $C_v$  and substituting  $k$  for  $C_p/C_v$ ,

$$kP dV + V dP = 0 \quad (4:16)$$

In order to separate the variables, all terms will be divided by an integrating factor  $PV$ .

$$k \frac{dV}{V} + \frac{dP}{P} = 0 \quad (4:17)$$

Integrating,

$$k \log_e V + \log_e P = K \quad (4:18)$$

in which  $K$  is a constant of integration. Equation (4:18) may be written

$$\log_e PV^k = K \quad (4:19)$$

or

$$PV^k = C \quad (4:20)$$

in which  $C$  is a constant such that  $K = \log_e C$ . If the coordinates of any point on the curve are known, the value of  $C$  for the curve may be calculated since  $C = P_1 V_1^k$ . Equation (4:20) is the equation of a frictionless adiabatic for a gas system. It is evident that the external work performed may again be found by integration of the area under the curve represented by this equation and illustrated in Fig. 4:4.

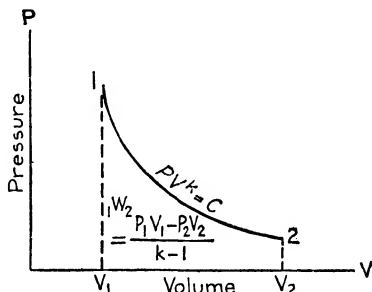


FIG. 4:4.—Adiabatic change for a gas.

$$P = \frac{C}{V^k} = \frac{P_1 V_1^k}{V^k}$$

$$\begin{aligned} {}_1W_2 &= \int_{V_1}^{V_2} P dV = P_1 V_1^k \int_{V_1}^{V_2} \frac{dV}{V^k} \\ &= P_1 V_1^k \left[ \frac{V^{-k+1}}{-k+1} \right]_{V_1}^{V_2} \\ &= \frac{P_1 V_1^k}{1-k} (V_2^{1-k} - V_1^{1-k}) \end{aligned}$$

and, since  $P_1 V_1^k = P_2 V_2^k$ ,

$${}_1W_2 = \frac{P_2 V_2^k V_2^{1-k} - P_1 V_1^k V_1^{1-k}}{1-k} = \frac{P_2 V_2 - P_1 V_1}{1-k}$$

or

$${}_1W_2 = \frac{P_1 V_1 - P_2 V_2}{k-1} = \frac{MR(T_1 - T_2)}{k-1} \quad (4:21)$$

Equation (4:21) may be derived by another method. Since  ${}_1Q_2 = 0$ , for a frictionless adiabatic  $A({}_1W_2) = -(U_2 - U_1)$  and, from Eq. (3:19),

$${}_1W_2 = -\frac{P_2 V_2 - P_1 V_1}{k-1} = \frac{P_1 V_1 - P_2 V_2}{k-1}$$

*Example.* Three pounds of air at 50 p.s.i. abs. and 350°F. expands adiabatically and without friction to a final pressure of 20 p.s.i. abs. What is the change of internal energy, and how much external work is performed? What is the final temperature of the air?

*Solution.*

$$V_1 = \frac{MRT_1}{P_1} = \frac{3 \times 53.3 \times 809.7}{144 \times 50} = 18 \text{ cu. ft.}$$

Since  $P_1 V_1^k = P_2 V_2^k$ ,

$$V_2 = V_1 \left( \frac{P_1}{P_2} \right)^{\frac{1}{k}} = 18 \left( \frac{50}{20} \right)^{\frac{1}{1.405}} = 18 \times 1.92 = 34.6 \text{ cu. ft.}$$

The external work performed is

$$\begin{aligned} {}_1W_2 &= \frac{P_1 V_1 - P_2 V_2}{k - 1} = \frac{144(50 \times 18 - 20 \times 34.6)}{1.405 - 1} = 74,000 \text{ ft.-lb.} \\ &= 95 \text{ B.t.u.} \end{aligned}$$

Since there is no heat added or abstracted, this work is performed at the expense of the internal energy of the system; therefore the *decrease* of internal energy is also 95 B.t.u.

The final temperature of the system is

$$T_2 = \frac{P_2 V_2}{MR} = \frac{144 \times 20 \times 34.6}{3 \times 53.3} = 622^\circ\text{F. abs. or } 162.3^\circ\text{F.}$$

The change of internal energy may also be calculated by Eq. (3:18) as follows:

$$U_2 - U_1 = MC_v(T_2 - T_1) = 3 \times 0.169 \times (162.3 - 350) = -95 \text{ B.t.u.}$$

**4:6. Variation of Pressure, Volume, and Temperature.** In order to calculate the various energy amounts and changes that take place during a frictionless adiabatic process for a gas system, it is necessary to know the coordinates  $P$  and  $V$ , or  $T$ , of the points 1 and 2 which represent the beginning and end of the change of state. In the case of the constant-pressure, constant-volume, and isothermal processes, where one of the coordinates does not change, the missing information may be obtained by a simple application of the laws of Boyle and Charles. When all three vary, as in the frictionless adiabatic case, the following relationships may be developed by a combination of the equation of the typical curve  $PV^k = C$  with the characteristic equation of state  $PV = MRT$ . Thus

$$\frac{P_2}{P_1} = \left( \frac{V_1}{V_2} \right)^k = \left( \frac{T_2}{T_1} \right)^{\frac{k}{k-1}} \quad (4:22)$$

$$\frac{V_2}{V_1} = \left( \frac{P_1}{P_2} \right)^{\frac{1}{k}} = \left( \frac{T_1}{T_2} \right)^{\frac{1}{k-1}} \quad (4:23)$$

$$\frac{T_2}{T_1} = \left( \frac{P_2}{P_1} \right)^{\frac{k-1}{k}} = \left( \frac{V_1}{V_2} \right)^{k-1} \quad (4:24)$$

**4:7. Polytropic Processes.** There are an infinite number of other curves which can be drawn in addition to those described above. A

curve which has the general form  $PV^n = C$ , in which  $n$  is a constant having any value between zero and infinity, is called a *polytropic*. All the changes listed in the preceding paragraphs represent special cases of the polytropic. In the constant-pressure change,  $n = 0$ ; for constant volume,  $n = \infty$ ; during an isothermal for a gas,  $n = 1$ ; and, of course, the value of  $n$  for an adiabatic change of a gas system is  $k$ . This family of curves is illustrated in Fig. 4:5. With the exception of the constant-temperature, adiabatic, and constant-volume phases, none of the terms of the nonflow energy equation is zero for a polytropic change, and the calculation of at least two of them becomes necessary in order to solve the equation completely.

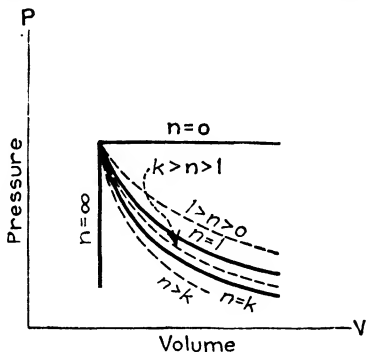


FIG. 4:5.—Polytropic changes.

The external work performed during a polytropic can be found by a process of integration similar to that by which Eq. (4:21) was developed, with the substitution of  $n$  for  $k$ , the result being

$${}_1W_2 = \frac{P_1V_1 - P_2V_2}{n - 1} = \frac{MR(T_1 - T_2)}{n - 1} \quad (4:25)$$

The change of internal energy may be calculated by means of Eq. (3:7) or Eq. (3:19), and the heat added or abstracted can then be found by the algebraic addition of the heat equivalent of the external work and the change of internal energy.

An interesting and important characteristic of this polytropic process for a perfect gas having constant specific heats can be developed as follows:

$$\begin{aligned} {}_1Q_2 &= U_2 - U_1 + A({}_1W_2) = MC_v(T_2 - T_1) - \frac{MAR}{n - 1}(T_2 - T_1) \\ &= MAR \left( \frac{1}{k - 1} - \frac{1}{n - 1} \right) (T_2 - T_1) \\ &= MAR \left( \frac{n - k}{(k - 1)(n - 1)} \right) (T_2 - T_1) \\ &= MC_v \left( \frac{n - k}{n - 1} \right) (T_2 - T_1) \end{aligned} \quad (4:26)$$

Or, since  $C_v$ ,  $n$ , and  $k$  are all constant during the polytropic process,

$${}_1Q_2 = MC_n(T_2 - T_1)$$

in which

$$C_n = C_v \left( \frac{n - k}{n - 1} \right)$$

and represents the *constant* specific heat during the process. This means that during the polytropic process 1 B.t.u. of heat flow will bring about the same temperature change at any temperature level and that the polytropic process for a perfect gas can now be redefined, on a new basis, as a process during which the specific heat remains constant.

The reader will note that the equations of Art. 4:6 will apply to the polytropic process if the general value  $n$  is substituted for the special value  $k$  in terms of which they have been stated.

A summary of the equations developed in Arts. 4:1 to 4:7 is available for the convenience of the reader in Table 5:1.

*Example.* Five cubic feet of air expands polytropically with  $n = 1.3$  from an initial pressure of 80 p.s.i. abs. until the final volume is 8 cu. ft. How much work is done? What are the change of internal energy and the heat added or abstracted during the process?

*Solution.* Since  $P_1 V_1^n = P_2 V_2^n$ ,

$$P_2 = P_1 \left( \frac{V_1}{V_2} \right)^n = 80 \left( \frac{5}{8} \right)^{1.3} = 80 \times 0.543 = 43.5 \text{ p.s.i. abs.}$$

$$A({}_1W_2) = \frac{A(P_1 V_1 - P_2 V_2)}{n - 1} = \frac{144(80 \times 5 - 43.5 \times 8)}{778(1.3 - 1)} = 32.1 \text{ B.t.u.}$$

$$U_2 - U_1 = \frac{A(P_2 V_2 - P_1 V_1)}{k - 1} = \frac{144(43.5 \times 8 - 80 \times 5)}{778(1.405 - 1)} = -23.8 \text{ B.t.u.}$$

and

$${}_1Q_2 = U_2 - U_1 + A({}_1W_2) = -23.8 + 32.1 = 8.3 \text{ B.t.u.}$$

Making use of Eq. (4:26), an alternate method of calculation of heat flow is as follows:

$$MT_1 = \frac{P_1 V_1}{R} = \frac{(144)(80)(5)}{53.3} = 1,080 \quad \text{and} \quad MT_2 = \frac{(144)(43.5)(8)}{53.3} = 940$$

and

$${}_1Q_2 = 0.169 \left( \frac{1.3 - 1.405}{1.3 - 1} \right) (940 - 1,080) = +8.3 \text{ B.t.u.}$$

Therefore, during the process, 8.3 B.t.u. of heat is added to the air, its internal energy is decreased 23.8 B.t.u., and 32.1 B.t.u., or 25,000 ft.-lb. of external work is performed.

**4:8. Heat Transfers during a Polytropic.** Reference to Eq. (4:26) makes it possible to analyze polytropic changes on the basis of the direction and magnitude of the heat flows that take place along them. Thus, if  $n$  is less than 1,  $C_n$  is positive and positive heat flow takes place during an increase of temperature of the system; if  $n$  has a value between

1 and  $k$ ,  $C_n$  is negative and an increase of temperature is accompanied by negative heat flow; if  $n$  has a value greater than  $k$ ,  $C_n$  is again positive and positive heat flow is again associated with a temperature rise.

When a system expands, positive external work is performed. When  $n$  is less than 1, an expansion will be accompanied by an increase of temperature, as indicated by Eq. (4:24), whereas if  $n$  has a value greater than 1 the temperature will fall. Thus, since for a perfect gas the direction in which the temperature changes indicates directly the sign of the change of internal energy, it is clear that, for values of  $n$  less than 1, positive external work is accompanied by an increase of internal energy and that, for values of  $n$  greater than 1, a decrease of internal energy accompanies an expansion of the system.

A review of the development of Eq. (4:26) will indicate that the following relationships will exist in a polytropic:

$$\frac{\text{Heat flow}}{\frac{n - k}{n - 1}} = \frac{\text{change of internal energy}}{1} = \frac{\text{external work (B.t.u.)}}{\frac{k - 1}{1 - n}}$$

**4:9. Determination of the Value of  $n$ .** If the coordinates  $P_1V_1$  and  $P_2V_2$  of two points lying on a polytropic curve are known, the exponent  $n$  of the curve can be found by the following method:

$$\frac{P_2}{P_1} = \left( \frac{V_1}{V_2} \right)^n$$

and, taking the logarithm of both sides of the equation,

$$\log \frac{P_2}{P_1} = n \log \frac{V_1}{V_2}$$

or

$$n = \frac{\log (P_2/P_1)}{\log (V_1/V_2)} \quad (4:27)$$

The value of  $n$  for other combinations of initial and final conditions can be found from Eqs. (4:22) to (4:24).

*Example.* During a polytropic the pressure changes from 75 to 25 p.s.i. abs. and the volume from 2 to 5 cu. ft. What is the exponent of the curve?

*Solution.*

$$n = \frac{\log (P_2/P_1)}{\log (V_1/V_2)} = \frac{\log \frac{25}{75}}{\log \frac{2}{5}} = \frac{-0.4765}{-0.398} = 1.198$$

**4:10. Change of Enthalpy for a Gas.** The change of enthalpy is of less importance in the study of the thermodynamics of gases than in

dealing with vapors; however, a short discussion is in order. It will be remembered that the change of enthalpy is the same as the heat added or abstracted *when a frictionless constant-pressure path is followed*. The heat added or removed from a gas during a constant-pressure change has been shown to be

$${}_1Q_2 = MC_p(T_2 - T_1)$$

and this expression will also denote the change of enthalpy. Since enthalpy is a function of the state of the system, the change of enthalpy may be developed by another method.

$$\begin{aligned}\text{Change of enthalpy} &= H_2 - H_1 \\ &= (U_2 + AP_2V_2) - (U_1 + AP_1V_1) \\ &= U_2 - U_1 + A(P_2V_2 - P_1V_1) \\ &= MC_v(T_2 - T_1) + AMR(T_2 - T_1) \\ &= M(C_v + AR)(T_2 - T_1) \\ &= MC_p(T_2 - T_1)\end{aligned}\tag{4:28}$$

$$= \frac{kA}{k-1}(P_2V_2 - P_1V_1)\tag{4:29}$$

These equations may be used to evaluate the change of enthalpy of the system without regard to the nature of the process through which the change was accomplished. Thus the enthalpy of a perfect gas, like the internal energy, is independent of its pressure or volume and depends only on its temperature. Moreover, the change of enthalpy of a perfect-gas system will be  $k$  times the change of its internal energy during a specified process.

*Example.* What is the change of enthalpy when 10 cu. ft. of air at 15 p.s.i. abs. is compressed to a volume of 4 cu. ft. and a pressure of 45 p.s.i. abs?

*Solution.*

$$\begin{aligned}H_2 - H_1 &= \frac{kA}{k-1}(P_2V_2 - P_1V_1) = \frac{1.405 \times 144 \times (45 \times 4 - 15 \times 10)}{0.405 \times 778} \\ &= 19.25 \text{ B.t.u.}\end{aligned}$$

### Problems

1. Find the heat flow, external work, and change of internal energy of the system as 5 lb. of air is increased in temperature from 70 to 120°F. at constant pressure and without friction.

2. The volume of an air system decreases from 10 to 6 cu. ft. during a frictionless process which takes place at a constant pressure of 50 p.s.i. abs. What is the heat flow? The change of internal energy?

3. During a frictionless constant-pressure process, a carbon-dioxide system receives 200 B.t.u. of heat. What is the external work? The change of internal energy?

4. The internal energy of an oxygen system decreases by 70 B.t.u. as it undergoes a frictionless constant-pressure process. What is the heat flow? The external work?

5. A system composed of a monatomic perfect gas expands frictionlessly at a constant pressure of 100 p.s.i. abs. from a volume of 3 cu. ft. to a final volume of 5 cu. ft. What is the heat flow? The change of internal energy?

6. Five pounds of air confined in a closed tank is heated from an initial temperature of 70°F. to a final temperature of 120°F. What is the heat flow? The external work? The change of internal energy of the air during the process?

7. Three cubic feet of carbon dioxide at 50 p.s.i. abs. is cooled at constant volume owing to the removal of 25 B.t.u. of heat. What is the final pressure? What is the change of internal energy?

8. Thirty B.t.u. of heat is added to the air in a closed tank having an internal volume of 6 cu. ft. During the process the temperature increased from 60 to 120°F. What are the original and final pressures?

9. The increase of internal energy during a constant-volume process for a gas system is 60 B.t.u. What is the heat flow?

10. A certain mixture of diatomic perfect gases has a gas constant of 30. During a constant-volume process, 20 B.t.u. of heat is added to each pound of the mixture. What change of temperature accompanies the process?

11. Five cubic feet of air at a pressure of 100 p.s.i. abs. expands frictionlessly at constant temperature to a final volume of 12 cu. ft. What is the external work during the process? The change of internal energy? The heat flow?

12. Two pounds of carbon dioxide is compressed isothermally and frictionlessly. The initial pressure and volume are 15 p.s.i. gage and 9 cu. ft., respectively; the final pressure is 75 p.s.i. gage. Calculate the external work, the change of internal energy, and the heat flow which accompanies the process.

13. The pressure of 3 lb. of oxygen is doubled at a constant temperature of 150°F. If the process is frictionless, what is the heat flow? The change of internal energy? The external work?

14. During a process which takes place frictionlessly and at a constant temperature of 200°F., the volume of an air system is tripled. The heat flow during the process is 100 B.t.u. What is the weight of the system?

15. A gas system is compressed frictionlessly with  $PV = \text{constant}$ . The final pressure and volume of the system are 60 p.s.i. abs. and 3 cu. ft., respectively, and 27,000 ft.-lb. of work are expended in effecting the compression. What is the original volume? What is the change of internal energy? What is the heat flow?

16. Five cubic feet of air at 120 p.s.i. abs. and 300°F. expands adiabatically and without friction to a final pressure of 40 p.s.i. abs. What is the final volume? The final temperature? How much external work is performed, what is the heat flow, and what is the change of internal energy of the system during the process?

17. An air system is compressed adiabatically and without friction from an original volume of 5 cu. ft. to a final volume of 2 cu. ft. If the original pressure is standard atmospheric, what is the final pressure? What are the heat flow, the external work, and the change of internal energy during the process?

18. Five cubic feet of a monatomic perfect gas at a pressure of 120 p.s.i. abs. and a temperature of 300°F. expands with  $PV^k = \text{constant}$  to a final pressure of 40 p.s.i. abs. What is the final volume? The final temperature? The change of internal energy during the process? If the process is frictionless, what are the heat flow and external work?

19. During a frictionless adiabatic process, the temperature of an oxygen system having a weight of 2 lb. decreases by 150°F. What is the heat flow? The change of internal energy? The external work?



20. A gas system undergoes a frictionless process during which it neither receives nor rejects heat. During the process the internal energy of the system increases by 100 B.t.u. What is the external work during the process?

21. Ten cubic feet of air at 100 p.s.i. abs. and 200°F. expands without friction and with  $n = 1.25$  to a final pressure of 20 p.s.i. abs. What is the final volume? The final temperature? How much work is performed, what is the heat flow, and what is the change of internal energy during the process? Calculate the (polytropic) specific heat during the process.

22. Ten cubic feet of air at 14 p.s.i. abs. and a temperature of 70°F. is compressed polytropically and frictionlessly to a final volume of 4 cu. ft. During the process the polytropic specific heat is  $-0.06$ . What is the value of the exponent  $n$ ? What is the final pressure? The final temperature? What are the heat flow, the external work, and the change of internal energy during the process?

23. Three pounds of oxygen undergoes a frictionless polytropic process during which the temperature decreases by 150°F. During the process the total heat flow is 8 B.t.u. *away from* the system. What is the value of the exponent  $n$  for this process? Is it an expansion or a compression, and what is the external work?

24. A monatomic perfect-gas system is compressed frictionlessly and polytropically with  $n = 1.45$ . The work supplied to accomplish the compression is 75,000 ft.-lb. What is the change of internal energy of the system? What is the heat flow during the process?

25. A carbon dioxide system undergoes a frictionless and polytropic process with  $n = 1.35$ . During the process the internal energy decreases by 40 B.t.u. What is the heat flow? What is the external work?

26. Derive Eqs. (4:22) to (4:24).

27. During the polytropic and frictionless expansion of a perfect-gas system, the temperature decreases, and heat leaves the system. Is the value of  $n$  between 0 and 1, 1 and  $k$ , or  $k$  and  $\infty$ ?

28. During the polytropic and frictionless compression of a perfect-gas system, the temperature rises and heat leaves the system. Is the value of  $n$  between 0 and 1, 1 and  $k$ , or  $k$  and  $\infty$ ?

29. During the polytropic and frictionless expansion of an air system, the heat flow into the system is one-third of the heat equivalent of the external work. What is the value of  $n$ ?

30. During the polytropic and frictionless compression of a system composed of carbon dioxide, the internal energy increases at a rate equal to 75 per cent of the rate at which energy in the form of work is supplied to accomplish the compression. What is the value of  $n$ ?

31. Ten cubic feet of air undergoes a frictionless and polytropic process during which it rejects heat at a rate equal to one-quarter of the rate at which its internal energy increases. What is the value of  $n$ ? Is this an expansion or a compression?

32. One pound of air has an initial pressure of 100 p.s.i. abs. and volume of 2.5 cu. ft. At the end of a frictionless process which is represented by a straight line on a  $PV$  diagram, the pressure becomes 20 p.s.i. abs. and the volume 9 cu. ft. What is the change of internal energy during the process? What is the heat flow? Between what values does  $n$  vary during the process?

33. Five cubic feet of air at 140 p.s.i. abs. expands polytropically and without friction until the pressure is 40 p.s.i. abs. and the volume 14 cu. ft. What is the value of  $n$ ?

34. Fifteen cubic feet of a gas at 60°F. is compressed frictionlessly and polytropically until the volume becomes 5 cu. ft. and the temperature 280°F. What is the value of  $n$ ?

35. A gas system at 200 p.s.i. abs. and 250°F. undergoes a frictionless and polytropic process until its pressure is 40 p.s.i. abs. and its temperature 70°F. What is the value of  $n$ ?

36. One pound of a monatomic perfect gas expands polytropically and without friction from an initial pressure of 90 p.s.i. abs. and volume of 2 cu. ft. to a final pressure of 20 p.s.i. abs. and volume of 7.5 cu. ft. During the process the temperature decreases by 113°F. What is its gas constant? What are the specific heats at constant volume and at constant pressure for this gas?

37. During a process which is neither frictionless nor polytropic, the pressure of an air system decreases from 120 to 20 p.s.i. abs. while the volume increases from 3 cu. ft. to a final volume of 15 cu. ft. What is the change of enthalpy?

38. Three pounds of oxygen increase in temperature by 75°F. What is the change of enthalpy?

39. During a certain process, the internal energy of an air system is increased by 40 B.t.u. What is its change of enthalpy?

40. A body of air at a pressure of 40 p.s.i. abs. and a volume of 2 cu. ft. receives 20 B.t.u. during a constant-volume process. What is the change of enthalpy?

41. A system consists of 1 lb. of air. Its initial pressure is 30 p.s.i. abs., and initial volume is 7 cu. ft. If, during each of the following processes, its volume increases to a final value of 10 cu. ft., find the heat flow, change of internal energy, and the external work in each case:

- (a) Frictionless constant pressure
- (b) Frictionless constant temperature
- (c) Frictionless adiabatic
- (d) Adiabatic expansion into an exhausted chamber.
- (e) Frictionless with  $PV^{-1} = \text{constant}$
- (f) Frictionless constant internal energy
- (g) Frictionless constant enthalpy

## CHAPTER V

### THE CARNOT CYCLE—ENTROPY

**5:1. The Availability of Energy.** It has already been noted that, when heat is continuously changed into mechanical energy, complete conversion cannot take place. It has also been stated as the second law of thermodynamics that, for the continuous production of work from heat, the heat must flow from a source at a relatively high temperature and the portion not converted into work must be rejected to a refrigerator at some lower temperature. The *maximum* proportion of the heat energy leaving the source which can be changed into mechanical energy by means of a cyclical process under stated conditions of temperature of source and refrigerator is known as the *available* portion of the heat received from the source under these conditions; the balance of the heat extracted from the source, *i.e.*, the *minimum* proportion which must be rejected to the refrigerator, is called the *unavailable* portion of the heat received from the source. Mechanical energy, although it may not be completely recoverable for useful purposes, will, for our purposes, be considered completely "available." It will be the purpose of this chapter to indicate what proportions of the total energy flowing from the source are available under varying conditions. The ideal availability is of great value as a measure of the performance of an actual engine.

**5:2. Reversible and Irreversible Processes.** In thermodynamic usage, a process is designated as *reversible* if it can be performed in both directions, the state path during the original change of condition being exactly retraced during the return to the original state of the system, and if all other systems involved in any way with either the direct or the reversed process are returned to their original condition. True reversibility will involve no change in the availability of energy at any point during either the direct or the reversed process.

The presence of friction will always make a process irreversible, since it will have the effect of changing work, a form in which energy is completely available, into thermal energy, of which only a part can be available. Any process that involves the flow of heat between bodies due to a finite difference in temperature between them must also be irreversible,

since energy must be supplied from an external system to return the thermal energy to its original location.

A familiar example of a process that approaches reversibility as a limit is found in a ball thrown upward. As we remove the retarding influence of air resistance (friction) and approach perfect elasticity as a property of the ball, the characteristics of a truly reversible process are more nearly attained. A similar example of an irreversible process is the fall of fruit from a tree.

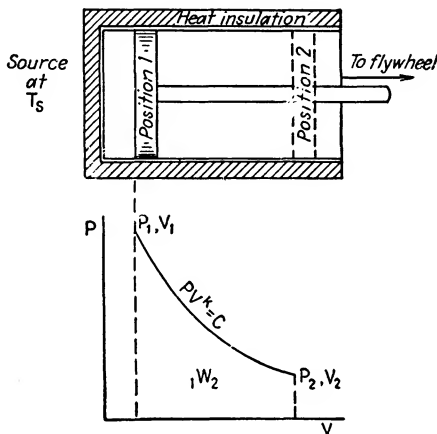


FIG. 5:1.—The frictionless adiabatic as a reversible process.

Two of the processes discussed in the preceding chapter can be considered reversible when the temperature of the source of heat remains at a constant level,  $T_s$ . One of these is the frictionless adiabatic. Assume that a body of gas is confined in a cylinder which is fitted with a frictionless piston. The head of the cylinder is in contact with the source of heat, but heat is prevented from entering or leaving the cylinder by insulation which is perfectly nonconducting. These assumptions are illustrated in Fig. 5:1. The motion of the piston and rod is resisted by the inertia of a very large frictionless flywheel (not shown), and the connecting linkage is also assumed to be frictionless.

In position 1; the mechanism is stationary with the flywheel crank on its dead-center position. However, if the flywheel is moved very slightly past this position, the pressure acting on the face of the piston will cause a slow movement of the piston and rod and the flywheel will begin to rotate. As it does so, its parts will accumulate kinetic energy exactly equivalent to the work done by the expanding gas system. It

has been shown in Art. 4:5 that, during this movement of the piston, the state path of the gas system confined within the cylinder will follow the track represented by the equation  $PV^k = C$  [Eq. (4:20)]. The work area  ${}_1W_2$  will represent the energy which has been received by the flywheel and stored in it as kinetic energy by the time the end of the stroke of the piston (position 2) has been reached.

As the flywheel passes the outer dead-center position, the motion of the piston is reversed and the pressure-volume path is slowly retraced until,

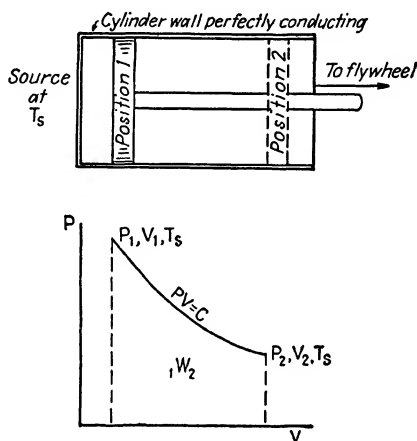


FIG. 5:2.—The frictionless isothermal as a reversible process.

when position 1 has again been reached, all the kinetic energy which was stored in the flywheel on the outward stroke has been expended as work performed in compressing the gas system and the flywheel has slowed to a stop as it reaches the inner dead-center position. The source of heat has remained unaffected throughout the double process since the thermal insulation around the cylinder prevented any interchange of heat and all other systems have returned to their original condition. It may be noted that work was required to move the flywheel off the dead-center position and begin the operation but, since the mechanism was assumed to be frictionless and since this displacement could take place as slowly as desired, the amount of work required was infinitesimal (could be made as small as desired). The requirements for reversibility have been met.

The second process which can be shown to approach reversibility as a limit while the temperature of the source remains constant is the frictionless isothermal. This is illustrated in Fig. 5:2. In this case, the insula-

tion around the cylinder is discarded, and the cylinder walls are assumed to be perfect conductors of heat. This means that the temperature of the system confined within the cylinder could differ in temperature from the source outside only infinitesimally. The source of heat, at temperature  $T_s$ , again surrounds the cylinder, and a very large frictionless flywheel is provided as before. At position 1, the flywheel is on its dead-center position, and the condition of the confined gas system is represented by its properties  $P_1$ ,  $V_1$ , and  $T_s$ . It will be noted that the temperature is the same as that of the source which surrounds the cylinder and that, consequently, no heat will flow in either direction between the source and the system.

The condition of equilibrium which existed originally is disturbed, as before, by moving the flywheel very slightly from its dead-center position. The gas system within the cylinder is now free to expand, doing work against the inertia of the flywheel. The first element of work so performed is at the expense of the internal energy of the gas system and a slight drop in temperature,  $dT$ , will take place. This lowering of temperature is only infinitesimal, because the walls of the cylinder have been assumed to be perfect conductors of heat, but is sufficient to induce a compensating heat flow from the source. During the outward expansion of the system, its temperature will therefore be constant at  $T_s - dT$ , in which  $dT$  is an infinitesimal, and the state path followed during this expansion is represented by the equation  $PV = C = MR(T_s - dT)$  (Art. 4:4). At the end of the outward stroke, the work area between this state path and the  $V$  axis (Fig. 5:2) will represent the increase in kinetic energy of the flywheel and also the amount of energy which has left the source as heat.

As the piston and flywheel reach outer dead center, the former comes momentarily to a halt and heat flow from the source immediately raises the temperature of the system to  $T_s$ . Then, as the momentum of the flywheel carries it past dead center, the piston reverses its motion and a compression of the system begins. The first small increment of this compression raises the temperature of the system to  $T_s + dT$ , where  $dT$  is again an infinitesimal difference in temperature, and during the balance of the return stroke it remains constant at this level as heat flows from the system to the source at a rate equal to that at which work is returned by the flywheel to accomplish the compression. The state path followed during the compression is  $PV = C = MR(T_s + dT)$ , but  $dT$  is an infinitesimal, the temperature of the system during the compression differs from that during the expansion by only  $2 dT$  (also an infinitesimal), and the two state paths may both be represented by a single curve.

The energy stored in the flywheel during the outward stroke will be just sufficient to return the system to its original condition, and the heat supplied the system from the source during the expansion will be returned to the source during the compression. Since the energy necessary to displace the flywheel originally from its dead-center position may be made as small as desired and since all systems involved in the double process have returned to their original condition, the frictionless isothermal has been shown to be reversible when the temperature of the source of heat is constant.

A study of the conditions under which the adiabatic and the isothermal processes can be considered reversible, as outlined in the paragraphs immediately above, will make it clear that the absence of friction and of heat flow across finite temperature differences is not the only requirement for reversibility. In addition, no unrestrained expansion must be permitted; any work which leaves the system must be received and stored as mechanical energy so that it will be available for return as work in its full amount. Also, if the cylinder wall had not been assumed to be a *perfect* heat conductor (*i.e.*, to offer zero resistance to the flow of heat) in the analysis of the isothermal process, the process would necessarily have to take place with infinite slowness if the temperature difference  $dT$  were to remain infinitesimal.

All the other processes studied in Chap. IV can be handled in like manner and shown to approach reversibility as a limit, but an additional condition must be imposed. For all but the adiabatic and isothermal, a source must be assumed which *varies* in temperature in conformity with the temperature of the expanding and contracting system. Ordinarily, heat sources are considered to remain at constant temperature, and for our purposes the two types of process first analyzed are therefore better examples of the reversible process.

**5:3. The Heat-engine Cycle.** A heat engine is the name given in thermodynamics to a device for converting heat into work. All heat engines operate by passing the thermodynamic system or working substances through a number of thermodynamic processes, returning it periodically to the same condition. These processes, which are necessary to return the system to its original condition, taken as a whole and in order, constitute the *heat-engine cycle*.

**5:4. The Carnot cycle,** proposed in 1824 by a Frenchman of that name, is designed to meet the conditions of reversibility when the temperatures of the source and the refrigerator remain at a constant level. It consists of the following processes in order:

1. An isothermal expansion at the temperature  $T_s$  of the source.

2. An adiabatic expansion to the temperature  $T_R$  of the refrigerator.
3. An isothermal compression at the temperature  $T_R$  of the refrigerator.
4. An adiabatic compression to the original condition.

The cycle, as it applies to a perfect gas system, is illustrated in Fig. 5:3 on  $PV$  coordinates. It is evident that, to construct a practical engine to operate on this cycle, the following conditions must be met:

1. The head of the cylinder in which the expansions take place must be made alternately conducting and nonconducting according to whether an isothermal or an adiabatic process is taking place within.

2. Provision must be made to apply the source and the refrigerator alternately during the cycle.

Because of the difficulty in evolving a design to meet these conditions, no practical engine to operate on this cycle has ever been constructed. In spite of its impractical characteristics, the Carnot cycle has great impor-

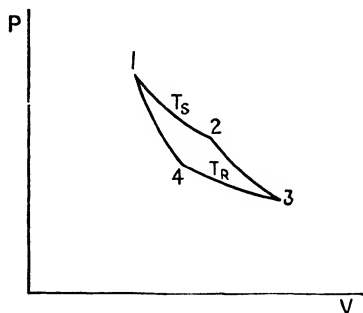


FIG. 5:3.—The Carnot cycle.

tance in thermodynamic theory for reasons to be developed in Art. 5:5.

The efficiency of the Carnot cycle may be determined by returning to the basic expression for efficiency,  $e = (Q_S - Q_R)/Q_S$  [Eq. (2:11)] in which  $Q_S$  is the heat taken from the source and  $Q_R$  the heat rejected to the refrigerator. With reference to Fig. 5:3, by the definition of an adiabatic no heat flow takes place during the processes which connect point 2 with point 3 or point 4 with point 1. The temperature of the system remains constant at  $T_S$  during the process 1-2 and is constant at  $T_R$  during the process 3-4; during the upper isothermal,  $Q_S$  heat units are supplied from the source, and, during the lower,  $Q_R$  heat units are rejected to the refrigerator. Assuming that the medium employed in a Carnot engine is a perfect gas, the following expressions may be written:

$$Q_S = AP_1V_1 \log_e \frac{V_2}{V_1} = AMRT_S \log_e \frac{V_2}{V_1} \quad (5:1)$$

$$Q_R = AP_3V_3 \log_e \frac{V_3}{V_4} = AMRT_R \log_e \frac{V_3}{V_4} \quad (5:2)$$

But since

$$\frac{T_S}{T_R} = \left( \frac{V_3}{V_2} \right)^{k-1} = \left( \frac{V_4}{V_1} \right)^{k-1} \quad (5:3)$$



therefore

$$V_3/V_2 = V_4/V_1 \quad (5:4)$$

and, transposing,

$$V_3/V_4 = V_2/V_1 \quad (5:5)$$

By substituting its equivalent  $V_2/V_1$  for  $V_3/V_4$  in Eq. (5:2) the efficiency of the cycle may be calculated as

$$\begin{aligned} e &= \frac{Q_S - Q_R}{Q_S} = \frac{AMRT_S \log_e (V_2/V_1) - AMRT_R \log_e (V_2/V_1)}{AMRT_S \log_e (V_2/V_1)} \\ &= \frac{T_S - T_R}{T_S} \end{aligned} \quad (5:6)$$

If the Carnot cycle is traversed in the opposite direction (counter-clockwise in Fig. 5:3), an analysis will indicate that heat will be absorbed at the lower temperature  $T_R$  and rejected at the higher level of temperature  $T_S$ ; the energy that was delivered per cycle in the form of work *to* an external system now must be supplied *from* that system in order to operate the Carnot cycle in reverse or as a *heat-pump or refrigeration cycle*. Further study will show that the relative amounts of heat flow and of shaft work will remain the same as in operation as an engine—only the *directions* of energy flow have been completely reversed. Thus we may write, for the Carnot *heat pump*, that the heat *absorbed* from the refrigerator plus the heat equivalent of the shaft work supplied by an external system is equal to the heat *discharged* at the higher temperature. If the heat flow at the lower temperature, though reversed in direction, is still designated as  $Q_R$  and that at the higher temperature as  $Q_S$ , the following proportions will evidently exist for the Carnot heat pump just as they did for the Carnot-engine cycle:

$$\frac{Q_S}{T_S} = \frac{Q_R}{T_R} = \frac{AW}{T_S - T_R}$$

### 5:5. The Carnot principle states:

*No engine working continuously between given temperatures of source and refrigerator can have a greater efficiency than a reversible engine operating between the same temperatures.*

By “reversible engine” is meant one for which all the processes making up the cycle on which it operates are reversible in the sense discussed in Art. 5:2. The Carnot “engine,” operating on the Carnot cycle, is a reversible engine, and the characteristic features of that cycle may there-

fore be used to represent those of the engine to which reference is made in the statement of the Carnot principle. It should be noted that the term “reversible” does not here refer to the possibility of changing the direction of rotation *as an engine* but to a reversal of the order in which the various processes making up the cycle are effected. Thus, as explained in the preceding article, the “reversed” engine becomes a “heat pump,” removing heat from the refrigerator and depositing heat in the source at the expense of work which is supplied continuously from an external system. Moreover, the respective *amounts* of such heat and work flows are in the same proportion in reversed as in direct operation, only the *direction* being changed. An “irreversible” engine, on the other hand, while it might change its direction of rotation as an engine, could not be converted into a heat pump since at least one of the processes making up its cycle could be carried out in only one direction.

For the proof of the Carnot principle, it will be assumed that an irreversible engine *A* exists which has an efficiency greater than that of a reversible engine *B* when

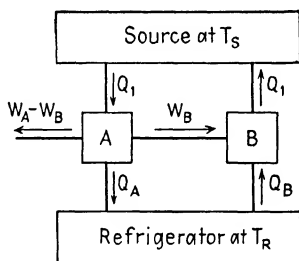


FIG. 5:4.—Proof of Carnot principle.

the two engines operate between the same source and refrigerator temperatures. Both engines will be placed between the same source and the same refrigerator, as illustrated in Fig. 5:4. If it can be shown that the assumption which has just been made leads to results which are inconsistent with careful and time-tried observation of the operation of natural laws, we shall be justified in accepting the Carnot principle and making use of it.

Engine *A* will be operated at a rate such that it will draw  $Q_1$  heat units from the source and discharge  $Q_A$  heat units to the refrigerator. If reversible engine *B* were operated direct at an equivalent rate, it would receive  $Q_1$  heat units from the source and reject  $Q_B$  heat units to the refrigerator, where  $Q_B$  is greater than  $Q_A$  since the efficiency of the irreversible engine is assumed to be the greater. However, engine *B* will be operated in reverse (as a heat pump), and the work developed by engine *A* will be used to drive it.

If the two are operated simultaneously at the given rates and under the stated conditions, engine *A* takes  $Q_1$  heat units from the source, develops  $W_A$  units of work, and rejects  $Q_A$  heat units to the refrigerator while engine *B*, operated in reverse, draws  $Q_B$  heat units from the refrigerator, requires the expenditure of  $W_B$  units of work to drive it, and discharges

$Q_1$  heat units to the source. Based on Eqs. (2:10) and (2:11), the following expressions may now be written:

$$Q_1 - Q_A = AW_A \quad \text{and} \quad Q_1 - Q_B = AW_B$$

But it has already been demonstrated that, if the efficiency of irreversible engine  $A$  is greater than that of reversible engine  $B$ , then  $Q_B$  must be greater than  $Q_A$ . Therefore  $W_A$  is greater than  $W_B$ , which means that the work developed by engine  $A$  will be more than sufficient to drive engine  $B$  and there will be a surplus of work,  $W_A - W_B$ , available. Since the condition of the source has not changed (it has given and received  $Q_1$  units of heat), this work has evidently been performed at the expense of the energy of the refrigerator and has not come from a source at higher temperature. This is a violation of the second law of thermodynamics. If it were possible to have an irreversible engine of greater efficiency than a reversible engine operating between the same temperature limits, it would be possible to produce power from the energy in the atmosphere without the expenditure of fuel or to reduce the temperature of a refrigerator of finite size without limit, at the same time developing a surplus of work. Experience has shown this to be impossible, and therefore the assumption must be abandoned and the truth of Carnot's principle confirmed. The reader will note that it has not been proved that the reversible engine must have an efficiency *higher* than that of any irreversible engine operating between the same limiting temperatures; it has been shown only that the efficiency of the reversible engine must be *at least as high* as that of such an engine. Based on this proof, it is entirely possible also for the efficiency of the irreversible engine to be the greater if the temperatures of source and refrigerator differ for the two engines.

An important corollary of the Carnot principle may be stated as follows:

*All reversible engines, when operating between the same temperatures of source and receiver, will have the same efficiency.*

The method of proof of this statement parallels that of the Carnot principle, the engines being assumed to differ in efficiency and this assumption then being proved to be invalid. The detailed demonstration is left to the reader.

This corollary is of great significance since it authorizes the substitution of the Carnot cycle to represent *any* reversible cycle as the "pattern" cycle, the cycle of highest possible performance. It will be remembered that the efficiency of the Carnot cycle was shown to be  $(T_S - T_R)/T_S$ ; this must also be the efficiency of *all* reversible cycles and, when  $T_S$

represents the constant temperature of the source and  $T_R$  the constant temperature of the refrigerator, the highest efficiency attainable in the production of work from heat. Referring to the definitions of available and unavailable energy at the beginning of this chapter, it is evident that if a Carnot engine is placed between any source and a given refrigerator the work output of this engine will be a measure of the availability of the energy taken from the source. Of a total energy  $Q_s$  removed from the source, the available portion  $E_A$  is therefore

$$E_A = Q_s \left( \frac{T_s - T_R}{T_s} \right) = Q_s \left( 1 - \frac{T_R}{T_s} \right) \quad (T_s \text{ and } T_R \text{ constant}) \quad (5:7)$$

and the unavailable portion  $E_B$  is

$$E_B = Q_s \left( \frac{T_R}{T_s} \right) = T_R \left( \frac{Q_s}{T_s} \right) \quad (5:8)$$

**5.6. Entropy.** The term “source” as used in preceding discussions has referred to a body or system from which energy *in the form of heat alone* may be withdrawn. Unless otherwise stated, the energy which can be taken from a source is assumed to be infinite, *i.e.*, continuous abstractions of heat may be made from it without lowering its temperature. In the practical case, this condition is met by continuously supplying energy from fuel to maintain the temperature. In the same manner, the refrigerator is considered to have an infinite capacity for the absorption of heat without increase of temperature; the atmosphere furnishes a reservoir approximately meeting these requirements, and atmospheric temperature is therefore commonly accepted as the basic refrigerator temperature for power cycles (cycles which have as their purpose the continuous development of work from heat).

When a definite quantity of heat is withdrawn from a vast source at constant temperature, the total amount of heat so passing can be divided into its available and unavailable portions by applying Eqs. (5:7) and (5:8). If this heat enters a body at lower temperature than that of the source, the available portion of the energy in its new location is decreased and the unavailable portion increases although their total remains unchanged. This change in proportion takes place because the process is irreversible since it involves heat flow across a finite temperature difference.

*Example A.* One thousand B.t.u. of heat leaves the hot gases of a boiler firebox at a constant temperature of 2500°F. This heat enters the steam within the boiler at a constant temperature of 400°F. The temperature of the atmosphere is 60°F.

Divide this energy into its available and unavailable portions ( $a$ ) as it leaves the hot gases and ( $b$ ) in its new location in the steam.

*Solution.*

$$(a) \text{ Available portion } E_A = 1000 \left( \frac{2959.7 - 519.7}{2959.7} \right) = 824 \text{ B.t.u.}$$

$$\text{Unavailable portion } E_B = 1000 \left( \frac{519.7}{2959.7} \right) = \frac{176 \text{ B.t.u.}}{1000 \text{ B.t.u.}}$$

$$(b) \text{ Available portion } E_A = 1000 \left( \frac{859.7 - 519.7}{859.7} \right) = 396 \text{ B.t.u.}$$

$$\text{Unavailable portion } E_B = 1000 \left( \frac{519.7}{859.7} \right) = \frac{604 \text{ B.t.u.}}{1000 \text{ B.t.u.}}$$

It will be noted that, if the temperature of the atmosphere (the refrigerator) had been changed in the preceding example, the proportions of available and unavailable energy would also have been altered. On the other hand, the *quotient* obtained by dividing the unavailable portion  $E_B$  by the refrigerator temperature  $T_R$  would be equal, as indicated by Eq. (5:8), to the quotient obtained by dividing the amount of heat transferred by the (constant) temperature at which it was transferred. This quotient is called the change of *entropy* and is designated by the symbol  $\Delta S$ . When the heat left the hot gases in the foregoing example, the total energy of the gases was decreased by 1000 B.t.u.; and, when the refrigerator temperature was 60°F., their unavailable energy was reduced by 176 B.t.u. The corresponding change of entropy was

$$\Delta S = \frac{E_B}{T_R} = \frac{Q_S}{T_S} = \frac{-176}{519.7} = \frac{-1000}{2959.7} = -0.338$$

Similarly, for the steam,

$$\Delta S = \frac{604}{519.7} = \frac{1000}{859.7} = 1.162$$

If the hot gases and the steam are both included in a single system, the heat flow becomes simply an interchange of thermal energy within the system and the process is adiabatic with respect to other systems. The net change of entropy of this system is the sum of the changes of its parts, or  $1.162 - 0.338 = 0.824$ . Because of the irreversibility of the process which has taken place, the unavailable energy of the system has increased by an amount which may be calculated as the product  $T_R \Delta S = (519.7)(0.824) = 428 \text{ B.t.u.}$  This should be compared with the calculated value from the example. If the temperature of the refrigerator has some other value, it is necessary only to multiply its absolute temperature by the same entropy change as calculated above to obtain the corresponding change of unavailable energy. Since the

temperature of the atmosphere, which conventionally forms the refrigerator, is extremely variable, the advantage in replacing the "change of unavailable energy" by the "change of entropy" in thermodynamic usage is readily apparent.

The reader will note that the discussion above is related to systems which can absorb or emit heat without change of temperature and that energy transfers in the form of heat alone are contemplated.

**5:7. Entropy of a Finite System.** A finite system may also possess energy dependent on its state, and, if the temperature of the system is higher than the temperature of the refrigerator, a part of this energy is available. In this case, however, the system, being limited in extent, can expand and contract and can therefore emit or receive energy not only as heat but also in the form of work. For a given process, designated in terms of its end points 1 and 2, the net amount of this energy transfer is equal to  ${}_1Q_2 - A_1W_2$ . It was shown in Chap. II that, although  ${}_1Q_2$  and  ${}_1W_2$  are dependent on the nature of the process connecting the end points, their difference is a function of the state of the system at those end points. Thus, if the total energy possessed by (stored in) the system is designated as  $E_1$  at the beginning and  $E_2$  at the end of the process, we may write

$${}_1Q_2 - A_1W_2 = E_2 - E_1 \quad (5:9)$$

The reader should note that the symbol  $E$  is measured in heat units and includes not only stored thermal energy (internal energy) and potential mechanical energy but also, if the system is in motion, net flow energy and mechanical kinetic energy as well. In fact, if our study of energy transfers had not been limited to interchanges between the forms of thermal and mechanical energy only, it would be necessary to include all other forms of energy stored in the system (as, for instance, chemical energy) in the symbol  $E$ .

Of the total energy transferred to or from the system, a part is available and the balance unavailable. We may therefore write

$${}_1Q_2 - A_1W_2 = E_2 - E_1 = (E_{A_2} - E_{A_1}) + (E_{B_2} - E_{B_1}) \quad (5:10)$$

in which  $E_{A_2} - E_{A_1}$  represents the change of available energy and  $E_{B_2} - E_{B_1}$  the change of unavailable energy of the system during the process. The part of the total transfer which is in the form of work is wholly available since it is already in the form of high-grade energy. The balance,  ${}_1Q_2$ , may be divided into available and unavailable portions by applying a reversible cycle which will operate between the temperature of the system and that of the refrigerator. Since the system is limited in extent, its temperature will often change during the process

and it is necessary to divide the total heat transfer into segments  $dQ$  so small that each may be considered to take place at essentially constant temperature. The refrigerator temperature, on the other hand, can be considered constant. Thus the change of available energy of the system which takes place during the reversible process is

$$E_{A_2} - E_{A_1} = \int_1^2 dQ \left( \frac{T - T_R}{T} \right) - A_1 W_2 \quad (5:11)$$

and the change of unavailable energy is

$$E_{B_2} - E_{B_1} = \int_1^2 dQ \left( \frac{T_R}{T} \right) = T_R \int_1^2 \frac{dQ}{T} \quad (5:12)$$

Therefore the change of entropy of the system during this reversible process may be calculated over a reversible path which connects the two end points as

$$S_2 - S_1 = \frac{E_{B_2}}{T_R} - \frac{E_{B_1}}{T_R} = \int_1^2 \frac{dQ}{T} \quad (5:13)$$

But the change of unavailable energy of the system is, for a given refrigerator temperature, a function of the state of the system (independent of the character of the process which connects the end states), and therefore *any* process, reversible or irreversible, which connects the two end states will show the same entropy change. Thus, in applying Eq. (5:13) in calculating the change of entropy of a given system during a given change in its state, *any* reversible path which connects the two end points may be selected,  $dQ$  expressed in terms of  $T$ , the temperature of the system, along this path, and the indicated integration performed. For instance, if for a gas system these two points could be connected by a constant-pressure path,  $dQ$  could be replaced by its equivalent  $MC_p dT$  and, with  $C_p$  constant, the change of entropy *along any path connecting these two end states* calculated as

$$S_2 - S_1 = MC_p \int_{T_1}^{T_2} \frac{dT}{T}$$

Obviously, if  $P_2$  had not been equal to  $P_1$ , the two end points could not have been connected by a process which took place at constant pressure and some other process, or combination of processes, would necessarily have been selected, with consequent alteration of the foregoing equation.

A review of the argument presented above will indicate that entropy has the following characteristics:

1. The entropy of a system is a function of the state of that system

The total entropy of the system is designated by the symbol  $S$ , the lower case  $s$  being used for specific entropy, or entropy per pound.

2. The change of entropy of a system is an index to the change in its store of unavailable energy; for any process during which the unavailable energy of the system increases, the entropy of the system must increase and vice versa. Although a change in the temperature of the refrigerator will alter proportionately the unavailable energy of the system, no change in entropy will result as long as the system does not change its state; thus its entropy may be used to describe the state of the system without regard to the refrigerator temperature.

3. Entropy, since it is a function of the state of the system, may be used as a coordinate in plotting the state of the system.

4. Entropy is a relative quantity since the expression for the entropy of a system will contain an unknown constant of integration. In the study of thermodynamics, the *change* of entropy alone is of interest, and these constants of integration cancel. When it is convenient to tabulate numerical values to represent the entropy, as in the steam tables, the entropy at some standard condition is assumed to be zero.

5. The characteristics of entropy are the same for all types of substances, whether solids, liquids, gases, or vapors.

6. The total entropy of a system is the sum of the entropies of its parts.

7. The unit of entropy has no generally accepted name but is measured in units of B.t.u. per degree of absolute temperature. It has sometimes been called the rank.

**5:8. Entropy Changes for Gas Systems.** The general method of calculating the change of entropy of a thermodynamic system during a given change in its state has been indicated in the preceding article. For gas systems, a reversible path which connects these two end states having first been selected, the appropriate substitution for  $dQ$  is made, based on the formulas developed in Chap. IV, and the integration indicated in Eq. (5:13) performed.

If the process which connects the two end points is a frictionless adiabatic, a process which has been shown to be reversible,  $dQ$  is zero and

$$S_2 - S_1 = \int_1^2 \frac{dQ}{T} = \int_1^2 \frac{0}{T} = 0 \quad (5:14)$$

Thus the change of entropy is zero for the frictionless adiabatic, and a line on any diagram defining a process which takes place adiabatically and without friction is therefore a line of constant entropy.

The reversible character of the frictionless isothermal has also been demonstrated. In this case, the temperature of the system being con-



stant, a simple division of the total heat flow during the process will serve our purpose. Thus

$$S_2 - S_1 = \int_1^2 \frac{dQ}{T} = \frac{AMRT \log_e (V_2/V_1)}{T} = AMR \log_e \frac{V_2}{V_1} \quad (5:15)$$

*Example A.* Three pounds of air doubles in volume at constant temperature. What is the change of entropy?

*Solution.* Substituting in Eq. (5:15),

$$S_2 - S_1 = AMR \log_e \frac{V_2}{V_1} = \frac{3 \times 53.3 \times \log_e 2}{778} = 0.142$$

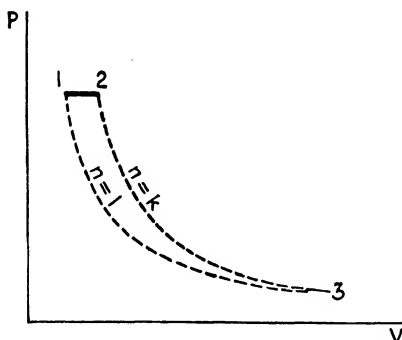


FIG. 5:5.—Change of entropy at constant pressure.

For any other type of change, the state line or path which defines the process may be replaced by an adiabatic and an isothermal and the change of entropy calculated as the sum of the change of entropy during the adiabatic (which is zero) and the change of entropy during the isothermal. For instance, for a change which takes place at constant pressure from the state which is defined by the coordinates  $P_1$ ,  $V_1$ , and  $T_1$  to the state where these quantities are, respectively,  $P_2$ ,  $V_2$ , and  $T_2$  (where  $P_2 = P_1$ ), an isothermal and an adiabatic, meeting at the point  $P_3$ ,  $V_3$ ,  $T_3$ , are substituted as illustrated on  $PV$  coordinates in Fig. 5:5. The change of entropy during the constant-pressure change from 1 to 2 is therefore equal to that taking place along the isothermal from 1 to 3, since the points 2 and 3 are joined by an adiabatic (a line of constant entropy). The change of entropy between 1 and 2 is

$$S_2 - S_1 = S_3 - S_1 = AMR \log_e \frac{V_3}{V_1} = AMR \log_e \frac{P_1}{P_3}$$

and, since

$$\frac{P_1}{P_3} = \frac{P_2}{P_3} = \left(\frac{T_2}{T_3}\right)^{\frac{k}{k-1}} = \left(\frac{T_2}{T_1}\right)^{\frac{k}{k-1}}$$

and

$$\frac{k}{k-1} AR = C_p$$

therefore,

$$\begin{aligned} S_2 - S_1 &= AMR \log_e \left(\frac{T_2}{T_1}\right)^{\frac{k}{k-1}} = \frac{k}{k-1} AMR \log_e \frac{T_2}{T_1} \\ &= MC_p \log_e \frac{T_2}{T_1} \end{aligned} \quad (5:16)$$

The same result may be secured more quickly by substituting the value of  $dQ$  during a constant-pressure process directly in Eq. (5:13).

$$\begin{aligned} S_2 - S_1 &= \int_1^2 \frac{dQ}{T} = MC_p \int_1^2 \frac{dT}{T} \\ &= MC_p \log_e \frac{T_2}{T_1} \end{aligned}$$

*Example B.* Three pounds of air increases in temperature from 40 to 200°F. at constant pressure. What is the change of entropy?

*Solution.* From Eq. (5:16),

$$S_2 - S_1 = MC_p \log_e \frac{T_2}{T_1} = 3 \times 0.2375 \times \log_e \frac{659.7}{499.7} = 0.198$$

If the same methods are applied to a constant-volume change, the change of entropy may be shown to be

$$S_2 - S_1 = \int_1^2 \frac{dQ}{T} = MC_v \int_1^2 \frac{dT}{T} = MC_v \log_e \frac{T_2}{T_1} \quad (5:17)$$

*Example C.* Three pounds of air increases in temperature from 40 to 200°F. at constant volume. What is the change of entropy?

*Solution.* From Eq. (5:17),

$$S_2 - S_1 = MC_v \log_e \frac{T_2}{T_1} = 3 \times 0.169 \times \log_e \frac{659.7}{499.7} = 0.141$$

For a polytropic change, from Art. 4:6,

$$\begin{aligned} dQ &= dU + A dW = MC_v dT + \frac{MAR(-dT)}{n-1} \\ &= \frac{MAR}{k-1} dT - \frac{MAR}{n-1} dT \\ &= \frac{(n-k)MAR}{(k-1)(n-1)} dT \end{aligned}$$

and the change of entropy will be

$$\begin{aligned}
 S_2 - S_1 &= \int_1^2 \frac{dQ}{T} = \frac{(n-k)MAR}{(k-1)(n-1)} \int_1^2 \frac{dT}{T} \\
 &= \frac{(n-k)MAR}{(k-1)(n-1)} \log_e \frac{T_2}{T_1} \\
 &= M \left( \frac{n-k}{n-1} \right) C_v \log_e \frac{T_2}{T_1} \quad (5:18)
 \end{aligned}$$

*Example D.* Three pounds of air increases in temperature from 40 to 200°F. polytropically with  $n = 1.25$ . What is the change of entropy?

*Solution.* From Eq. (5:18),

$$S_2 - S_1 = \frac{(n-k)MAR}{(k-1)(n-1)} \log_e \frac{T_2}{T_1} = \frac{(-0.155) \times 3 \times 53.3 \times 0.2775}{0.405 \times 0.25 \times 778} = -0.0875$$

A general expression for the change of entropy of a perfect-gas system which may be applied when the end points are known and which does not require the preliminary determination of a reversible path to connect these states may be developed as follows: Differentiating Eq. (5:13),

$$dS = \frac{dQ}{T} = \frac{dU}{T} + \frac{AP dV}{T} \quad (5:19)$$

If  $dU$  is replaced by its equivalent  $MC_v dT$  and since  $P/T$  equals  $MR/V$ , we may write

$$dS = MC_v \frac{dT}{T} + MAR \frac{dV}{V} \quad (5:20)$$

Also, since the product  $PV$  must vary with the absolute temperature,

$$dS = MC_v \frac{d(PV)}{PV} + MAR \frac{dV}{V} \quad (5:21)$$

Further, because  $d(PV)$  in the preceding equation is equal to  $P dV + V dP$  and since  $C_v + AR = C_p$ ,

$$dS = MC_p \frac{dV}{V} + MC_v \frac{dP}{P} \quad (5:22)$$

Lastly,  $(MR dT/P) - (V dP/P)$  may be substituted for  $dV$  in Eq. (5:20). Simplifying and combining terms,

$$dS = MC_p \frac{dT}{T} - MAR \frac{dP}{P} \quad (5:23)$$

The detailed demonstration of the methods by means of which Eqs. (5:20) to (5:23) were obtained is left to the reader.

The integration between limits of these five equations makes possible the statement of the corresponding equations as given below:

$$S_2 - S_1 = MC_v \log_e \frac{T_2}{T_1} + MAR \log_e \frac{V_2}{V_1} \quad (5:24)$$

$$S_2 - S_1 = MC_v \log_e \frac{P_2 V_2}{P_1 V_1} + MAR \log_e \frac{V_2}{V_1} \quad (5:25)$$

$$S_2 - S_1 = MC_p \log_e \frac{V_2}{V_1} + MC_v \log_e \frac{P_2}{P_1} \quad (5:26)$$

$$S_2 - S_1 = MC_p \log_e \frac{T_2}{T_1} - MAR \log_e \frac{P_2}{P_1} \quad (5:27)$$

*Example E.* An air system has an initial pressure of 20 p.s.i. abs., a volume of 10 cu. ft., and a temperature of 70°F. At the end of a compression, the pressure has become 60 p.s.i. abs., and the volume is 4 cu. ft. Calculate the change of entropy of the system.

*Solution.* The weight of the system is

$$M = \frac{P_1 V_1}{RT_1} = \frac{(20)(144)(10)}{(53.3)(529.7)} = 1.02 \text{ lb.}$$

Either Eq. (5:25) or Eq. (5:26) may be employed to determine the change of entropy as follows:

$$\begin{aligned} S_2 - S_1 &= (1.02)(0.169) \log_e \frac{(60)(4)}{(20)(10)} + \frac{(1.02)(53.3)}{778} \log_e \frac{4}{10} \\ &= 0.0315 - 0.0640 = -0.0325 \end{aligned} \quad (5:25)$$

$$\begin{aligned} S_2 - S_1 &= (1.02)(0.2375) \log_e \frac{4}{10} + (1.02)(0.169) \log_e \frac{60}{20} \\ &= -0.2220 + 0.1895 = -0.0325 \end{aligned} \quad (5:26)$$

Checking these results by applying Eq. (5:18) along an assumed polytropic (reversible) process connecting the two end states, the value of  $n$  for this polytropic is calculated as in Art. 4:10.

$$n = \frac{\log (P_2/P_1)}{\log (V_1/V_2)} = \frac{\log 3.0}{\log 2.5} = 1.2$$

and, substituting this value in Eq. (5:18),

$$S_2 - S_1 = \frac{(1.02)(-0.205)(0.169)}{0.2} \log_e \left( \frac{60}{20} \right)^{\frac{0.2}{1.2}} = -0.0325$$

**5:9. The Temperature-entropy Diagram.** The amount of heat or work developed or absorbed during a thermodynamic process has been shown to be dependent on the path followed during the process. The amount of work so developed or absorbed can be found by an integration of the area under the state path on  $PV$  coordinates (see Chap. I). It would provide a very useful tool if a similar method could be developed for evaluating the amount of *heat flow*. It has been shown that the temperature and the entropy are both classified as properties of the

system and that therefore, if an equation expressing the pressure in terms of the volume during a specified change of state is available as in the calculation of work amounts, it will follow that it will also be possible to develop a parallel equation representing the absolute temperature in terms of the entropy of the system during that same change of state. The curve representing this relationship can then be plotted on coordinates of absolute temperature and entropy as in Fig. 5:6. Further, if Eq. (5:13) is written in its infinitesimal form

$$dS = \frac{dQ}{T} \quad \text{or} \quad T dS = dQ \quad (5:28)$$

it is immediately observed that the element of area  $T dS$  below this curve

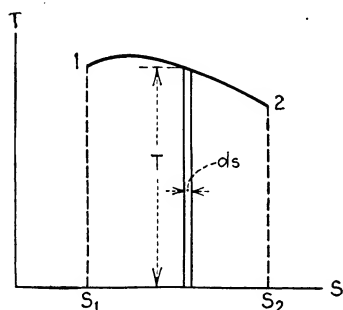


FIG. 5:6.—Significance of area on  $TS$  diagram.

is an element of *heat flow* and that the total heat flow can be measured by a summation of these elementary areas. It must be carefully noted that this is the heat flow from or to an external system only if the system is in equilibrium and if the process takes place without friction; the reason for these qualifications will be found in the method by which Eq. (5:13) was developed. Further, the ordinate of the element of heat flow  $T dS$  must be

measured in terms of the absolute temperature.

In Fig. 5:7 is shown a temperature-entropy diagram on which are drawn curves representing the various types of processes for a gas system which were discussed in the preceding chapter. This figure should be compared with Fig. 4:5, which illustrates these changes on  $PV$  coordinates. The area between the curve and the  $S$  axis will in each instance represent the heat flow during the process if the system is in equilibrium throughout the process and if no friction is present. On  $TS$  coordinates, an adiabatic becomes a vertical and an isothermal a horizontal line. Three polytropics are shown and the same regular progression of values of  $n$  from zero to infinity will be observed as in Fig. 4:5; the limiting curves are still those representing the constant-pressure and constant-volume processes. It will be noted that, for values of  $n$  between 1 and  $k$ , the heat flow is positive (into the system) as the temperature decreases and, for values greater than  $k$  or less than 1, it is negative.

In Fig. 5:8, let the line 1-2 represent any process, reversible or irreversible, which may connect the end points 1 and 2. Note that the

area under this line no longer necessarily represents the heat absorbed during the process. The temperature level  $T_R$  represents the (constant) temperature of the refrigerator. Since, from Eq. (5:13),

$$S_2 - S_1 = \frac{E_{B_2} - E_{B_1}}{T_R}$$

it follows that

$$E_{B_2} - E_{B_1} = T_R(S_2 - S_1) \quad (5:29)$$

and the change of unavailable energy of the system during the process is measured by the area of the rectangle  $abdc$  in Fig. 5:8. The gain or loss of available energy of the system during this change in its state can

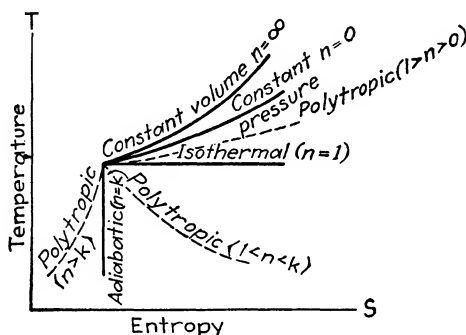


FIG. 5:7.—Gas changes on  $TS$  coordinates.

be calculated by subtracting the energy represented by this area from the total amount by which the stored energy of the system was increased,  ${}_1Q_2 - A_1W_2$ . For a nonflow process,  ${}_1Q_2 - A_1W_2 = U_2 - U_1$ , and therefore

$$E_{A_2} - E_{A_1} = U_2 - U_1 - T_R(S_2 - S_1) \quad (5:30)$$

*Example.* The initial pressure of a system composed of 2 lb. of air is 100 p.s.i. abs., and its temperature is 60°F. At the end of a nonflow expansion, its pressure is 90 p.s.i. abs., and its temperature is 250°F. The temperature of the refrigerator is 40°F. Calculate (a) the change of unavailable energy of the system during the process and (b) the change of available energy of the system during the same period.

*Solution.* (a) Applying Eq. (5:27),

$$\begin{aligned} S_2 - S_1 &= (2)(0.2375) \log_e \frac{709.7}{519.7} - \frac{(2)(53.3)}{778} \log_e \frac{90}{100} \\ &= 0.1482 + 0.0145 = 0.1627 \end{aligned}$$

$$E_{B_2} - E_{B_1} = T_R(S_2 - S_1) = 499.7(0.1627) = 81.2 \text{ B.t.u.}$$

$$\begin{aligned} \text{(b) } E_{A_2} - E_{A_1} &= U_2 - U_1 - T_R(S_2 - S_1) = (2)(0.169)(250 - 60) - 81.2 \\ &= 64.2 - 81.2 = -17.0 \text{ B.t.u.} \end{aligned}$$

Analysis of these results indicates that, although the system received a net energy transfer of 64.2 B.t.u. during the process, it contained 17 B.t.u. less available energy at the end than at the beginning of the change. If this result appears to the reader to be inconsistent with the higher final temperature of the system, it is suggested that he carry out calculations based on an assumed reversible polytropic process which will connect the given end states. The value of  $n$  for this process will be found to be 0.254, the heat flow 99 B.t.u., and the heat equivalent of the external work 34.8 B.t.u.

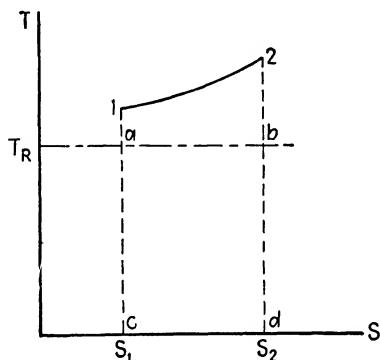


FIG. 5:8.—Change of unavailable energy.

For this assumed process, the net energy transfer of 64.2 B.t.u. into the system has been divided into its components. It should be observed that the energy entering the system was in the form of heat, low-grade energy, and therefore only partly available, while the energy leaving the system was in the form of work and therefore completely available.

The  $TS$  diagram as applied to the Carnot cycle is illustrated in Fig. 5:9.

In the figure, if the cycle operates direct, the area 5126 represents the heat absorbed from the source, area 5436 the heat rejected to the refrigerator, and area 1234 the heat transformed into work during the cycle. Since 2-3 and 4-1 are vertical, the areas are proportional to their heights and the efficiency of the Carnot cycle is

$$e = \frac{AW}{Q_S} = \frac{\text{area } 1234}{\text{area } 5126} = \frac{T_S - T_R}{T_S}$$

Since entropy is defined by the equation

$$dS = \frac{dQ}{T}$$

for a reversible process, without regard to the character of the medium employed, the characteristic appearance of the Carnot cycle on  $TS$  coordinates is the same for all mediums and the typical expression for the efficiency of the Carnot cycle remains unchanged for a system of any composition. As first treated, the efficiency of this cycle was developed based on the use of a perfect gas as the thermodynamic medium. The analysis based on the  $TS$  diagram broadens the application to include all mediums.

With reference to Fig. 5:9, it has been stated above that the enclosed area of the figure represents the heat equivalent of the shaft work developed per cycle. It must be clearly understood that this is true only

because all the processes making up this cycle are frictionless, since the areas under 1-2 and 3-4 measure heat flow only when friction is absent.

In the strict sense, area 1234 even in this diagram represents only the *difference* between the heat added and that abstracted from the system during the cycle; this is *numerically* equal to the net external work delivered.

It now becomes evident that the  $TS$  diagram is of great value in the analysis of an engine cycle since, if all the processes composing the cycle are frictionless (as they are commonly assumed in the theoretical treatment), the amounts of heat added, heat rejected, and the heat equivalent of the shaft work developed will all be represented by proportionate areas. The thermal efficiency of the cycle can thus be determined directly from the  $TS$  diagram as the ratio of the enclosed area of the cycle to the area representing the amount of heat supplied per cycle. This is an advantage not associated with representation on  $PV$  coordinates.

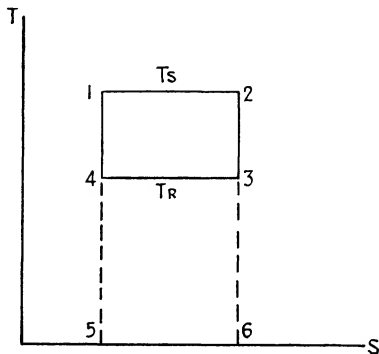


FIG. 5:9.—The Carnot cycle on  $TS$  coordinates.

### Problems

1. Are the following processes reversible or irreversible? Discuss. Neglect heat flow except as specifically stated to take place.

- A weight slides down a frictionless inclined plane.
- A weight slides down a rough inclined plane; it is slowed by friction.
- A perfectly elastic spring is elongated.
- A fluid expands slowly behind a frictionless piston in an insulated cylinder.
- Water is confined under a piston which exerts a constant pressure on its surface. The temperature of the water is raised by stirring.
- The process outlined in (e) is continued until the water evaporates.
- Water confined as in (e) increases in temperature as the result of heat received from a source which is at constant temperature.
- A perfectly elastic ball is dropped through a void, striking a rigid floor.
- The ball of (h) drops through the air.
- A gas system is confined in an insulated cylinder beneath a weighted piston which is supported on a stop. The stop is removed, and the piston drops suddenly until the pressure of the system becomes sufficient to support its weight.
- Steam flows through a frictionless nozzle, decreasing in pressure and increasing in velocity.
- Steam flows through a small orifice from a chamber at high pressure to a chamber at considerably lower pressure.



(m) The steam jet leaving the nozzle in (k) is expelled into a large chamber in which eddies are formed and which it leaves at low velocity.

(n) Air flows through a Venturi tube without friction, the pressure at the throat being lower and the velocity higher than at the entrance to the tube.

2. Prove that all reversible engines must have the same efficiency if they operate between the same (constant) temperatures of source and refrigerator.

3. A Carnot engine receives 100 B.t.u. per minute from a source at  $900^{\circ}\text{F}$ . The refrigerator temperature is  $60^{\circ}\text{F}$ . What horsepower does it develop?

4. A Carnot engine has an efficiency of 40 per cent and rejects 4000 B.t.u. per hour to a refrigerator at  $50^{\circ}\text{F}$ . What is the temperature of the source? What horsepower does the engine develop?

5. A reversible engine receives 150 B.t.u. per minute from a source at  $1000^{\circ}\text{F}$ . and has an efficiency of 65 per cent. What horsepower does it develop? What is the temperature of the refrigerator?

6. A Carnot engine utilizes a perfect gas as the thermodynamic medium. At the beginning of the isothermal expansion the pressure is 120 p.s.i. abs., and the volume is 1 cu. ft. At the end of the adiabatic expansion the pressure is 20 p.s.i. abs., and the volume is 3 cu. ft. What is its efficiency?

7. A reversible engine discharges 100 B.t.u. per minute to a refrigerator at  $70^{\circ}\text{F}$ . It develops 2 hp. What is the temperature of its source?

8. A reversible engine receives 50,000 B.t.u. per hour from a source at  $800^{\circ}\text{F}$ . and develops 11 hp. What is the temperature of its refrigerator?

9. A Carnot engine contains 0.4 lb. of air. At the beginning of the isothermal expansion, the pressure is 100 p.s.i. abs., and the volume is 2 cu. ft. At the end of the isothermal expansion, the volume has doubled. The refrigerator temperature is  $120^{\circ}\text{F}$ . What is the efficiency of the engine? What net work will it deliver per cycle? What are the pressure and the volume at the end of the adiabatic expansion?

10. The efficiency of a reversible engine which receives heat from a source at  $1000^{\circ}\text{F}$ . is 60 per cent. What is the effect on its efficiency if the temperature of the source is raised  $100^{\circ}\text{F}$ .? What effect on efficiency results if the source remains at  $1000^{\circ}\text{F}$ . but the refrigerator temperature is lowered by  $100^{\circ}\text{F}$ .? What factors limit the possibility of increasing the temperature of the source? Lowering the temperature of the refrigerator?

11. What percentage of the heat taken from a source at  $900^{\circ}\text{F}$ . is *available* if the temperature of the atmosphere is  $80^{\circ}\text{F}$ .?

12. A reversed Carnot engine removes 400 B.t.u. per minute from a refrigerator at  $0^{\circ}\text{F}$ . At what rate will heat be rejected to a hot body at  $100^{\circ}\text{F}$ .? What horsepower will be required to operate the machine?

13. A reversible engine is operated as a heat pump and requires 10 hp. to remove heat from a cold body at  $10^{\circ}\text{F}$ . and discharge heat to a hot body at  $80^{\circ}\text{F}$ . At what rate is heat removed from the cold body? Discharged to the hot body?

14. A building that requires 150,000 B.t.u. per hour to maintain its temperature at  $70^{\circ}\text{F}$ . when the outside temperature is  $20^{\circ}\text{F}$ . is to be heated by means of a reversed Carnot engine. What horsepower will be required to drive it? At what rate is heat removed from the outside air?

15. A Carnot engine receives heat from a source at  $900^{\circ}\text{F}$ . and rejects part of it to a building at a temperature of  $70^{\circ}\text{F}$ . This engine drives a reversed Carnot engine which operates between the outside air temperature of  $10^{\circ}\text{F}$ . and the building temperature of  $70^{\circ}\text{F}$ . The heat required to keep the building at  $70^{\circ}\text{F}$ . (300,000 B.t.u.

per hour) comes from two sources, the exhaust of the direct-acting and the exhaust of the reversed Carnot engine. What horsepower does the direct-acting engine develop and the reversed engine absorb? How much heat leaves the 900°F. source every hour? How much heat is removed from the atmosphere per hour? Is any violation of the second law involved?

16. It requires 714 B.t.u. to evaporate a pound of water at a constant temperature of 500°F. This heat is radiated from a boiler furnace in which a temperature of 2400°F. is maintained. The temperature of the atmosphere is 80°F.

(a) Divide the energy leaving the furnace into available and unavailable portions.

(b) What are the available and unavailable portions of this energy after it has been stored in the steam?

(c) What is the change of entropy of the hot gases in the furnace?

(d) What is the change of entropy of the water as it became steam?

(e) What is the change of entropy of the combined system?

(f) What is the change of unavailable energy during the process calculated as the product of  $T_R \Delta S$ . Compare with the result as obtained from (a) and (b).

17. In a feed-water heater, steam condenses at a constant temperature of 220°F. The feed water is heated from 60 to 150°F. at constant pressure.

(a) Assuming the specific heat at constant pressure of the feed water to be unity, how many B.t.u. are absorbed by each pound in its passage through the heater?

(b) What is the change of entropy of the condensing steam per pound of feed water heated?

(c) What is the change of entropy of 1 lb. of the feed water as it passes through the heater?

(d) What is the change of entropy of the combined system?

(e) What is the change of unavailable energy of the combined system based on a refrigerator temperature of 40°F.?

18. In an air preheater, the temperature of the combustion air is increased at constant pressure from 80 to 385°F., and the flue-gas temperature decreases from 650 to 410°F.

(a) Assuming its specific heat to be 0.24, how much heat is added to each pound of combustion air?

(b) What is the change of specific entropy of the air?

(c) Assuming the specific heat of the flue gas to be 0.24, what is the change of entropy of the flue gas per pound of combustion air?

19. The volume of 2 lb. of air is doubled at constant pressure. What is the change of entropy of the air system?

20. The temperature of 3 lb. of air decreases from 200 to 70°F. at constant pressure. What is the change of entropy of the air?

21. During a constant-pressure process, the entropy of a 5-lb. gas system decreases by 0.7. The temperature drops from 350 to 60°F. during the process. What is the (constant) specific heat of the gas?

22. A gas system is compressed frictionlessly at a constant temperature of 200°F. To accomplish the compression, 120,000 ft.-lb. of work is expended. What is the change of entropy of the system?

23. A gas system expands at a constant temperature of 200°F. from an initial pressure of 70 p.s.i. abs. and volume of 7 cu. ft. to a final volume of 10 cu. ft. What is the change of entropy of the system?

24. Two pounds of air undergoes an isothermal process, during which the pressure increases from 20 to 45 p.s.i. abs. What is the change of entropy of the system?

25. The temperature of 3 lb. of air contained in a closed tank drops from 210 to 90°F. What is the change of entropy of the air?

26. The pressure of 5 lb. of oxygen which is stored in a closed tank increases from 30 to 42 p.s.i. gage. Barometric pressure is standard. What is the change of entropy of the oxygen?

27. The temperature of a perfect-gas system increases from 120 to 400°F. as 350 B.t.u. of heat enters the system during a constant-volume process. What is the change of entropy?

28. No change of entropy takes place as a system consisting of a monatomic perfect gas undergoes a reversible process. If the original pressure is 15 p.s.i. abs., the original volume is 10 cu. ft., and the final pressure is 40 p.s.i. abs., what is the volume at the end of the process?

29. Two pounds of air expands polytropically with  $n = 1.45$  from an original pressure of 70 p.s.i. abs. to a final pressure of 20 p.s.i. abs. What is the change of entropy of the system?

30. During a polytropic compression with  $n = 1.32$ , the volume of 2 lb. of air is halved. What is the change of entropy of the air?

31. The temperature of 3 lb. of carbon dioxide decreases from 250 to 30°F. during a polytropic process with  $n = 1.25$ . What is the change of entropy of the system?

32. During a reversible process which is represented by a straight line on  $T$ - $S$  coordinates, the temperature decreases from 300 to 120°F. while the entropy increases by 0.12. Is heat added to or removed from the system during the process? What is the heat flow?

33. At the beginning of a process, the temperature and volume of a system composed of 4 lb. of air are 110°F. and 52 cu. ft., respectively. The final values are 230°F. and 13 cu. ft., respectively. What is the change of entropy of the system?

34. The initial pressure and volume of a system composed of 2 lb. of carbon dioxide are 140 p.s.i. abs. and 2.5 cu. ft., respectively. At the end of a certain change, the final values are 20 p.s.i. abs. and 12 cu. ft., respectively. What is the change of entropy of the system?

35. A system composed of 3 lb. of oxygen changes irreversibly from an original pressure of 30 p.s.i. abs. and temperature of 170°F. to a final condition at which the pressure is 45 p.s.i. abs. and the temperature 220°F. What is the change of entropy of the system?

36. A certain process is represented on  $T$ - $S$  coordinates by a straight line. During the process the temperature decreases from 400 to 100°F., and the decrease of entropy of the system is 0.15. The temperature of the refrigerator is 60°F. What is the change of unavailable energy of the system during the process? If the process is reversible, what is the heat flow which accompanies it?

37. The temperature of 3 lb. of air is increased from 90 to 320°F. at constant volume. The temperature of the atmosphere is 70°F. What is the change of unavailable energy for the system during the process? The change of available energy?

38. The temperature of 2 lb. of oxygen decreases from 280 to 110°F. during a constant-pressure process. The temperature of the refrigerator is 60°F. What is the change of unavailable energy of the system during the process? What is the change of available energy of the system?

39. During a reversible expansion of a perfect-gas system which takes place at a

TABLE 5:1.—SUMMARY OF NONFLOW FRICTIONLESS PROCESSES FOR GAS SYSTEMS

Type of process	Value of exponent $n$	Final pressure $P_2$	Final temperature $T_2$	Final volume $V_2$	Heat added to system $Q_2$	Change of internal energy $U_2 - U_1$	Thermal equivalent of work performed by system $A/(W_2)$	Change of entropy of system $S_2 - S_1$
Constant pressure	0	$P_1$	$T_1 \left( \frac{V_2}{V_1} \right)$	$V_1 \left( \frac{T_2}{T_1} \right)$	$MC_p(T_2 - T_1)^*$	$MC_p(T_2 - T_1)^*$ or $A(P_2V_2 - P_1V_1)/k - 1$	$AP(V_2 - V_1)$	$MC_p \log_e \frac{T_2}{T_1}$ or $MC_p \log_e \frac{V_2}{V_1}$
Constant volume	$\infty$	$P_1 \left( \frac{T_2}{T_1} \right)$	$T_1 \left( \frac{P_2}{P_1} \right)$	$V_1$	$MC_v(T_2 - T_1)^*$	$MC_v(T_2 - T_1)^*$ or $A(P_2V_2 - P_1V_1)/k - 1$	0	$MC_v \log_e \frac{T_2}{T_1}$ or $MC_v \log_e \frac{P_2}{P_1}$
Constant temperature —isothermal	1	$P_1 \left( \frac{V_1}{V_2} \right)$	$T_1$	$V_1 \left( \frac{P_1}{P_2} \right)$	$AP_1V_1 \log_e \frac{V_2}{V_1}$ <i>Note:</i> $MRT$ may be substituted for $P_1V_1$ or $P_2V_2$ for $V_2/V_1$ in expression above if more convenient	0	$AP_1V_1 \log_e \frac{V_2}{V_1}$ <i>Note:</i> $MRT$ may be substituted for $P_1V_1$ or $P_2V_2$ for $V_2/V_1$ in expression above if more convenient	$MAR \log_e \frac{T_2}{T_1}$ or $MAR \log_e \frac{P_2}{P_1}$
Reversible adiabatic —isentropic	$k$	$P_1 \left( \frac{V_1}{V_2} \right)^k$ or $P_1 \left( \frac{T_2}{T_1} \right)^{\frac{k}{k-1}}$	$T_1 \left( \frac{V_1}{V_2} \right)^{k-1}$ or $T_1 \left( \frac{P_2}{P_1} \right)^{\frac{k}{k-1}}$	$V_1 \left( \frac{P_1}{P_2} \right)^{\frac{1}{k-1}}$ or $V_1 \left( \frac{T_1}{T_2} \right)^{\frac{1}{k-1}}$	0	$MC_v(T_2 - T_1)^*$ or $A(P_2V_2 - P_1V_1)/k - 1$	$MC_v(T_1 - T_2)^*$ or $A(P_1V_1 - P_2V_2)/k - 1$	0
Polytropic	Any constant value	$P_1 \left( \frac{V_1}{V_2} \right)^n$ or $P_1 \left( \frac{T_2}{T_1} \right)^{\frac{n}{n-1}}$	$T_1 \left( \frac{V_1}{V_2} \right)^{n-1}$ or $T_1 \left( \frac{P_2}{P_1} \right)^{\frac{n}{n-1}}$	$V_1 \left( \frac{P_1}{P_2} \right)^{\frac{1}{n-1}}$ or $V_1 \left( \frac{T_1}{T_2} \right)^{\frac{1}{n-1}}$	$M \left( \frac{n-k}{n-1} \right) C_v (T_2 - T_1)^*$	$MC_v(T_2 - T_1)^*$ or $A(P_2V_2 - P_1V_1)/k - 1$	$MAR(T_1 - T_2)^*$ or $A(P_1V_1 - P_2V_2)/k - 1$	$M \left( \frac{n-k}{n-1} \right) C_v \log_e \frac{T_2}{T_1}$ <i>Note:</i> $(V_1/V_2)^{n-1}$ or $(P_2/P_1)^{\frac{n-1}{n}}$ may be substituted for $T_2/T_1$ in expression above if more convenient

\* Based on the assumption that specific heat is constant.

constant temperature of  $600^{\circ}\text{F}$ ., 75,000 ft.-lb. of external work is performed. The temperature of the atmosphere is  $40^{\circ}\text{F}$ . What is the change of unavailable energy of the system? What is the change of available energy of the system?

40. A Carnot engine uses steam (a vapor) as the thermodynamic medium. One thousand B.t.u. per minute is supplied by a source at  $500^{\circ}\text{F}$ . The temperature of the refrigerator is  $120^{\circ}\text{F}$ . What is the efficiency of the engine? What horsepower does it develop?

## CHAPTER VI

### HEAT-ENGINE CYCLES—GAS SYSTEMS

**6:1. Analysis of Heat-engine Cycles.** The analysis of all heat-engine cycles is directed toward substitution in the efficiency equation in either of its two forms,  $e = AW/Q_S$  or  $e = (Q_S - Q_R)/Q_S$  in which  $Q_S$  is understood as representing the heat entering the system during the cycle from an external source or system, and  $Q_R$  as the heat rejected to an external system. Some cycles, such as the Stirling and the Ericsson, employ a regenerative principle so that heat rejected during one portion of the cycle is restored during some later portion without affecting any external system; the quantity  $Q_S$ , as defined above, would not include heat added under such conditions, nor would  $Q_R$  include heat rejected to a regenerator in this manner. Another example is heat flow to and from the water jacket of an internal combustion engine throughout the cycle. In the latter case, the heat flow has the effect of slightly changing the conformation of the cycle (*e.g.*, for a change which would otherwise be adiabatic the exponent  $n$  of the curve has a value somewhat different from  $k$ , the adiabatic value) and, ultimately, of reducing somewhat the efficiency of the cycle.

In the Carnot cycle, the weight and composition of the charge remained constant throughout the cycle. Thus the Carnot cycle was both the cycle of the condition of the *working fluid* and that of the *machine* in which these changes of condition took place. In the internal-combustion engine, both the weight and the composition of the charge vary greatly throughout the cycle, and the cycle can be considered a machine cycle only.

The analysis of a gas cycle, taking into account all the various factors such as variable specific heat, variable weight and composition of the gas system throughout the cycle, the time element necessary for explosion, and the effect of heat flow to and from the water jacket, is a very difficult and laborious problem thermodynamically and is not within the scope of this text. The methods for making such analyses may be found in advanced works on thermodynamics or the internal-combustion engine. However, the efficiency of an actual engine may be readily calculated

as the result of tests upon the engine by dividing the heat equivalent of the work performed during the test by the heat value of the fuel consumed.

**6:2. The ideal, or air-standard, efficiency** of the cycle under consideration is based upon the assumptions of constant specific heat, constant weight and composition of the gas system which acts as the medium in the manufacture of work from heat (the values for air are used), and instantaneous (constant-volume) addition of heat during an explosion. It is also assumed that the head of the cylinder which contains the system can be made either perfectly conducting during an isothermal change or perfectly nonconducting for an adiabatic change. The process of combustion which actually takes place is replaced by the simple addition of heat in equivalent quantities. The removal of the products of combustion from the cylinder ("exhaust," in the cycle of the actual engine) is similarly replaced by the extraction of heat sufficient to bring about the required change of state. The air-standard efficiency may be very easily calculated thermodynamically in advance of the construction of the engine and indicates the maximum conceivable efficiency for an engine operating on that cycle. The actual engine cannot attain so high an efficiency as the air standard because of the factors enumerated above, but the ratio between the air-standard efficiencies of two cycles will serve as a valuable guide in the comparison of the two cycles. Also, the ratio of the actual efficiency of the completed engine to the air-standard efficiency of the cycle on which it operates is of some assistance in measuring the ratio of actual to perfect performance for that engine. This ratio will usually be about one-half to two-thirds; if it is much less than the lower figure, it is possible that improvement in design or operating conditions may be made.

**6:3. The Otto cycle**, proposed in 1862 by Beau de Roches, is shown in Fig. 6:2. A schematic diagram, showing the components of an internal-combustion-engine power plant operating on the Otto cycle, is presented in Fig. 6:1. The plant and cycle illustrated are of the four-stroke-cycle type in which four strokes of the piston (or two revolutions of the engine shaft) are required to complete the cycle; two-stroke-cycle operation is also possible by certain changes in the position of the valves and their method of operation and in the method of introducing the fuel mixture into the cylinder. Thermodynamically, as will be explained below, the two- and four-stroke cycles are analyzed in an identical manner.

Referring to Fig. 6:1, the downward motion of the piston on the suction stroke, with intake valve open, draws in a charge of air through the in-

take line. On its way to the cylinder, a charge of fuel is drawn into this stream of air in the carburetor or mixing valve. This fuel may be either a volatile liquid, such as gasoline, or a fuel gas, such as natural gas; if the former, a carburetor is employed; if the latter, a simple mixing valve. During the remainder of its passage to the cylinder, the fuel and air become thoroughly mixed.

At the proper point in the cycle which takes place within the cylinder, as described below, a spark is introduced into this mixture by an electrical system consisting of a battery and coil, or magneto, a timer, or distrib-

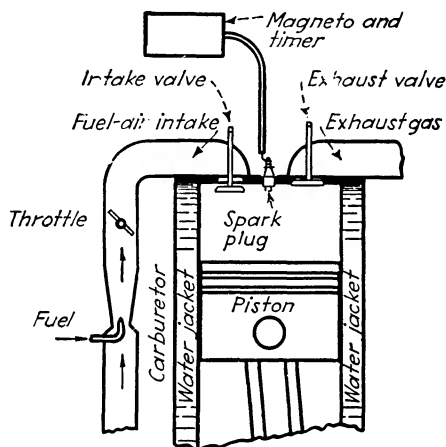


FIG. 6:1.—Internal-combustion engine—Otto cycle.

utor, and a spark plug set into the top of the cylinder. Since all the fuel and all the air necessary to its combustion are present, an *explosion* takes place. After the cycle has been completed within the cylinder, the burned products of combustion move out into the atmosphere through the exhaust valve, and a new charge enters the cylinder. The reciprocating motion of the piston is converted into rotary motion by means of the connecting rod and crank, and some sort of flywheel is required to store energy during the single power stroke and return it during the balance of the cycle.

The water jacket shown here may be replaced by thin fins for direct air cooling. Cooling is necessary to protect the materials of which the cylinder and piston are fabricated at the high temperatures developed within the cylinder; it is not necessary or even desirable in a strictly thermodynamic sense. The difficulty of properly cooling the piston accounts for the prevalence of the single-acting engine in internal-combustion design. The engine is governed by changing the position



of the throttle valve at the top of the carburetor; this changes the weight of fuel mixture taken into the cylinder.

The detailed operations that take place within the cylinder are outlined below with reference to Fig. 6:2. They comprise

1. A suction stroke, 5-1, during which a mixture of air and fuel is drawn into the cylinder at constant pressure.

2. A compression stroke, 1-2, during which the charge is compressed adiabatically.

3. The ignition and consequent explosion of the charge, 2-3, occurring at constant volume (instantaneously), and causing the temperature and pressure to rise owing to the addition of thermal energy developed by combustion.

4. An expansion stroke, 3-4, providing for the adiabatic expansion of the burned gases.

5. Rejection of the burned gases from the cylinder at constant volume, 4-1, and constant pressure, 1-5, and consequent return to the starting point of the cycle.

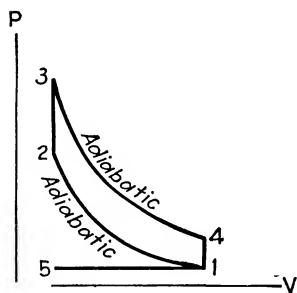


FIG. 6:2.—Otto cycle.

This cycle may be completed in two strokes of the piston instead of the four outlined above by providing for rejection of the burned gases and injection of a new charge at the end of the expansion and the beginning of the compression strokes. In actual engines operating on the Otto cycle, certain practical considerations, such as weight, size, cost, simplicity, and fuel economy, may cause preference to be given either the four-stroke or the two-stroke cycle.

In the air-standard Otto cycle, the charge is air alone, the combustion process 2-3 is replaced by the addition of heat in sufficient amount to bring about this rise in pressure at constant volume, and the pressure drop 4-1 is accomplished by the extraction of heat in appropriate amount. Since the weight and composition of the charge does not change, the exhaust stroke 1-5 and the suction stroke 5-1 are not necessary and may be omitted; the air-standard cycle is therefore a two-stroke cycle, but the limiting efficiencies developed will apply equally as well to four-stroke operation.

Examination of the air-standard Otto cycle will show that heat enters from the source only during the constant-volume process 2-3 and is rejected to the refrigerator only during the process 4-1, also at constant volume. The air-standard efficiency may be calculated as follows:

$$Q_S = MC_v(T_3 - T_2) \quad (6:1)$$

$$Q_R = MC_v(T_4 - T_1) \quad (6:2)$$

$$\begin{aligned} e &= \frac{Q_S - Q_R}{Q_S} = \frac{MC_v(T_3 - T_2) - MC_v(T_4 - T_1)}{MC_v(T_3 - T_2)} \\ &= 1 - \frac{T_4 - T_1}{T_3 - T_2} \end{aligned} \quad (6:3)$$

But

$$\frac{T_1}{T_2} = \left( \frac{V_2}{V_1} \right)^{k-1} = \frac{T_4}{T_3}$$

and, by taking advantage of the mathematical properties of a proportion, it may be shown that

$$\frac{T_4 - T_1}{T_3 - T_2} = \frac{T_1}{T_2}$$

and

$$e = 1 - \frac{T_1}{T_2} \quad (6:4)$$

Substituting equivalent expressions for  $T_1/T_2$  in terms of pressure and volume, the following equations may also be written:

$$e = 1 - \left( \frac{V_2}{V_1} \right)^{k-1} \quad (6:5)$$

$$e = 1 - \left( \frac{P_1}{P_2} \right)^{\frac{k-1}{k}} \quad (6:6)$$

The ratio  $V_1/V_2$  is called the *compression ratio* of the engine. It will be noted that the air-standard efficiency depends solely on the compression ratio for this cycle and that it increases as the compression ratio increases. In practice, the compression ratio is limited by the ignition characteristics of the fuel used. During the compression stroke, not only the pressure but also the temperature rises. If the temperature increases above the ignition temperature of the fuel mixture which is under compression, a premature explosion will take place. For this reason, practical values of the compression ratio are dependent on the fuel used and seldom exceed 7.

The ratio of the volume  $V_2$  to the volume  $V_1 - V_2$  swept through by the piston (the *piston displacement*) is known as the *clearance* of the engine.

$$\text{Clearance, per cent} = \frac{V_2 \times 100}{V_1 - V_2} \quad (6:7)$$

The mean effective pressure (m.e.p.) of the cycle is the average net pressure acting on the piston throughout the cycle and is represented in Fig. 6:2 by the average vertical distance between the upper and lower lines of the diagram. It may be calculated by dividing the area of the cycle as plotted on  $PV$  coordinates by its length. The area of the diagram is equal to the work performed during the cycle and therefore to  $J(Q_S - Q_R)$ ; the length is represented by  $V_1 - V_2$ , the piston displacement in cubic feet. When the work is expressed in foot-pounds and the piston displacement in cubic feet, the result will be in pounds per square foot.

$$\text{M.e.p.} = \frac{W_{\text{cycle}}}{V_1 - V_2} = \frac{J(Q_S - Q_R)}{V_1 - V_2} \quad (6:8)$$

The Otto cycle is represented on  $TS$  coordinates in Fig. 6:3. The area  $a23b$  represents the heat  $Q_S$  taken from the source during the cycle, and the area  $a14b$  the heat  $Q_R$  rejected to the refrigerator. Their difference is the heat equivalent of the net work performed and, in this case, is represented by the enclosed area of the cycle (area 1234). Note that it has here been necessary to introduce the concept of a variable temperature source (varying between  $T_2$  and  $T_3$ ) and a variable-temperature refrigerator (between  $T_4$  and  $T_1$ ) in order to include reversibility as one of the characteristics of the cycle.

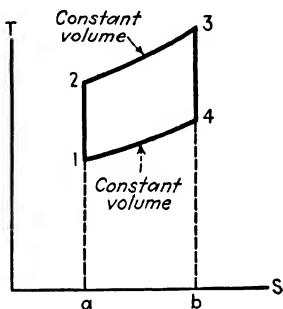


FIG. 6:3.—Otto air-standard cycle.

*Example.* An Otto cycle has a temperature of 90°F., a pressure of 14 p.s.i. abs., and a volume of 10 cu. ft. at the beginning of the compression stroke. At the end of compression the pressure is 140 p.s.i. abs., and 100 B.t.u. is added during the explosion. Calculate the pressures, temperatures, and volumes throughout the cycle, the percentage of clearance, compression ratio, efficiency, net work, and mean effective pressure of the air-standard cycle.

*Solution.* The weight of air contained in the cylinder is

$$M_1 = \frac{P_1 V_1}{RT_1} = \frac{14 \times 14 \times 10}{53.3 \times 549.7} = 0.688 \text{ lb.}$$

The volume at the end of compression (clearance volume) is

$$V_2 = V_3 = V_1 \left( \frac{P_1}{P_2} \right)^{\frac{1}{k}} = 10 \left( \frac{14}{140} \right)^{\frac{1}{1.405}} = 10 \times 0.1945 = 1.945 \text{ cu. ft.}$$

The temperature at the end of compression is

$$T_2 = T_1 \left( \frac{P_2}{P_1} \right)^{\frac{k-1}{k}} = 549.7 \left( \frac{140}{14} \right)^{0.388} = 549.7 \times 1.941 = 1068^\circ\text{F. abs. or } 608^\circ\text{F.}$$

or, by another method,

$$T_2 = \frac{P_2 V_2}{MR} = \frac{144 \times 140 \times 1.945}{0.688 \times 53.3} = 1068^\circ\text{F. abs.}$$

The increase of temperature during the explosion is

$$T_3 - T_2 = \frac{Q_3}{MC_v} = \frac{100}{0.688 \times 0.169} = 860^\circ$$

and, therefore,

$$T_3 = T_2 + 860 = 1,038 + 860 = 1928^\circ\text{F. abs. or } 1468^\circ\text{F.}$$

Since  $T_1/T_2 = T_4/T_3$ , the temperature at the end of the adiabatic expansion is

$$T_4 = T_3 \left( \frac{T_1}{T_2} \right) = 1,928 \left( \frac{549.7}{1,068} \right) = 993^\circ\text{F. abs. or } 533^\circ\text{F.}$$

The pressure at the end of explosion (a constant-volume process) is

$$P_3 = P_2 \left( \frac{T_3}{T_2} \right) = 140 \left( \frac{1,928}{1,068} \right) = 253 \text{ p.s.i. abs.}$$

and, since  $P_4/P_3 = P_1/P_2$ ,

$$P_4 = P_3 \left( \frac{P_1}{P_2} \right) = 253 \left( \frac{14}{140} \right) = 25.3 \text{ p.s.i. abs.}$$

$$\text{Clearance} = \frac{V_2 \times 100}{V_1 - V_2} = \frac{1.945 \times 100}{10 - 1.945} = 24.1 \text{ per cent}$$

$$\text{Compression ratio} = \frac{V_1}{V_2} = \frac{10}{1.945} = 5.14$$

$$\text{Efficiency} = 1 - \frac{T_1}{T_2} = 1 - \frac{549.7}{1,068} = 0.485 \text{ or } 48.5 \text{ per cent}$$

$$\text{Net work per cycle} = e \times Q_s \times 778 = 0.485 \times 100 \times 778 = 37,750 \text{ ft.-lb.}$$

$$\text{M.e.p.} = \frac{W}{V_1 - V_2} = \frac{37,750}{10 - 1.945} = 4,680 \text{ p.s.f. or } 32.5 \text{ p.s.i.}$$

**6:4. The Diesel engine** is diagramed in Fig. 6:4, and the cycle on which it operates is illustrated on  $PV$  and  $TS$  coordinates in Fig. 6:5. In this engine, the charge drawn into the cylinder through the intake valve is air alone, and the fuel is injected mechanically only after this charge of air has been compressed. The practical limit, which was, in the case of the Otto cycle, placed upon the compression ratio by the kindling temperature of the fuel, no longer applies, and this ratio is limited only by the practical limits of temperature and pressure that the cylinder can withstand. Indeed, the limit on compression ratio now becomes a lower limit, since the temperature of the compressed charge

of air is relied upon to ignite the fuel as it enters the cylinder. Compression ratios used in practice are about 15:1.

The fuel is forced into the cylinder under high pressure due to the action of a *fuel-injection pump*, driven from the engine shaft, and enters through a *fuel-injection nozzle*, designed to atomize it thoroughly as it

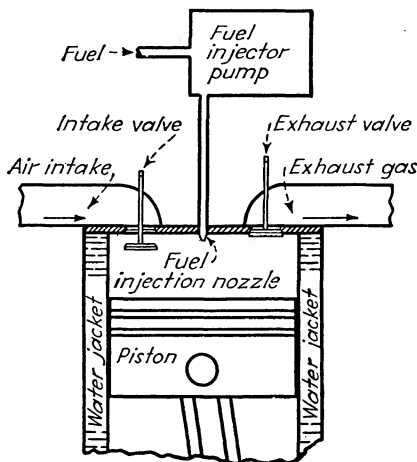


FIG. 6:4.—Internal-combustion engine—Diesel cycle.

enters the cylinder head. It burns progressively as it enters and comes in contact with the heated air in the cylinder. The rate of delivery must be very carefully proportioned to the change of volume of the charge as the piston moves downward on its power stroke if the pressure is to remain even approximately constant during the period of combustion. Governing is effected by designing the fuel-injection system so that the time period over which injection takes place can be varied either manually or automatically. The valves and valve-operating mechanism may be quite similar to Otto-cycle engine practice.

Referring to Fig. 6:5, which is presented for a two-stroke-cycle engine, the successive operations carried out in the cycle of the ideal Diesel engine are as follows:

1. The adiabatic compression of a charge of air to a high pressure and temperature, represented by the line 1-2.

2. The injection of the fuel and its combustion (note that this is not an explosion) at a rate sufficient to maintain a constant pressure within the cylinder, as shown by the line 2-3. At point 3 the fuel supply is cut off.

3. Adiabatic expansion of the products of combustion takes place, 3-4, to the end of the stroke.

4. The burned gases are rejected, 4-1, at a constant volume.

The same difference exists between the two-stroke and four-stroke cycles as in the Otto cycle.

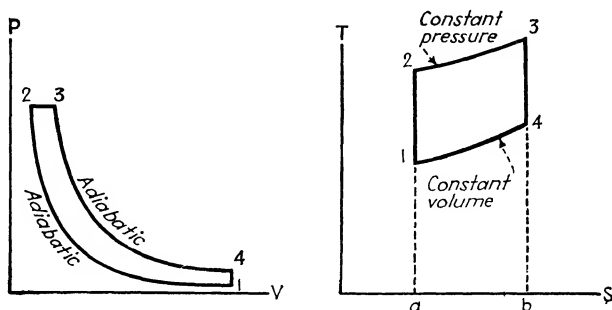


FIG. 6:5.—Diesel air-standard cycle.

The air-standard efficiency of the Diesel cycle may be calculated by the following steps:

$$Q_S = MC_p(T_3 - T_2) \quad (6:9)$$

$$Q_R = MC_v(T_4 - T_1) \quad (6:10)$$

$$\begin{aligned} e &= \frac{Q_S - Q_R}{Q_S} = \frac{MC_p(T_3 - T_2) - MC_v(T_4 - T_1)}{MC_p(T_3 - T_2)} \\ &= 1 - \frac{1}{k} \left( \frac{T_4 - T_1}{T_3 - T_2} \right) \end{aligned} \quad (6:11)$$

The methods by which the temperatures, pressures, and volumes at all points of a specific cycle may be calculated have been outlined in Chap. IV. The same definitions of compression ratio, clearance, piston displacement, and mean effective pressure apply as for the Otto cycle. Another ratio of importance for this cycle is the percentage of cutoff, which is expressed as

$$\text{Cutoff, per cent} = \frac{V_3 - V_2}{V_1 - V_2} \times 100$$

The effect of a variation in the percentage of cutoff on the efficiency of the Diesel cycle may be shown as follows:

$$e = 1 - \left( \frac{T_1}{T_2} \right) \frac{[(T_4/T_1) - 1]}{k[(T_3/T_2) - 1]}$$

and, since  $T_3/T_2 = V_3/V_2$ ,

$$T_4 = T_3 \left( \frac{V_3}{V_4} \right)^{k-1}, \quad T_1 = T_2 \left( \frac{V_2}{V_1} \right)^{k-1}, \quad \text{and} \quad V_4 = V_1$$

therefore

$$\frac{T_4}{T_1} = \frac{T_3(V_3/V_4)^{k-1}}{T_2(V_2/V_1)^{k-1}} = \left( \frac{V_3}{V_2} \right) \left( \frac{V_3}{V_2} \right)^{k-1} = \left( \frac{V_3}{V_2} \right)^k$$

Substituting these values in the efficiency expression,

$$e = 1 - \left( \frac{V_2}{V_1} \right)^{k-1} \frac{[(V_3/V_2)^k - 1]}{k(V_3/V_2 - 1)} \quad (6:12)$$

*Example.* For the Diesel cycle illustrated in Fig. 6:5, let  $p_1 = 14$  p.s.i. abs.,  $t_1 = 90^\circ\text{F}$ .,  $V_1 = 10$  cu. ft.,  $V_2 = 0.8$  cu. ft., and  $t_3 = 2100^\circ\text{F}$ . Calculate the pressures and temperatures throughout the cycle, the heat added and rejected, the efficiency, the net work, clearance percentage, compression ratio, cutoff percentage, and the mean effective pressure of the air-standard cycle.

*Solution.*

$$M = \frac{P_1 V_1}{RT_1} = \frac{144 \times 14 \times 10}{53.3 \times 549.7} = 0.688 \text{ lb.}$$

$$p_2 = p_1 \left( \frac{V_1}{V_2} \right)^k = 14 \left( \frac{10}{0.8} \right)^{1.405} = 14 \times 34.8 = 487 \text{ p.s.i. abs.}$$

$$T_2 = T_1 \left( \frac{V_1}{V_2} \right)^{k-1} = 549.7 \left( \frac{10}{0.8} \right)^{0.405} = 549.7 \times 2.78 = 1530^\circ\text{F. abs. or } 1070^\circ\text{F.}$$

$$p_3 = p_2 = 487 \text{ p.s.i. abs.}$$

$$V_3 = V_2 \left( \frac{T_3}{T_2} \right) = 0.8 \left( \frac{2,100 + 459.7}{1,530} \right) = 1.34 \text{ cu. ft.}$$

$$V_4 = V_1 = 10 \text{ cu. ft.}$$

$$T_4 = T_3 \left( \frac{V_3}{V_4} \right)^{k-1} = 2,559.7 \left( \frac{1.34}{10} \right)^{0.405} = 2,559.7 \times 0.443 = 1135^\circ\text{F. abs. or } 675^\circ\text{F.}$$

$$p_4 = \frac{MRT_4}{144V_4} = \frac{0.688 \times 53.3 \times 1,135}{144 \times 10} = 28.9 \text{ p.s.i. abs.}$$

$$Q_S = MC_p(T_3 - T_2) = 0.688 \times 0.2375 \times (2,559.7 - 1,530) = 168.2 \text{ B.t.u.}$$

$$Q_R = MC_v(T_4 - T_1) = 0.688 \times 0.169 \times (1,135 - 549.7) = 68.0 \text{ B.t.u.}$$

$$\text{Efficiency} = \frac{Q_S - Q_R}{Q_S} = \frac{168.2 - 68.0}{168.2} = 0.595 \text{ or } 59.5 \text{ per cent}$$

$$\text{Net work} = J(Q_S - Q_R) = 778(168.2 - 68.0) = 78,000 \text{ ft.-lb.}$$

$$\text{Clearance} = \frac{V_2 \times 100}{V_1 - V_2} = \frac{0.8 \times 100}{10 - 0.8} = 8.7 \text{ per cent}$$

$$\text{Compression ratio} = \frac{V_1}{V_2} = \frac{10}{0.8} = 12.5$$

$$\text{Cutoff} = \frac{V_3 - V_2}{V_1 - V_2} = \frac{1.34 - 0.8}{10 - 0.8} = 0.0587 \text{ or } 5.87 \text{ per cent}$$

$$\text{M.e.p.} = \frac{W}{V_1 - V_2} = \frac{78,000}{10 - 0.8} = 8,480 \text{ p.s.f. or } 58.8 \text{ p.s.i.}$$

A variation of the Diesel cycle, to which considerable importance attaches because of its use in practice, is found in the Dual (or combination) cycle illustrated in Fig. 6:6. This cycle is followed when the injection of fuel begins slightly earlier than in the Diesel, just before the end of the compression stroke. The effect of the earlier portion of the combustion is to raise the pressure at nearly constant volume, since the piston is practically at a standstill; combustion continues through a portion of the power stroke as in the Diesel. It is suggested that the student derive an expression for the efficiency of the Dual cycle in terms of the temperatures at the points 1, 2, 3, 4, p and 5 by applying the methods employed in Arts. 6:3 and 6:4.

#### 6:5. Comparison of the Otto and Diesel Cycles.

The efficiency of the Diesel cycle increases theoretically not only with increasing compression ratio but also with decreasing cutoff percentage. The limiting value for a given compression ratio is, of course, reached when the cutoff becomes zero and the work done and heat added both equal zero. The compression ratio of the Diesel engine is not limited by the ignition temperature of the fuel since fuel does not enter the cylinder until after compression has been completed. The inherent advantage of the Diesel cycle over the Otto cycle operating between equal temperature limits, as far as efficiency is concerned, is illustrated in Fig. 6:7, in which  $12_034$  represents the Otto cycle and  $12_p34$  the Diesel cycle. The efficiency of the Otto cycle is the ratio of area  $2_0341$  to area  $a2_03b$  and that of the Diesel cycle is the ratio of area  $2_p341$  to area  $a2_p3b$ . Since the second ratio is evidently greater than the first and since the upper and lower limits of temperature are the same for both cycles, the Diesel approaches more closely the efficiency of the ideal reversible (Carnot) cycle.

Comparison of Eq. (6:5) with Eq. (6:12) will show that, *for the same compression ratio*, the Otto cycle will have a higher air-standard efficiency than the Diesel. Balanced against this is the fact that the actual Diesel engine may operate at and is designed for considerably higher compression ratio than it is possible to utilize in Otto-cycle-engine practice. This factor gives a net practical advantage in attainable operating efficiency to the Diesel.

**6:6. Cycles of Historical Interest.** Although the Otto and Diesel cycles are of the greatest present interest, other cycles are of importance either for historical reasons and their place in the development of the

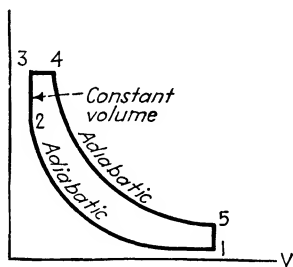


Fig. 6:6.—Dual cycle.



present types or because they seem to offer possible improvement in performance if the principles involved can be applied in practice.

The Lenoir engine, operating on the cycle illustrated in Fig. 6:8, was the first successful practical internal-combustion engine. The charge, which was a mixture of fuel and air, was drawn into the cylinder (1-2) during the first part of the forward stroke of the piston, exploded (2-3) without previous compression, expanded adiabatically for the rest of the forward stroke (3-4), and the burned gases were rejected (4-1) during the return stroke. The partial strokes 1-2 and 2-1 may be disregarded

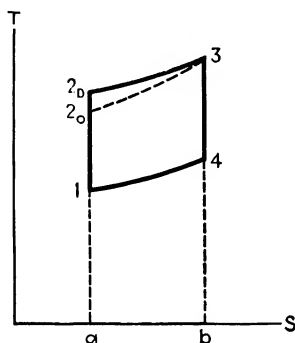


FIG. 6:7.—Comparison of Diesel and Otto cycles.

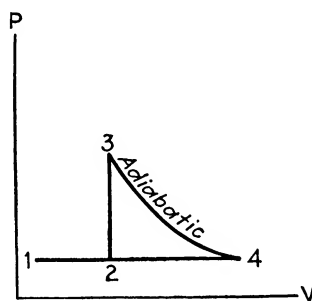


FIG. 6:8.—The Lenoir cycle.

since, in the air-standard cycle, they take place entirely below the refrigerator temperature, and the efficiency is

$$e = 1 - k \left( \frac{T_4 - T_2}{T_3 - T_2} \right) \quad (6:13)$$

This cycle was abandoned when Otto demonstrated that higher efficiencies could be obtained by the introduction of a compression stroke before explosion.

The hot-air-engine cycles are illustrated in Fig. 6:9. Both these cycles employ regenerators for the purpose of restoring the heat rejected between 2 and 3 to the cycle again as it passes from point 4 to point 1. In the Stirling cycle, the regenerative process takes place at constant volume, in the Ericsson at constant pressure. Considering that all the heat rejected from 2 to 3 can be restored between 4 and 1, which, of course, is possible, even theoretically, only as a limiting condition, the efficiencies of both these cycles become  $(T_s - T_R)/T_s$  or equal to the Carnot efficiency. Engines of both these types have been constructed and, indeed, are in use in very small units at the present time, but the

practical difficulties in regeneration and in application of the isothermal process cause the actual efficiencies to be extremely low. Engines of this type are very large compared with their output rating. Both types employ two cylinders and pistons, a working cylinder and a displacer cylinder. They are of little practical importance.

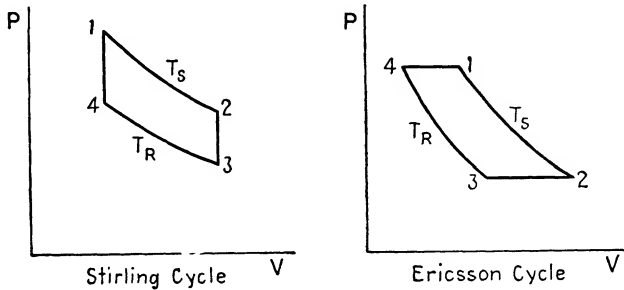


FIG. 6:9.

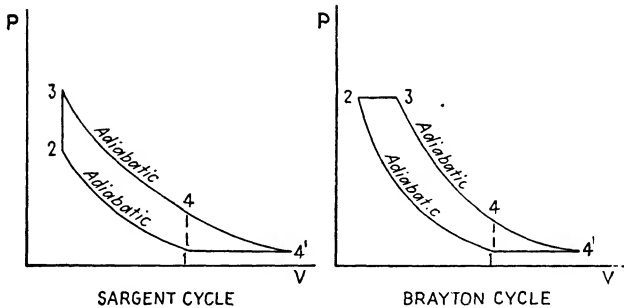


FIG. 6:10.

**6:7. Proposed Cycles.** It will be observed that, if the adiabatic expansion could be prolonged in either the Otto or the Diesel cycle until the pressure  $p_1$  were reached as shown in Fig. 6:10, the amount of work performed would be increased by the triangular area  $44'1$  at no additional expenditure of heat added to the cycle. The application of this principle to the Otto cycle is known as the Sargent cycle, and the application to the Diesel cycle is called the Brayton cycle. The Sargent cycle has not as yet been successfully applied to a practical engine. The Brayton is the cycle on which the gas turbine is based. Its application cannot yet be said to be completely successful, for reasons to be discussed below, but considerable effort is being devoted to its development. Units operating on this cycle have passed out of the strictly experimental

stage and have a number of advantages under certain special conditions of operation.

The general features of the design of a Brayton-cycle power plant are diagrammed in Fig. 6:11 and can best be understood by a comparison of this figure with Fig. 6:12, which shows the work areas associated with the compressor and turbine, the two major units. Atmospheric air is drawn continuously into an axial-flow (rotary) air compressor; the induction of this air is represented in Fig. 6:12 by the line  $a-1$ . The air is then compressed adiabatically to condition 2 (line 1-2 in Fig. 6:12)

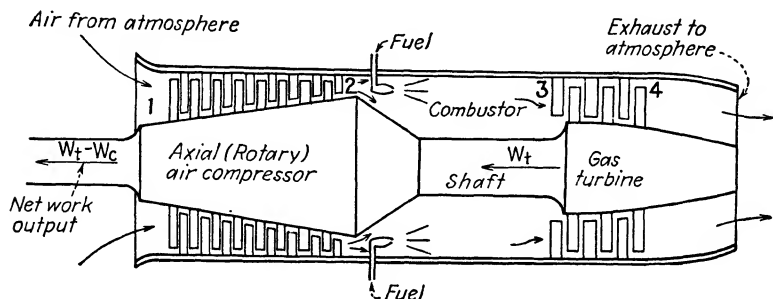


FIG. 6:11.—Schematic diagram of gas-turbine power plant.

and rejected at this elevated pressure and temperature into a receiver (between the compressor and turbine of Fig. 6:11). The work theoretically required to accomplish this compression, for the weight of air defined by the properties  $P_1$ ,  $V_1$ , and  $T_1$ , is represented by the area 12ba of Fig. 6:12, as will be shown in Chaps. VII and X.

As the air moves toward the turbine unit, fuel is injected and burns, increasing the volume to  $V_3$  at constant pressure. The fuel supplied need not be volatile and the combustion may be spontaneous as in the Diesel since air alone is compressed and the temperature  $T_2$  can be above the kindling temperature of the fuel. The induction of the hot gases into the turbine is represented by the line  $b-3$ , their expansion in that unit by  $3-4$ , and their ejection at atmospheric pressure by  $4-a$ . The ideal work output of the turbine,  $W_t$ , is represented by the area  $b34a$ ; a part,  $W_c$ , of this work is utilized to drive the compressor, and the net work  $W_t - W_c$  is therefore represented by the area 1234a of the Brayton cycle. This excess may be used to drive a generator, a propeller, or for any desired purpose and is transmitted along the compressor-turbine shaft as indicated in the diagram.

The air-standard efficiency of the Brayton cycle can be shown to be, like that of the Otto cycle, dependent only on the compression ratio and

equal to  $1 - (T_1/T_2)$ ; it is suggested that the reader carry out the calculation necessary to prove this. Moreover, since air alone is compressed as has been shown above, the compression ratio is not limited by the danger of preignition; in theory, therefore, it should be possible to construct a Brayton-cycle plant to operate at higher compression ratio and consequently higher efficiency than can be attained through the use of an engine based on the Otto cycle. A further advantage rests in the fact that the flow through a rotary compressor and gas turbine is steady and continuous, producing a constant flow of power to the shaft instead of the periodic impulses associated with the cycle applications previously discussed in this chapter.

In practice, the efficiency of the gas-turbine power plant is as yet considerably below (less than half) that characteristic of the engine based on the Otto cycle. The net work output is, in either case, the difference between the (positive) work performed as the hot gases

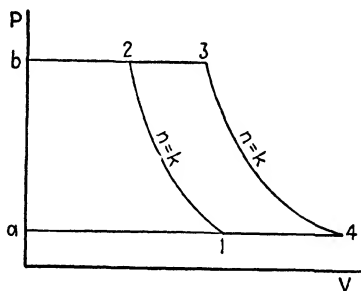


FIG. 6:12.—Work areas—Brayton cycle.

expand and the (negative) work required for the compression of the charge. In the Otto engine, the maximum temperature of the gases attained due to combustion may exceed 3000°F. without damage to the metal parts of the engine since temperatures of this order exist over only a small part of the cycle and the temperature of the cylinder walls and piston can be kept far below this level. Some parts of the gas-turbine unit, on the other hand, are *continuously* subjected to the highest temperatures reached during the cycle, and no materials are at present commercially available which will both resist corrosion and retain the necessary strength when the temperature of the hot gases is even as high as 2000°F. Thus the increase of temperature during combustion in the gas-turbine plant must be held to a much smaller value than during explosion in the Otto engine.

The principal effect of this limitation on the maximum temperature is to reduce the size of the net work area in comparison with the work required for compression of the charge and the gross work delivered by the turbine unit. If the compressor and turbine operated without loss, the efficiency of the plant, since it depends only on the compression ratio, would not be affected. Such effects as mechanical friction and turbulence, however, increase the work required for compression a minimum of about 15 per cent above the theoretical and account for a reduction

of similar percentage in the work output of the turbine. These losses are both at the direct expense of the net work area enclosed by the Brayton cycle. Thus the reduction in relative size of this area which is forced by limiting maximum temperatures to 2000°F. or less means that these losses are relatively more important than in the Otto-cycle engine, and the gas-turbine plant is at a disadvantage when presently attainable efficiencies are compared with those of Otto-cycle engines. It is obvious that we may confidently expect the present interest in this type of power plant to result in the development of metals more suitable for its construction than any at present available and a consequent steady improvement in efficiency. Other phases of investigation which are being carried on concurrently with this metallurgic research include the development of axial-flow compressors and gas turbines which will give better performance.

It will be realized that there are many situations where fuel economy is less important, owing to a surplus of fuel which would otherwise be wasted, than other features such as compactness and simplicity of design, the ability to use a nonvolatile fuel, and low maintenance. This will account for some of the interest in the gas turbine in the present stage of its development. A special application to the airplane power plant, where fuel economy is certainly of major importance, has directed considerable attention to the gas turbine recently. In this field, reciprocating engines operating on the Otto or Diesel cycles have been employed, their power output being used to drive a propeller which, in turn, supplies the "thrust" necessary to propel the airplane. As airplane design has progressed, airplane speeds have increased until, at present, speeds in level flight above 400 m.p.h. are not uncommon; above this speed the propeller becomes very inefficient, and the use of some other method of providing thrust is desirable. If, in Fig. 6:12, the expansion in the gas turbine (along line 3-4) is halted at some intermediate point, so chosen that the power provided by the turbine will be just sufficient to drive the compressor, the gases as they leave the turbine will have a pressure and temperature well above that of the atmosphere. Their velocity can then be greatly increased as they are ejected at the rear of the airplane through a properly designed converging channel (see Chap. X for the design of nozzles). This increases the momentum of the gases and results in a reaction in the opposite direction (a *forward-acting* force on the airplane) which provides the necessary thrust. Moreover, the efficiency of this method of providing thrust, while low at moderate airplane speeds, can be shown to increase as those speeds increase. The greater fuel consumption is principally a detriment not because of cost

but because of the greater weight of fuel which must be carried, but partial compensation is effected since a considerable saving in power-plant weight can be made. Thus, for *ultra-high-speed airplanes*, the combination of the Brayton (gas-turbine) cycle with “jet propulsion” has definite advantages in removing one of the barricades which blocks the progress of airplane design.

Jet propulsion is also used in combination with propeller drive for airplanes. In this case the expansion in the turbine is allowed to proceed beyond the point at which power just sufficient to operate the compressor unit is available at the shaft of the turbine, and the excess shaft power so developed is used to drive a conventional propeller through suitable gearing. The energy remaining in the hot exhaust gases, although not as great as before, is still sufficient to provide considerable thrust as they are ejected to the rear of the airplane. This combination gives better take-off, low-speed, and low-altitude performance than jet propulsion alone, at the expense of performance at high speeds and high altitudes. The field of application of this variant of jet propulsion at present seems to be airplanes designed to travel at top speeds of 400 to 600 m.p.h.

#### Problems

1. Calculate the air-standard efficiency of an Otto cycle with a compression ratio of 4.5; with a compression ratio of 6.

2. If the temperature at the beginning of compression is  $80^{\circ}\text{F.}$ , what is the temperature at the end of compression in an air-standard Otto cycle if the compression ratio is 4.5? If the compression ratio is 6?

3. An Otto cycle has a clearance of 24 per cent. What is its compression ratio? What is its air-standard efficiency?

4. The pressure at the beginning of the compression stroke in an air-standard Otto cycle is 14 p.s.i. abs. At the end of this stroke, it is 130 p.s.i. abs. What is the compression ratio? What is the clearance percentage? What is the air-standard efficiency of the cycle?

5. At the beginning of compression, the temperature in an air-standard Otto cycle is  $70^{\circ}\text{F.}$  and, at its end,  $600^{\circ}\text{F.}$  What is the air-standard efficiency? What is the compression ratio? The clearance percentage?

6. Discuss the effects of the following practical factors on the efficiency and performance of the actual Otto-cycle engine as compared with air-standard performance. Illustrate the effects of these factors where possible by comparing the indicator card taken from an actual engine with the equivalent air-standard diagram.

(a) Variable specific heats, increasing with temperature

(b) The water jacket

(c) Time lag in explosion

(d) Inertia of gases

(e) Friction

(f) Excess air and incomplete combustion

7. An air-standard Otto cycle contains 1 lb. of air,  $t_1 = 90^{\circ}\text{F.}$ ,  $p_1 = 14$  p.s.i. abs.,  $p_2 = 140$  p.s.i. abs., and  $t_3 = 2300^{\circ}\text{F.}$  Calculate the heat added, heat rejected,

and the net work of this cycle. What are the air-standard efficiency, clearance percentage, compression ratio, and mean effective pressure of this cycle?

8. At the beginning of compression in an air-standard Otto cycle,  $V_1 = 1$  cu. ft.,  $t_1 = 100^\circ\text{F.}$ ,  $p_1 = 15$  p.s.i. abs. The temperature at the end of compression,  $t_2$ , is  $650^\circ\text{F.}$ , and the explosion raises the pressure  $p_3$  to 300 p.s.i. abs. What are the heat added, heat rejected, net work, air-standard efficiency, mean effective pressure, clearance, and compression ratio for this cycle?

9. An air-standard Otto cycle with a compression ratio of 6 has  $V_1 = 2$  cu. ft.,  $t_1 = 90^\circ\text{F.}$ ,  $p_1 = 14.5$  p.s.i. abs.,  $t_3 = 2500^\circ\text{F.}$  Calculate the heat added, the heat rejected, the net work, air-standard efficiency, mean effective pressure, and clearance of the cycle.

10. Show that, if the compression ratio is constant, the air-standard efficiency of the Diesel cycle will decrease with increasing percentages of cutoff.

11. For an air-standard Diesel cycle containing 1 lb. of air,  $p_1 = 14.5$  p.s.i. abs.,  $t_1 = 90^\circ\text{F.}$ ,  $p_2 = 575$  p.s.i. abs.,  $t_3 = 2500^\circ\text{F.}$  Calculate the heat added, heat rejected, net work, air-standard efficiency, clearance per cent, cutoff per cent, compression ratio, and mean effective pressure for this cycle.

12. For an air-standard Diesel cycle with a compression ratio of 15,  $p_1 = 14$  p.s.i. abs.,  $V_1 = 2$  cu. ft.,  $t_1 = 100^\circ\text{F.}$ ,  $V_3 = 0.25$  cu. ft. Calculate the heat added, heat rejected, net work, air-standard efficiency, clearance per cent, cutoff per cent, and mean effective pressure for this cycle.

13. For an air-standard Diesel cycle,  $p_1 = 14.2$  p.s.i. abs.,  $V_1 = 1$  cu. ft.,  $t_1 = 100^\circ\text{F.}$ ,  $t_2 = 1250^\circ\text{F.}$ ,  $t_3 = 2600^\circ\text{F.}$  Calculate the heat added, heat rejected, net work, air-standard efficiency, clearance per cent, cutoff per cent, compression ratio, and mean effective pressure of this cycle.

14. For an air-standard Diesel cycle which uses 1 lb. of air,  $t_1 = 110^\circ\text{F.}$ ,  $t_2 = 1300^\circ\text{F.}$ , and 250 B.t.u. of heat is added during combustion. Calculate the heat rejected, net work, compression ratio, clearance per cent, cutoff per cent and air-standard efficiency of this cycle.

15. A Diesel cycle has a cutoff of 5 per cent. What is its air-standard efficiency if the compression ratio is 10? If it is 15?

16. A Diesel cycle has a compression ratio of 15. Calculate the air-standard efficiency if the cutoff is 5 per cent; if it is 10 per cent.

17. Calculate the air-standard efficiencies of Diesel cycles having a compression ratio of 10 and cutoffs of 15, 10, and 5 per cent. What is the efficiency of an air-standard Otto cycle having a compression ratio of 10? Plot efficiencies as ordinates against cutoff percentages as abscissas. Toward what limit does the air-standard efficiency of the Diesel cycle approach as its cutoff approaches zero?

18. Derive an expression for the air-standard efficiency of the Dual cycle in terms of the temperatures.

19. An air-standard Dual cycle uses 1 lb. of air and has a compression ratio of 12. For this cycle,  $p_1 = 14.5$  p.s.i. abs.,  $t_1 = 80^\circ\text{F.}$ ,  $t_3 = 2100^\circ\text{F.}$ ,  $t_4 = 2800^\circ\text{F.}$  Calculate the heat added, heat rejected, net work, air-standard efficiency, clearance per cent, cutoff per cent, and mean effective pressure of this cycle.

20. For an air-standard Dual cycle,  $p_1 = 14$  p.s.i. abs.,  $t_1 = 90^\circ\text{F.}$ ,  $V_1 = 2$  cu. ft.,  $V_2 = 0.15$  cu. ft. During combustion, 20 B.t.u. is added at constant volume and 30 B.t.u. at constant pressure. What are the pressure, temperature, and volume of the charge at the end of the adiabatic expansion? Calculate the heat rejected, net work, air-standard efficiency, compression ratio, clearance per cent, cutoff per cent, and mean effective pressure of this cycle.

21. Compare, using  $TS$  diagram, Otto and Dual cycles having the same compression ratio and the same (total) change of entropy of the charge during combustion. State your conclusions. Toward what limit does the efficiency of the Dual cycle approach as its cutoff approaches zero?

22. Derive Eq. (6:13) for the efficiency of the Lenoir cycle.

23. Compare, using the  $TS$  diagram, Otto and Lenoir cycles having the same change of entropy of the charge during explosion. State your conclusions.

24. At the beginning of the explosion in an air-standard Lenoir cycle which uses 1 lb. of air, the pressure is 14.5 p.s.i. abs., and the temperature is  $100^{\circ}\text{F}$ . During explosion, 150 B.t.u. is added to the air. What is the air-standard efficiency of the cycle?

25. For an air-standard Lenoir cycle,  $P_2 = 15$  p.s.i. abs.,  $t_2 = 80^{\circ}\text{F}$ .,  $p_3 = 35$  p.s.i. abs. What is the efficiency of the cycle?

26. Show that the efficiency of the Stirling cycle, assuming complete regeneration, is the same as that of the Carnot cycle.

27. Plot the Stirling and Ericsson cycles on  $TS$  coordinates.

28. Derive an expression for the efficiency of the air-standard Sargent cycle in terms of the temperatures. Show that the efficiency indicated by this expression will be higher than for the Otto cycle of the same compression ratio. What factor, in addition to compression ratio, determines Sargent-cycle efficiency?

29. For an air-standard Sargent cycle containing 1 lb. of air,  $p_1 = 13.5$  p.s.i. abs.,  $t_1 = 80^{\circ}\text{F}$ .,  $V_2 = 3$  cu. ft. Calculate the air-standard efficiency of the cycle if (a) 250 B.t.u., (b) 150 B.t.u., and (c) 50 B.t.u. of heat are added to the charge during explosion. Compare with the efficiency of an air-standard Otto cycle of the same compression ratio. State your conclusions.

30. For a Sargent cycle,  $p_1 = 14$  p.s.i. abs.,  $t_1 = 110^{\circ}\text{F}$ .,  $V_1 = 2$  cu. ft.,  $p_3 = 300$  p.s.i. abs., and the compression ratio is 5. Calculate the heat added, heat rejected, net work, air-standard efficiency, and mean effective pressure of this cycle. What is the efficiency of the "equivalent" air-standard Otto cycle?

31. Compare, using the  $TS$  diagram, Otto and Sargent cycles having the same compression ratio and the same change of entropy during the addition of heat. State your conclusions.

32. Derive an expression for the air-standard efficiency of the Brayton cycle in terms of the temperatures. Show that this expression can be simplified to indicate that the efficiency is dependent only on the compression ratio.

33. Compare, using the  $TS$  diagram, Diesel and Brayton cycles having the same compression ratio and the same change of entropy of the charge during combustion. State your conclusions as based on this diagram.

34. At the beginning of compression in an air-standard Brayton cycle, the pressure is 14.2 p.s.i. abs. and the temperature  $100^{\circ}\text{F}$ . At the end of compression, the pressure is 200 p.s.i. abs., and the temperature at the end of the combustion is  $2000^{\circ}\text{F}$ . How much heat is added, how much heat is rejected, and what is the net work per pound of air passing through the cycle? What is the air-standard efficiency? The compression ratio?

35. In an air-standard Brayton cycle, the absolute pressure at discharge from the compressor is eight times that at the suction end of this unit. What is its air-standard efficiency?

36. During compression in an air-standard Brayton cycle, the temperature of the air is raised from 90 to  $600^{\circ}\text{F}$ . What is the air-standard efficiency of the cycle?



## CHAPTER VII

### AIR COMPRESSION

**7:1. The Air Compressor.** Compressed air may be used as a source of power and, since it may be transmitted from a central plant over relatively long distances without excessive loss of energy, is of considerable practical value. The action of the piston type of air compressor is diagrammed in Fig. 7:1, in which, for simplicity, the compressor is represented as single-acting. Both intake and exhaust valves are automatic in their action and are opened and closed by differences in the air pressure acting on their opposite faces; when the pressures are balanced, they are

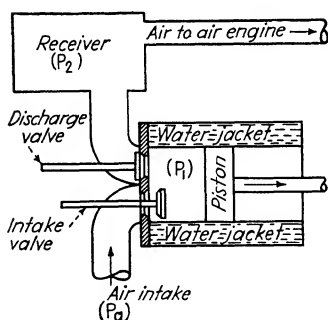


FIG. 7:1.—Piston air compressor.

kept closed by light springs (not shown in the diagram). In operation, the pressure of the atmosphere acts on the outside of the intake valve; that of the air at receiver (discharge) pressure, on the outside of the discharge valve. Thus, if the pressure within the cylinder is caused to fall below atmospheric pressure by an amount sufficient to overcome the small resistance offered by the valve spring plus the resistance to flow offered by the intake passage, the intake valve will open and air will be drawn from the atmosphere into the cylinder. Similarly, if, as the result of piston action, the pressure of the charge within the cylinder reaches a value sufficiently above the receiver pressure to overcome the resistance of the spring on the discharge valve plus exhaust-passage resistance, this valve will open and discharge will take place from the cylinder into the receiver.

In the position shown, the piston is moving to the right on its *suction* stroke. Referring to the *PV* diagram (Fig. 7:2), which has been drawn for a compressor with no clearance, a charge of air will be drawn into the cylinder at constant pressure  $P_1$ ; this pressure, as explained above, will be slightly below that of the atmosphere from which the flow takes place, and atmospheric pressure is represented by a dashed line in this diagram.

At point 1, the piston reaches the end of its suction stroke and reverses

its direction of motion. The intake valve immediately closes, since the pressure in the cylinder is no longer low enough to keep it open, and *compression* of the charge proceeds along line 1-2. At point 2, the pressure  $P_2$  within the cylinder has become slightly higher than the receiver pressure, and this differential of pressure is sufficient to open the discharge valve; the remainder of the stroke of the piston as it continues to move to the left is concerned with the *delivery* of the compressed charge to the receiver. As the piston reaches the left end of the cylinder and again starts on its suction stroke, it will be observed that the discharge valve will close, the intake valve will open in response to the lowered pressure within the cylinder, and the cycle of operation of the compressor will have been completed.

Figure 7:2 is a theoretical indicator diagram for the air compressor with no clearance. Referring to the discussion of Chap. I, the net work required for compression *and delivery* of the charge is represented by the area enclosed within the diagram or, in this case, the area between the compression curve 1-2 and the  $P$  axis. Integration of the elementary areas  $V dP$  between the limits 1 and 2 along this curve will therefore permit the calculation of the amount of shaft work which must be supplied by the driving engine or motor. However, it will be observed that this work must be negative if the conventions used to this point in the text are to be observed, whereas the area developed mathematically as the curve is traversed from point 1 to point 2 is positive. Therefore we write

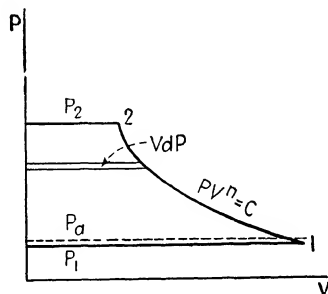


FIG. 7:2.—Cycle of an air compressor.

$$\text{Work per cycle} = W = - \int_{P_1}^{P_2} V dP = \int_{P_2}^{P_1} V dP \quad (7:1)$$

Necessarily, the amount of work expended will depend on the nature of the compression curve. If the compression occurs rapidly in a non-conducting cylinder, it will be practically adiabatic; if carried out slowly with proper provision, such as a water jacket, for the removal of heat from the cylinder, it will approach the isothermal. In Fig. 7:3, compression curve  $A$  represents an adiabatic,  $B$  an isothermal compression, and  $C$  shows the usual compromise between these two limiting conditions which applies in the actual compressor. Curve  $C$  is a polytropic with  $n$  equal to about 1.3 for the usual water-jacketed compressor cylinder.

It will be observed that the ideal condition from the standpoint of minimum work expended is the isothermal; the least desirable is the adiabatic. The saving in work resulting from isothermal as compared with adiabatic compression is represented by the area bounded by the

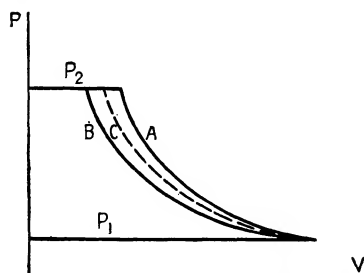


FIG. 7:3.—Comparison of adiabatic and isothermal air-compressor diagrams.

curves *A* and *B* and the constant-pressure line  $P_2$ . The water jacket about the cylinder causes an approach toward isothermal compression and a partial saving in the amount of work required for compression. Thus the water jacket, which was used for practical purposes only in the case of the internal-combustion engine, is shown to have a thermodynamic advantage in the case of the air compressor.

The total work per cycle required for isothermal compression is, since  $PV = C = P_1V_1$  along an isothermal and thus  $V = C/P = P_1V_1/P$ ,

$$\begin{aligned} W &= \int_{P_2}^{P_1} V \, dP = P_1V_1 \int_{P_2}^{P_1} \frac{dP}{P} = P_1V_1 \log_e \frac{P_1}{P_2} \\ &= MRT_1 \log_e \frac{P_1}{P_2} \end{aligned} \quad (7:2)$$

Since  $P_1$  is less than  $P_2$ , the result will be negative, indicating work expended on the air during the compression cycle.

For adiabatic compression, the equation of the compression curve is

$$PV^k = C = P_1V_1^k = P_2V_2^k \quad \text{or} \quad V = \frac{C^{\frac{1}{k}}}{P^{\frac{1}{k}}} = \frac{P_1^{\frac{1}{k}}V_1}{P^{\frac{1}{k}}} = \frac{P_2^{\frac{1}{k}}V_2}{P^{\frac{1}{k}}}$$

and the work expended during the cycle,

$$\begin{aligned} W &= \int_{P_2}^{P_1} V \, dP = C^{\frac{1}{k}} \int_{P_2}^{P_1} \frac{dP}{P^{\frac{1}{k}}} = C^{\frac{1}{k}} \frac{P_1}{P_2} \left\{ \left[ \frac{1}{1 - (1/k)} \right] P^{1 - \frac{1}{k}} \right\} \\ &= \left( \frac{k}{k-1} \right) C^{\frac{1}{k}} \left( P_1^{1 - \frac{1}{k}} - P_2^{1 - \frac{1}{k}} \right) \\ &= \frac{k}{k-1} \left[ P_1^{\frac{1}{k}} V_1 (P_1)^{1 - \frac{1}{k}} - P_2^{\frac{1}{k}} V_2 (P_2)^{1 - \frac{1}{k}} \right] \\ &= \frac{k}{k-1} (P_1V_1 - P_2V_2) \end{aligned} \quad (7:3)$$

The reader will note that this expression represents the foot-pound equivalent of the enthalpy increase between points 1 and 2 on the compression curve [cf. Eq. 4:29]. Equation (7:3) may be expressed in more convenient form by writing its equivalent

$$W = \frac{k}{k-1} MR(T_1 - T_2) \quad (7:4)$$

and, since

$$\begin{aligned} \frac{T_2}{T_1} &= \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} \\ W &= \frac{k}{k-1} MRT_1 \left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}\right] \\ &= \frac{k}{k-1} P_1 V_1 \left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}\right] \end{aligned} \quad (7:5)$$

*Example A.* Find the theoretical horsepower necessary for the isothermal compression of 800 cu. ft. of air per minute from 14.7 to 120 p.s.i. abs.

*Solution.*

$$\begin{aligned} \text{Hp.} &= \frac{\text{work per minute}}{33,000} = \frac{P_1 V_1 \log_e (P_1/P_2)}{33,000} = \frac{144 \times 14.7 \times 800 \times \log_e (14.7/120)}{33,000} \\ &= 51.35 \times (-2.10) = -107.9 \text{ hp.} \end{aligned}$$

The negative sign indicates that this horsepower must be expended on the air.

*Example B.* If the compression in Example A had been adiabatic, what horsepower would have been required?

*Solution.*

$$\begin{aligned} \text{Hp.} &= \frac{k}{k-1} \frac{P_1 V_1}{33,000} \left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}\right] \\ &= \frac{1.405 \times 144 \times 14.7 \times 800}{0.405 \times 33,000} \left[1 - \left(\frac{120}{14.7}\right)^{\frac{0.405}{1.405}}\right] \\ &= 178 (1 - 1.83) = -147.7 \text{ hp.} \end{aligned}$$

If the compression is polytropic with  $n$  having any value except 1 (the value for an isothermal), the work required is found by substitution in an expression equivalent to Eq. (7:5), replacing  $k$  by  $n$ . In this case

$$W = \frac{n}{n-1} P_1 V_1 \left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}\right] \quad (7:6)$$

*Example C.* What saving of power could be effected by proper waterjacketing of the cylinder of the air compressor of Example B, assuming that the exponent of the compression curve is reduced to 1.3?

*Solution.*

$$\begin{aligned}\text{Hp.} &= \frac{n}{n-1} \frac{P_1 V_1}{33,000} \left[ 1 - \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{n}} \right] \\ &= \frac{1.3 \times 144 \times 14.7 \times 800}{0.3 \times 33,000} \left[ 1 - \left( \frac{120}{14.7} \right)^{\frac{0.3}{1.3}} \right] \\ &= 222.5(1 - 1.623) = -138.7 \text{ hp.}\end{aligned}$$

Saving = 147.7 - 138.7 = 9 hp. or 6.1 per cent

Although the foregoing analyses have been based on a study of the pressure-volume changes which take place in a reciprocating-air-compressor cylinder, it may be shown that the same expressions would measure the work required for compression in a rotary compressor (cf. Art. 6:7 and Chap. X).

*Fans* are compressors which produce pressure changes which are normally very small in comparison with the suction pressure; thus the volume  $V_2$  is only very slightly smaller than  $V_1$ . If their cycle of operation is placed on a  $PV$  diagram, its enclosed area is seen to be nearly rectangular and equal, approximately, to  $(P_1 - P_2)V$ . Thus it has become customary to use the following simplified expression for the theoretical power input to a fan:

$$\text{Fan hp.} = \frac{(P_1 - P_2)V}{33,000} = \frac{(\Delta Z)V}{33,000v} = \frac{M \Delta Z}{33,000}$$

in which  $V$  is the volume of gas handled in cubic feet per minute,  
 $P_1 - P_2$  is the differential of pressure created by the fan, in pounds per square foot.

$\Delta Z$  is the head of gas equivalent to this pressure difference, in feet of gas,

$v$  is the specific volume of the gas, in cubic feet per pound,  
 and  $M$  is the weight of gas handled, in pounds per minute.

If  $\Delta Z$  is calculated based on the specific volume of the gas in the intake, the result will be slightly too large and, if based on discharge density, slightly too small; if the difference  $P_2 - P_1$  is less than 3 per cent of  $P_2$ , the error is usually considered negligible.

**7:2. The Clearance Factor and Volumetric Efficiency.** Equations (7:2), (7:5), and (7:6) state the theoretical work required for compression in a cylinder without clearance. The introduction of clearance, necessary in the actual compressor, will not theoretically increase the work of compression since the air which is trapped in the clearance space requires work during compression but restores an equivalent amount during expansion. The effect of clearance is to increase the necessary volume

of the cylinder and therefore to increase somewhat the work required in the actual compressor since the friction loss is slightly increased.

In Fig. 7:4 is shown the diagram of an ideal compressor with clearance.  $V_3$  is the clearance volume,  $V_1 - V_3$  is the piston displacement, and it is assumed that the value of  $n$  during expansion of the air trapped in the

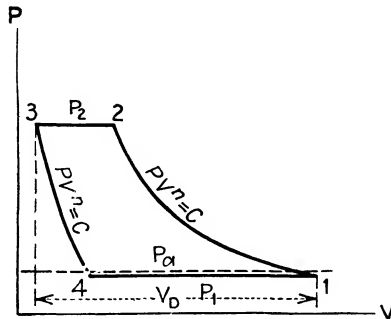


FIG. 7:4.—Effect of clearance on compression.

clearance space is the same as during the compression from  $V_1$  to  $V_2$ . The clearance is usually expressed in terms of the piston displacement, or

$$V_3 = CV_D$$

in which  $V_D$  represents the piston displacement and is equal to  $V_1 - V_3$ . The quotient  $(V_1 - V_4)/V_D$  measures the ratio of the volume of the fresh charge drawn into the cylinder per cycle, *measured at suction pressure  $P_1$* , to the piston displacement of the compressor and is called the *clearance factor*. It may be calculated in terms of the clearance as follows:

$$\begin{aligned} V_1 &= V_D + CV_D \\ P_3 V_3^n &= P_4 V_4^n \quad \text{or} \quad V_4 = V_3 \left( \frac{P_3}{P_4} \right)^{\frac{1}{n}} = CV_D \left( \frac{P_2}{P_1} \right)^{\frac{1}{n}} \\ \text{Clearance factor} &= \frac{V_1 - V_4}{V_D} = \frac{1}{V_D} \left[ V_D + CV_D - CV_D \left( \frac{P_2}{P_1} \right)^{\frac{1}{n}} \right] \\ &= \left[ 1 + C - C \left( \frac{P_2}{P_1} \right)^{\frac{1}{n}} \right] \quad (7:7) \end{aligned}$$

The term *free air* refers to the volume of air handled as measured at atmospheric pressure and temperature. This volume is less than as measured at suction pressure because of the differential of pressure,  $P_a - P_1$ , necessary to maintain flow from the atmosphere into the

cylinder during the suction stroke. The ratio of the volume of the fresh charge of air drawn into the cylinder per cycle, *measured at atmospheric pressure*  $P_a$  (on the free-air basis), to the piston displacement is called the *volumetric efficiency*. The volumetric efficiency is a ratio slightly smaller than the clearance factor of a given air-compressor cylinder; and, in order to find its value, it is necessary to calculate the free-air volume as a proportion of the volume as measured at suction pressure. It will be observed that the induction of the charge of air through the intake passages is a flow process. Equation (2:6) will therefore be applied between a section  $a$ , in the atmosphere just outside the intake passage, and a section 1, just inside the cylinder; these sections will be located far enough from the two ends of the passage so that the velocity at both stations is negligible. The heat flow between the two stations is certainly small and may be neglected, and there is, of course, no external work. Eliminating the corresponding terms from Eq. (2:6),

$$u_a + AP_a V_a = u_1 + AP_1 V_1 \quad \text{or} \quad h_a = h_1$$

But if the enthalpies are equal inside and outside the cylinder, the temperatures are also equal (cf. Art. 4:10), and

$$P_a V_a = P_1 V_1, \quad \text{or} \quad \frac{V_a}{V_1} = \frac{P_1}{P_a}$$

Thus if the volume drawn into the cylinder per cycle, as measured at suction pressure, is multiplied by the ratio  $P_1/P_a$ , the result will be the equivalent free-air volume, or

$$e_v = \frac{P_1}{P_a} \frac{(V_1 - V_4)}{V_D} = \frac{P_1}{P_a} \left[ 1 + C - C \left( \frac{P_2}{P_1} \right)^{\frac{1}{n}} \right] \quad (7:8)$$

in which  $e_v$  is the volumetric efficiency, a decimal fraction,

$P_1$  is the suction pressure (within the cylinder),

and  $P_a$  is the pressure of the atmosphere.

An analysis of Eqs. (7:7) and (7:8) will indicate that both the clearance factor and the volumetric efficiency will decrease with increasing values of the clearance and with increasing compression ratios.

Compressors are usually rated on the basis of their capacity in cubic feet of free air per minute. If the volume of free air to be handled per stroke is divided by the volumetric efficiency, the result will be the necessary displacement volume of the compressor. It will also be noted that  $P_a V_a$  may be substituted for  $P_1 V_1$  in Eqs. (7:3) to (7:6) in calculating the theoretical horsepower input to the compressor.

*Example.* Calculate the horsepower and cylinder volume required to compress 800 cu. ft. of free air per minute to 120 p.s.i. abs. Atmospheric pressure is 14.7 p.s.i. abs., and the pressure at the beginning of compression is 14 p.s.i. abs. The clearance of the compressor is 3 per cent, and the value of  $n$  during compression is 1.3. The compressor is double-acting and makes 100 r.p.m.

*Solution.*

$$\begin{aligned}\text{Compressor hp.} &= \frac{1.3 \times 144 \times 14.7 \times 800}{0.3 \times 33,000} \left[ 1 - \left( \frac{120}{14} \right)^{\frac{0.3}{1.3}} \right] \\ &= 222.0(1 - 1.642) = -142.8 \text{ hp.}\end{aligned}$$

Assuming a 20 per cent loss between steam engine and compressor, the horsepower required for the steam engine is

$$\text{Hp. steam engine} = \frac{142.8}{0.80} = 178.5 \text{ hp.}$$

The volumetric efficiency of the compressor is

$$\begin{aligned}e_v &= \frac{p_1}{p_a} \left[ 1 + C - C \left( \frac{P_2}{P_1} \right)^{\frac{1}{n}} \right] = \frac{14}{14.7} \left[ 1 + 0.03 - 0.03 \left( \frac{120}{14} \right)^{\frac{1}{1.3}} \right] \\ &= \frac{14}{14.7} [1 + 0.03 - 0.03(5.21)] = 0.832\end{aligned}$$

The displacement volume may be calculated by dividing the volume of free air by the volumetric efficiency.

$$V_D = \frac{800}{0.832} = 962 \text{ cu. ft. per minute}$$

The piston displacement of the compressor cylinder is

$$V_D \text{ per stroke} = \frac{V_D}{2N} = \frac{962}{2 \times 100} = 4.81 \text{ cu. ft.}$$

Allowing for the piston rod, this would correspond to a cylinder having a stroke of 30 in. and a diameter of 19 in.

**7:3. Multistage Compression.** If the ratio  $p_2/p_1$  exceeds 5, *i.e.*, if air is compressed from atmospheric pressure to 75 p.s.i. abs. or over, the compression is usually divided into two or more stages. Receivers are installed between the cylinders for the purpose of allowing the compressed air to cool to atmospheric temperature before entering the succeeding stage. If the compression could be carried out isothermally, no advantage would be gained by multistaging except as clearance losses would be reduced. The saving in work which results when the compression follows the curve  $PV^n = C$  is shown in Fig. 7:5 for a two-stage compressor. In the first stage, the air is compressed to an intermediate pressure  $p_i$ , the work done being

$$W_{\text{1st stage}} = \frac{n}{n-1} P_1 V_1 \left[ 1 - \left( \frac{P_i}{P_1} \right)^{\frac{n-1}{n}} \right] \quad (7:9)$$



In the receiver between the first and second stages, the air is cooled to atmospheric temperature so that point  $i$  will lie on an isothermal through point 1, and  $P_i V_i = P_1 V_1$ . The work performed during the second stage is

$$W_{2\text{d stage}} = \frac{n}{n-1} P_1 V_1 \left[ 1 - \left( \frac{P_2}{P_i} \right)^{\frac{n-1}{n}} \right] \quad (7:10)$$

The saving effected over single-stage compression is represented by the crosshatched area. The total work,

$$W_{\text{total}} = \frac{n}{n-1} P_1 V_1 \left[ 2 - \left( \frac{P_i}{P_1} \right)^{\frac{n-1}{n}} - \left( \frac{P_2}{P_i} \right)^{\frac{n-1}{n}} \right] \quad (7:11)$$

and this evidently becomes a minimum when the expression

$$\left( \frac{P_i}{P_1} \right)^{\frac{n-1}{n}} + \left( \frac{P_2}{P_i} \right)^{\frac{n-1}{n}} \quad (7:12)$$

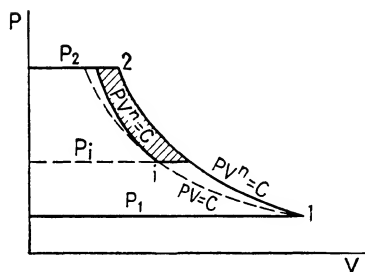


FIG. 7:5. —Effect of multistaging in air compression.

has its minimum value. If we designate this function of  $P_i$  ( $P_1$  and  $P_2$  are constants since they are fixed as the limiting pressures of the total compression) by the symbol  $G$ , then the value of  $P_i$  obtained when  $dG/dP_i$  equals zero will correspond to the minimum value of the function  $G$  and, therefore, the minimum total work. Thus

$$G = \left( \frac{P_i}{P_1} \right)^{\frac{n-1}{n}} + \left( \frac{P_2}{P_i} \right)^{\frac{n-1}{n}} = (P_1)^{\frac{1-n}{n}} (P_i)^{\frac{n-1}{n}} + (P_2)^{\frac{n-1}{n}} (P_i)^{\frac{1-n}{n}}$$

$$\frac{dG}{dP_i} = \frac{n-1}{n} (P_1)^{\frac{1-n}{n}} (P_i)^{\frac{-1}{n}} - \frac{n-1}{n} (P_2)^{\frac{n-1}{n}} (P_i)^{\frac{1-2n}{n}} = 0$$

or

$$\frac{n-1}{n} (P_1)^{\frac{1-n}{n}} (P_i)^{\frac{-1}{n}} = \frac{n-1}{n} (P_2)^{\frac{n-1}{n}} (P_i)^{\frac{1-2n}{n}}$$

$$(P_i)^{-\frac{1}{n} - \frac{1-2n}{n}} = (P_i)^{\frac{2n-2}{n}} = P_1 P_2^{\frac{n-1}{n}}$$

and

$$(P_i)^2 = P_1 P_2 \quad \text{or} \quad \frac{P_i}{P_1} = \frac{P_2}{P_i} = \left( \frac{P_2}{P_1} \right)^{\frac{1}{2}} \quad (7:13)$$

It will be noted that this ratio of pressures indicates that, for least total work, the work performed in the high- and low-pressure cylinders of a two-stage compressor will be equal. An analysis carried out for three-stage compression will indicate that this principle of equal work performance and therefore of equal pressure ratios for the individual stages will again prevail, and therefore

$$\frac{P'}{P_1} = \frac{P''}{P'} = \frac{P_2}{P''} = \left(\frac{P_2}{P_1}\right)^{\frac{1}{X}}$$

in which  $P'$  and  $P''$  are the absolute pressures at the end of the first and second stages, respectively. Similarly, for any number of stages of compression  $X$ , the pressure ratio in each stage will become  $(P_2/P_1)^{\frac{1}{X}}$ . Substituting  $(P_2/P_1)^{\frac{1}{X}}$  for  $P_2/P_1$  in Eq. (7:6), the work per stage can be calculated. But since equal work is performed in each stage and there are  $X$  stages, the total work of staged compression between the pressures  $P_1$  and  $P_2$  is

$$W = \frac{Xn}{n-1} P_1 V_1 \left[ 1 - \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{Xn}} \right] \quad (7:14)$$

*Example.* Calculate the saving which could be made in the power required by the compressor of the example of Art. 7:2 by compressing in two stages. What would be the pressure in the intercooler?

*Solution.*

$$\begin{aligned} \text{Hp}_{\cdot 2 \text{ stage}} &= \frac{2n}{n-1} \frac{P_1 V_1}{33,000} \left[ 1 - \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{2n}} \right] \\ &= \frac{2 \times 1.3 \times 144 \times 14.7 \times 800}{0.3 \times 33,000} \left[ 1 - \left( \frac{120}{14} \right)^{\frac{0.3}{2.6}} \right] \\ &= 444.5(1 - 1.281) = -125 \text{ hp.} \end{aligned}$$

The saving is therefore 17.8 hp. or 12.5 per cent.

The pressure in the intercooler is

$$p_i = (p_1 \times p_2)^{\frac{1}{2}} = (14 \times 120)^{\frac{1}{2}} = 41 \text{ p.s.i. abs.}$$

**7:4. The Air Engine.** The work cycle for an air engine is similar to that for an air compressor, with two important differences. One of these is that it is traced in the opposite direction, thus producing positive work, and the other is that the pressure at the end of expansion is usually higher than atmospheric pressure and, consequently, a rapid drop in pressure occurs when the exhaust valve opens. In Fig. 7:6, which is the theoretical indicator diagram for an air engine without clearance, the top line of the diagram represents the admission of high-pressure air into the air-engine cylinder as the piston moves to the right. At point 1,

the admission valve is closed (this closing point is referred to as the *cutoff*), and the charge of air expands along the polytropic curve  $PV^n = C$  until the piston reaches the end of its stroke at point 2. At this point, the pressure is dependent upon the relationship  $P_2/P_1 = (V_1/V_2)^n$  and may be determined if the air-supply pressure and the ratio of the volume at cutoff to the piston displacement (the cutoff percentage) is known. When point 2 is reached, the exhaust valve is opened and the charge flows out into the atmosphere, the pressure within the cylinder decreasing to  $P_3$ , or practically atmospheric pressure, as the piston returns to the opposite end of the cylinder preparatory to a repetition of the cycle.

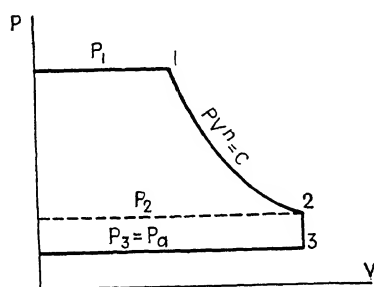


FIG. 7:6.—Air-engine cycle.

Both admission and exhaust valves are, in the case of the air engine, operated mechanically.

The work delivered per cycle is represented by the area enclosed within the diagram which may be conveniently divided into two parts by the dashed line representing the pressure  $P_2$ . The work area above this line can be evaluated by applying Eq. (7:6); the area below is a rectangle of height  $P_2 - P_3 (= P_2 - P_a)$  and length  $V_2$ , equal to the piston displacement of the air engine. Thus, work per cycle equals

$$W = \frac{n}{n-1} P_1 V_1 \left[ 1 - \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{n}} \right] + (P_2 - P_3) V_2 \quad (7:15)$$

in which the subscripts refer to the notations of Fig. 7:6.

The effect of clearance, which is always present in the practical air engine, is to reduce the area of the diagram. The work expended on this compression is not lost but returned, at least in part, during the succeeding cycle. The net effect, as in the compressor, is to reduce the capacity of an engine of given size and speed and, since the loss due to mechanical friction is the same as for an engine without clearance, to increase slightly the *relative* friction losses. Since compression in an engine is usually not complete, the indicator diagram is very different from a compressor diagram and no formula of general application may be developed to adjust for its effects.

**Example** Calculate the theoretical horsepower which would be developed by 10 cu. ft. of compressed air at 150 p.s.i. abs. expanded per minute in an air engine with 40 per cent cutoff if the expansion is adiabatic. Assume exhaust is to atmospheric pressure.

*Solution.* From the statement of the problem,

$$V_1 = 10 \text{ cu. ft.}$$

$$V_2 = \frac{1.00}{0.40} V_1 = 25 \text{ cu. ft.}$$

$$p_1 = 150 \text{ p.s.i. abs.; } p_2 = 14.7 \text{ p.s.i. abs.}$$

$$p_2 = p_1 \left( \frac{V_1}{V_2} \right)^k = 150 \left( \frac{10}{25} \right)^{1.405} = 150 \times 0.276 = 41.4 \text{ p.s.i. abs.}$$

$$\begin{aligned} \text{Hp.} &= \frac{kP_1V_1}{(k-1)(33,000)} \left[ 1 - \left( \frac{P_2}{P_1} \right)^{\frac{k-1}{k}} \right] + \frac{(P_2 - P_1)V_2}{33,000} \\ &= \frac{1.405 \times 144 \times 150 \times 10}{0.405 \times 33,000} \left[ 1 - \left( \frac{41.4}{150} \right)^{0.286} \right] + \frac{144(41.4 - 14.7)25}{33,000} \\ &= 22.7(1 - 0.692) + 2.91 = 9.91 \text{ hp.} \end{aligned}$$

### Problems

1. Contrast the purposes behind the use of a water jacket about the cylinders of an internal-combustion engine and an air compressor.

2. What horsepower is theoretically required for the isothermal compression of 1,000 cu. ft. of air per minute from 14.5 to 50 p.s.i. abs. in a single-stage compressor? What horsepower if the compression is adiabatic? If  $n = 1.3$ ?

3. What horsepower is theoretically required for the compression of 500 cu. ft. of carbon dioxide per minute from 14 to 60 p.s.i. abs. in a single-stage compressor if the compression is isothermal? If adiabatic? If  $n = 1.20$ ?

4. Compare the horsepower necessary to compress 800 cu. ft. per minute from 14.7 to 45 p.s.i. abs. in a single-stage compressor for oxygen and for hydrogen. What weight of gas is compressed per minute in each case if the temperature of the charge is 60°F. at the beginning of compression?

5. Fifteen hundred pounds of air is compressed per hour from 14.5 to 55 p.s.i. abs. in a single-stage compressor. The temperature of the air at the beginning of compression is 70°F. What horsepower is theoretically required if the compression is isothermal? If adiabatic? If  $n = 1.28$ ? What are the corresponding values if the initial air temperature is 100°F.?

6. Fifty pounds of oxygen is compressed per minute in a single-stage compressor from an initial temperature of 80°F. and pressure of 14 p.s.i. abs. to a final pressure of 60 p.s.i. abs. What horsepower is theoretically required if the compression is isothermal? If adiabatic? If  $n = 1.3$ ? What are the corresponding values if the gas is hydrogen?

7. Compare the theoretical horsepower required to quadruple the pressure of a given volume per minute of air and of a monatomic gas if the compression is isothermal; if the compression is adiabatic. Assume original pressures identical.

8. How much work is required to compress 12 cu. ft. of air at 14 p.s.i. abs. to a pressure of 50 p.s.i. abs. if the compression is adiabatic? How much work is absorbed in delivering this compressed charge to the receiver? How much work is returned to the piston on the suction stroke? What is the net work requirement? Compare with the result obtained by substitution in Eq. (7:5). Illustrate this problem on a PV diagram.

9. Repeat Prob. 8, assuming isothermal compression. Compare with result obtained by substitution in Eq. (7:2).

10. Show that the theoretical work required for the compression *cycle* is  $n$  times the work required for the compression *alone*, whether the compression is isothermal, adiabatic, or polytropic.

11. A centrifugal fan handles 5,000 cu. ft. of air per minute against a total head of 5 in. of water (density of water = 62.3 lb. per cubic foot). Calculate the theoretical fan horsepower.

12. The total head against which a fan operates is the sum of the static (pressure) head and the velocity head (the difference of the kinetic energies per pound at intake and at discharge from the fan, expressed in terms of the distance through which this energy would lift a unit weight). A propeller type of fan handles 25,000 cu. ft. of standard air per minute, increasing its velocity from 0 to 25 m.p.h. The static pressures at intake and discharge from this fan at the points where the velocities are measured are balanced. Calculate the theoretical fan horsepower (neglect friction between intake and outlet).

13. A compressor with 3 per cent clearance operates at a suction pressure of 14.2 p.s.i. abs. when atmospheric pressure is 14.7 p.s.i. abs. The discharge pressure is 60 p.s.i. abs., and  $n = 1.32$ . What is the clearance factor? The volumetric efficiency? What are the corresponding values if the compression is isothermal?

14. A certain compressor has a clearance factor of 0.84 and a volumetric efficiency of 0.815. If atmospheric pressure is standard, what is the suction pressure? If  $n = 1.3$  and  $p_2$  is 57 p.s.i. abs., what is the clearance?

15. One thousand cubic feet of free air is compressed per minute to a pressure of 75 p.s.i. abs. in a single-stage double-acting compressor with 3 per cent clearance. Atmospheric pressure is standard, and the suction pressure is 14.2 p.s.i. abs. What theoretical horsepower is required if  $n = 1.3$ ? If the compressor makes 150 double strokes per minute, what piston displacement is necessary?

16. The total cylinder volume of a double-acting compressor with equal clearances at the two ends of the cylinder is 5.3 cu. ft. Its piston displacement is 5 cu. ft., and it makes 80 r.p.m. The suction pressure is 14.3 p.s.i. abs., discharge pressure is 45 p.s.i. abs., atmospheric pressure is standard, and  $n = 1.3$  for the compression. What is its capacity in cubic feet of free air per minute? What is its theoretical horsepower?

17. Fifty pounds of oxygen is compressed per minute from a suction pressure of 14 p.s.i. abs. to a discharge pressure of 60 p.s.i. abs. in a single-stage double-acting compressor with 4 per cent clearance. The pressure of the atmosphere is 14.7 p.s.i. abs., and its temperature is 80°F. The exponent  $n$  for the compression is 1.31, and the compressor makes 90 double strokes per minute. What horsepower is theoretically required? What is the necessary piston displacement of the compressor?

18. An air compressor compresses air from 14 to 200 p.s.i. abs. in two stages. The intermediate pressure is 100 p.s.i. abs., the value of  $n$  in both stages is 1.3, and the temperature of the air entering both stages is the same. What is the ratio of the theoretical power requirement of the first stage to the power requirement of the second? What intermediate pressure would be ideal? What percentage saving in the total power requirement would be made possible by changing the pressure at discharge from the first stage to this new value?

19. What horsepower is theoretically required for the compression of 1,000 cu. ft. of air per minute from 14 to 200 p.s.i. abs. with  $n = 1.3$  if two stages of compression are used? What is the pressure at discharge from the first stage? What is the saving over single-stage compression?

20. What horsepower is theoretically required for the compression of 30 lb. of air per minute from 14 to 900 p.s.i. abs. if  $n = 1.32$ , three stages of compression are used, and the temperature of the atmosphere is 70°F.? What are the pressures at discharge from each stage?

21. Eight hundred cubic feet of air per minute is to be compressed from 14 to 2,000 p.s.i. abs. The value of  $n$  may be assumed as 1.3. How many stages of compression do you recommend? What are the ideal pressures at discharge from each stage as based on your recommendation? What is the saving in power as compared with single-stage compression?

22. What is the theoretical ratio of the piston displacement of the high-pressure to that of the low-pressure cylinder in two-stage compression of air from 14 to 180 p.s.i. abs.? What is the relative piston displacement for single-stage compression over the same pressure range? Assume zero clearance.

23. Referring to Fig. 7:6, show that the theoretical horsepower developed by an air engine without clearance, having polytropic expansion, cutoff at  $r$  per cent of the stroke, and using  $V_1$  cu. ft. of compressed air per minute, may be expressed as

$$\text{Hp.} = \frac{n}{n-1} \frac{P_1 V_1}{33,000} \left[ 1 - \left( \frac{r}{100} \right)^{n-1} \right] + \frac{100 V_1}{r 33,000} \left[ P_1 \left( \frac{r}{100} \right)^n - P_3 \right]$$

24. Calculate the theoretical work per cubic foot of compressed air supplied to an air engine without clearance at a pressure of 110 p.s.i. abs. Cutoff is at 30 per cent of the stroke,  $n = 1.37$  for the expansion, atmospheric pressure is 14.7 p.s.i. abs., and the temperature of the charge at cutoff is 120°F. What is the temperature of the air as it is discharged into the atmosphere?

25. An air engine uses 20 cu. ft. of compressed air per minute at a pressure of 125 p.s.i. gage. The cutoff is at 35 per cent of the stroke,  $n = 1.35$  for the expansion, and atmospheric pressure is standard. What theoretical horsepower does it develop?

26. Compressed air is supplied to an air engine from a large storage tank at a pressure of 130 p.s.i. gage and a temperature of 80°F. Cutoff is at one-third of the stroke, and  $n = 1.36$  for the expansion. Neglecting friction losses in the line connecting the tank and the engine, how much work can theoretically be performed per pound of air supplied the engine? If the engine develops 10 hp., at what rate in pounds per minute is air withdrawn from the tank?

27. In Prob. 26, the air passing through the line connecting the tank with the engine is heated to a temperature of 250°F. before entering the engine. How much heat is required per pound of air? How much work is performed by the engine per pound of air withdrawn from the tank?

## CHAPTER VIII

### PROPERTIES OF VAPORS--VAPOR TABLES

**8:1. Characteristics of Vapors.** A vapor may be described as a fluid which can be liquefied by minor changes in temperature or pressure. The pressure, volume, and temperature of a vapor are not connected by any simple relationship such as is expressed in the characteristic equation for a perfect gas. Instead, each vapor has its own characteristic equation which has been developed from experimental results. In discussing the methods of producing power by employing a gaseous medium, it is never necessary to consider the system in any but the gaseous state; on the other hand, in the study of vapor cycles almost invariably the state of aggregation of the system changes between the forms of vapor and liquid. Each state must be represented by its own characteristic relationships. In the study of the thermodynamics of vapors, tables giving the properties of the substance in each condition are usually employed. These tables are compiled from a study of the experimentally derived equations of the substance. Since experimental results vary somewhat, the tables for a given vapor such as steam, as prepared by different authors, will not exactly agree.

**8:2. Vapor Tables.** The quantities tabulated in vapor tables are the pressure and temperature and the specific volume, enthalpy, entropy, and, occasionally, internal energy of the fluid. All these are functions of the state of the fluid so that definite values of each may be recorded for each condition of the fluid. Pressure, temperature, and volume can be given their actual absolute values; but enthalpy and entropy are purely relative quantities so that some convenient reference condition must be established. For steam and some other vapors, this point is arbitrarily taken as the condition of the liquid at a temperature of 32°F. and equivalent pressure; *i.e.*, the enthalpy and the entropy of water at 32°F. are both zero. The internal energy will always be less than the enthalpy by the product  $APv$ ; therefore the zero point for the calculation of internal energy is very slightly above 32°F. The difference, however, is so small at this temperature that it is usually neglected.

The volume of a system is the sum of the volumes of its parts; the same is true of enthalpy, entropy, and internal energy. These may

therefore be classified as *extensive* properties. Pressure and temperature, it will be noted, are not extensive properties.

Any two *independent* properties are sufficient to establish the state of a homogeneous system of known weight and therefore to determine all other properties of the system. As will be seen later, the pressure and temperature are, in some cases, interdependent.

**8:3. The Process of Vaporization.** If heat is added at constant atmospheric pressure to 1 lb. of water at 32°F., the first effect which will be noticeable will be a gradual rise in temperature. Since the specific heat of water at this temperature level is practically unity, for each B.t.u. added the temperature will rise 1°F. After 180 B.t.u. has been added and the temperature has reached 212°F., a further addition of heat does not immediately accomplish any increase of temperature; instead, the water begins to boil. This vaporization process has the following characteristics:

1. If the pressure remains constant, the temperature will remain the same until vaporization is complete (*i.e.*, until the liquid is entirely boiled away).

2. As heat is added during vaporization, the liquid progressively changes its state from liquid to vapor.

3. If the pressure acting on the surface of the liquid is increased, the temperature at which vaporization occurs is increased, and vice versa.

4. As the temperature (and consequently the pressure) of vaporization increases, the heat required to accomplish the change of state decreases.

If temperature is plotted as ordinates against the heat added to the water and steam as abscissas, a diagram similar to Fig. 8:1 is obtained. With reference to this figure, to 1 lb. of water under atmospheric pressure and already at the boiling temperature of 212°F., 970.3 B.t.u. must be added in order to bring about complete vaporization. In any intermediate state the steam is said to have a certain *quality*. Quality, denoted by the symbol  $x$ , may be defined as the ratio of the weight of the vaporized portion to the total weight of the mixture and is proportional to the amount of heat added during vaporization. For instance, if 400 B.t.u. had been added to the pound of water after it had reached the boiling temperature (212°F.), the resulting quality of the steam would have been  $x = 400/970.3 = 0.412$ . This can be interpreted to mean that each pound of the substance, which is now a mixture of a liquid and a vapor, is composed of 0.412 lb. of vapor and 0.588 lb. of liquid, or it can be considered as a statement of the ratio of actual to complete vaporization. The heat added under the constant-pressure condition is equal to the change of enthalpy (see Chap. II); and therefore, since the enthalpy



of water at 32°F. has been arbitrarily assigned a zero value, the total amount of heat added above this point at constant pressure as in the illustration will represent the enthalpy of the steam in the final condition. For example, the enthalpy of a pound of water at 212°F. is 180.07 B.t.u., of a pound of completely vaporized steam at the same temperature is 1150.4 B.t.u. ( $180.07 + 970.3$ ), and of a pound of *wet* steam at this temperature, the quality of which is 0.412, is 580.07 B.t.u. ( $180.07 + 400$ ).

If the pressure on the surface of the water during the addition of heat is 100 p.s.i. abs. instead of atmospheric, it will be discovered that ebullition will not start at 212°F. but that the temperature must be increased to 327.8°F. before the liquid begins to boil. To do this requires a larger investment of thermal energy in the liquid (298.4 B.t.u.); but, the boiling temperature having been reached, it is found that only 888.8 B.t.u. is required for the vaporization process. Similarly, if the pressure is reduced to 5 p.s.i. abs., boiling takes place at 162.2°F., representing the addition of 130.1 B.t.u. to the liquid, and 1001.0 B.t.u. is required for vaporization.

A vapor in equilibrium with its liquid is called a *saturated vapor*. If the process of vaporization is complete but no further heat has been added, the vapor is said to be *dry and saturated*. The temperature at which vaporization takes place under a stated external pressure condition is termed the *saturation temperature* for that pressure. Thus the saturation temperature for steam at 100 p.s.i. abs. is 327.8°F., and the enthalpy of dry and saturated steam at the same pressure is 1187.2 ( $298.4 + 888.8$ ) B.t.u. When the liquid is at the saturation temperature corresponding to its pressure, it is referred to as a *saturated liquid*; the addition of further heat to a saturated liquid will, if the pressure is held constant, result in progressive vaporization rather than an increase of temperature of the liquid. The fluid may also exist as a liquid at temperatures below that equivalent to the pressure to which it is subjected; in this condition, it is variously termed a *subcooled*, *superpressure*, or *compressed liquid*.

All four of the characteristics of the process of vaporization as listed in Art. 8:3 may be confirmed by a reference to Fig. 8:1. A fifth conclusion might also be made from the figure if the lower pressures and temperatures alone are considered. This is that the total enthalpy of the dry saturated vapor increases with the saturation temperature. However, when the saturation temperature rises to about 450°F., the decrease in the heat required for vaporization becomes more rapid than the increase of the enthalpy of the liquid and the dry-saturated-steam

line in Fig. 8:1 becomes first vertical and then begins to bend over toward the left, indicating that the total enthalpy of the dry saturated steam decreases above this temperature.

**8:4. The Critical Temperature.** From the foregoing analysis, the convergence of the liquid line and the dry-saturated-vapor line is evident. At the point where they meet, the heat required for vaporiza-

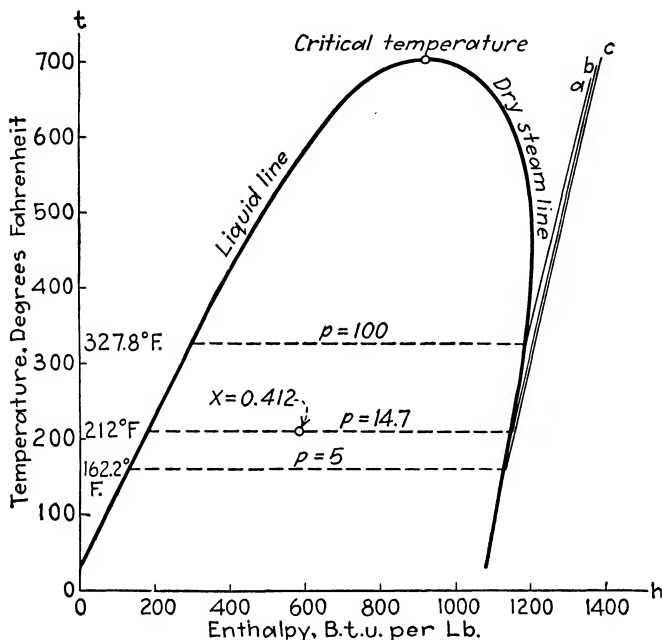


FIG. 8:1.—Temperature-enthalpy chart for steam.

tion has become zero, and the enthalpy of the liquid is equal to the enthalpy of the dry saturated vapor. The corresponding temperature is known as the *critical temperature* and may be defined as the point where the pressure-temperature relationship, outlined as the third characteristic of the process of vaporization, ceases to apply. All vapors have a characteristic critical temperature; for steam, it is 705.4°F. At this temperature, the saturation pressure has become 3,206.2 p.s.i. abs. Steam cannot exist as a *saturated* vapor above this temperature.

**8:5. The Superheated Vapor.** If, after the dry-saturated-vapor state is reached, further heat is added, the temperature again will gradually rise and the vapor is then said to be superheated. The temperature of the superheated vapor is always above its saturation temperature at the

designated pressure, and the difference between the actual and the saturation temperatures measures the number of *degrees of superheat*. In Fig. 8:1, *a*, *b*, and *c* are lines of constant pressure and represent the relation between the heat added and the temperature rise during the process of superheating a pound of steam at constant pressures of, respectively, 100, 14.7, and 5 p.s.i. abs. It will be noted that in the superheated condition the temperature is not determined by the pressure, as for the saturated vapor, except that it must always be above the saturation temperature corresponding to its pressure. As the degree of superheat increases, the behavior of the vapor approaches that of a perfect gas; but, for ordinary superheats, the application of the characteristic equation for a perfect gas does not give values of sufficient accuracy for practical calculations. Therefore, special experimental equations for the superheated vapor are derived by means of which its properties may be found for any stated condition of pressure and temperature. For convenience, these properties are usually tabulated; such tabulations must be keyed by both temperature and pressure since, for a superheated vapor, the two are independent.

**8:6. Volume.** The volume of a liquid is relatively small. As heat is added and the temperature rises, the volume, in general, will increase slightly. This increase is not, as in the case of the perfect gas, by any means proportional to the absolute temperature. For instance, as the temperature of a pound of water increases from 40.3°F. (500°abs.) to 540.3°F. (1000°abs.), the volume increases from 0.01602 to 0.0215 cu. ft. Since liquids are practically incompressible, the volume is often considered as a function of the temperature alone and independent of the pressure.

During vaporization the volume per pound increases very rapidly. For instance, the volume of 1 lb. of water at 212°F. is 0.01672 cu. ft., while 1 lb. of dry saturated steam at the same temperature has a volume of 26.80 cu. ft. The specific volume of the dry saturated vapor decreases with increase of saturation temperature because of the greater effect of increasing saturation pressures. When the critical temperature is reached, the specific volume of the liquid becomes equal to the specific volume of the dry saturated vapor so that vaporization at the critical temperature is not accompanied by an increase of volume just as, it has been observed, no addition of heat is required.

During the process of superheating a vapor at constant pressure, the volume will increase approximately (although not exactly) in proportion to the absolute temperature, indicating that the vapor is approaching the behavior of a perfect gas.

**8:7. The Steam Tables.** The values of the pressure, temperature, enthalpy, and volume of steam which have been used in the preceding numerical illustrations have been taken from the Keenan and Keyes tables for steam, an abstract of which will be found in Appendix I of this text. It is recommended that a copy of these or some other standard unabridged steam tables be used in connection with a study of the thermodynamics of vapors. Steam tables state in tabular form the relationships existing between the pressure, temperature, volume, enthalpy, internal energy, and entropy of steam in the liquid, saturated, and superheated vapor states. The Keenan and Keyes tables are divided into three sections, the first two of which tabulate the relationships which exist for the liquid and the dry saturated vapor and the third those for superheated steam. Table 1 is keyed by temperatures and Table 2 by pressures for easy reference; both give the specific volume, enthalpy per pound, and entropy per pound of both liquid and dry saturated vapor. Table 3 is keyed by both pressure and temperature independently and tabulates the values of specific volume, enthalpy, and entropy for superheated steam. In Tables 1 and 2, the pressure and temperature will always correspond since the pressure-temperature relationship applies in the saturated-steam condition. In Table 3, the two are independent of each other.

Table 4 of the unabridged tables (not included in the Appendix of this text) deals with the properties of the compressed liquid, *i.e.*, water confined under a pressure higher than its saturation pressure.

**8:8. Specific Volume of Saturated Steam.** The specific volume of saturated water is presented in Tables 1 and 2 and is denoted by  $v_f$ . The specific volume of the dry saturated vapor is represented by the symbol  $v_g$  and the *change* of specific volume that takes place during vaporization by  $v_{fg}$ ; in Table 2, the latter is not tabulated but may be found by subtracting the value of  $v_f$  from  $v_g$ . For saturated steam of quality  $x$ , the volume of water in 1 lb. of the mixture is  $(1 - x)v_f$ , and the volume of the vapor is  $xv_g$ . The total volume of a pound of the mixture in this condition is, therefore,

$$v = (1 - x)v_f + xv_g = v_f + xv_{fg} \quad (8:1)$$

or, since  $v_f$  is usually very small in comparison with  $v_g$ , the volume of the water may be neglected with negligible error and the expression simplified to the form

$$v = xv_g \quad (8:2)$$

It should be understood that the latter expression is an approximation but one which is sufficiently accurate for most practical calculations,

especially when the quality is high and the pressure well below the critical value.

If the liquid and dry-steam lines are represented on  $PV$  coordinates, Fig. 8:2 is obtained. In order to show the complete range more clearly,

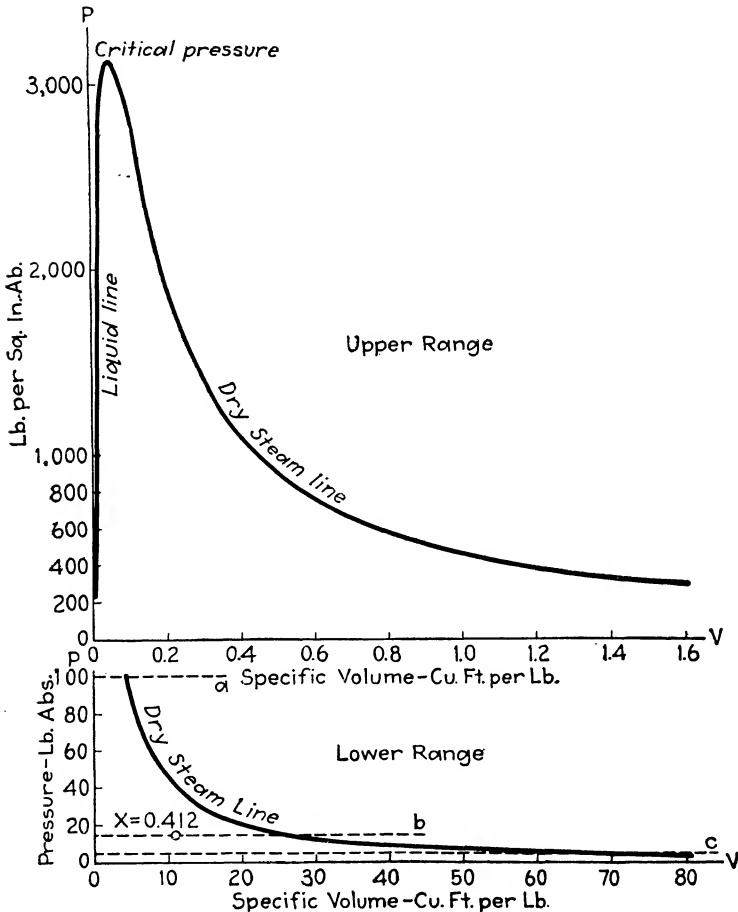


FIG. 8:2.—Pressure-volume chart for steam.

this chart has been divided into two sections, one for the upper and the other for the lower range of pressures. For the lower range the volume of the liquid is so small in comparison with the volume of the dry saturated steam that the liquid line practically coincides with the  $P$  axis. The lines indicating vaporization and superheating under the conditions of

the examples illustrated in Fig. 8:1 (at constant pressures of 100, 14.7, and 5 p.s.i. abs., respectively) have been added to this chart.

*Example.* What is the volume of a pound of steam with a quality of 0.9 and a pressure of 150 p.s.i. abs.?

*Solution.* Since this is saturated steam and the pressure is stated, the use of Table 2 is indicated. The *exact* method of calculation is

$$v = (1 - x)v_f + xv_g = 0.1(0.01809) + 0.9(3.015) = 2.7153 \text{ cu. ft. per pound}$$

The *approximate* method is

$$v = xv_g = 0.9(3.015) = 2.714 \text{ cu. ft. per pound}$$

In this case the error involved is less than one-tenth of 1 per cent.

**8:9. Enthalpy of Saturated Steam.** Under the column heading Enthalpy, the enthalpy of the saturated liquid  $h_f$ , of the dry saturated vapor  $h_g$ , and the change of enthalpy during vaporization  $h_{fg}$  are recorded.

The enthalpy of a pound of saturated steam of quality  $x$  is equal to the enthalpy of the water which it contains plus the enthalpy of the dry-vapor content, or

$$h = (1 - x)h_f + xh_g = h_f + xh_{fg} = h_g - (1 - x)h_{fg} \quad (8:3)$$

Any of the three forms may be used. Ordinarily, if the quality is low, the second expression will be employed; if high, the third will be used. Since the enthalpy of the liquid is not negligible in comparison with that of the dry vapor, no simplified expression such as was suggested for the calculation of volume may properly be utilized.

*Example.* (a) What is the enthalpy of a pound of saturated water at 150 p.s.i. abs.? (b) Of a pound of wet steam of 90 per cent quality at the same pressure?

*Solution.* (a) The tabulated value of  $h_f$  at 150 p.s.i. abs. is 330.51 B.t.u. This is the required value of the enthalpy of the saturated liquid. It will be noted that the temperature is 358.42°F. If the specific heat had been assumed to be unity over the entire range from 32°F., the calculated enthalpy would have been 358.42 - 32 or 326.42 B.t.u. The steam-table value should always be used when the temperature exceeds 212°F. as it takes into account the increasing specific heat above that point.

(b) The enthalpy of saturated steam at 150 p.s.i. abs. and quality of 90 per cent may be calculated from the values of  $h_g$  and  $h_{fg}$  given for that pressure in Table 2.

$$h = h_g - (1 - x)h_{fg} = 1194.1 - 0.1(863.6) = 1107.7 \text{ B.t.u. per pound}$$

**8:10. Entropy of Saturated Steam.** The entropy of the liquid  $s_f$ , the change of entropy during vaporization  $s_{fg}$ , and the total entropy of the dry saturated vapor  $s_g$  are also recorded in Tables 1 and 2. The entropy of a pound of water at 32°F. is arbitrarily zero. If the specific heat is

assumed to be unity, the increase of entropy for 1 lb. due to heating the water from 32°F. to temperature  $T_1$  is

$$\begin{aligned}s_{f1} - s_{f0} &= \int_{491.7}^{T_1} \frac{dQ}{T} = C \int_{491.7}^{T_1} \frac{dT}{T} = 1 \int_{491.7}^{T_1} \frac{dT}{T} \\ &= \log_e T_1 - \log_e 491.7 = \log_e \frac{T_1}{491.7}\end{aligned}$$

or, as the entropy at 32°F. is arbitrarily zero,

$$s_{f1} = \log_e \frac{T_1}{491.7} \quad (8:4)$$

Since the specific heat of the liquid is not quite constant but tends to increase with the temperature, the tabulated values of the entropy of the liquid will not show exact agreement with the calculated values based on Eq. (8:4).

*Example A.* Assuming the specific heat of water to be unity in each case, calculate the entropy of 1 lb. of water at 100 and at 300°F., and compare with the tabulated values.

*Solution.* (a) By Eq. (8:4),

$$s_f = \log_e \frac{559.7}{491.7} = 0.1293$$

The tabulated value is 0.1295.

$$(b) \quad s_f = \log_e \frac{759.7}{491.7} = 0.434$$

The tabulated value is 0.4369.

During vaporization, heat is added at constant temperature, and the increase of entropy for the process is therefore represented by the quotient of the heat required for vaporization divided by the absolute temperature at which vaporization takes place, or

$$s_g - s_f = s_{fg} = \frac{h_{fg}}{T} \quad (8:5)$$

*Example B.* The heat added during the vaporization of a pound of water at a temperature of 300°F. is 910.1 B.t.u. What is the increase of entropy during vaporization?

*Solution.*

$$s_{fg} = \frac{h_{fg}}{T} = \frac{910.1}{300 + 459.7} = 1.1980$$

The total entropy of the dry saturated vapor is the sum of the entropy of the liquid and the increase of entropy during vaporization, or

$$s_g = s_f + s_{fg} = s_f + \frac{h_{fg}}{T}$$

If only a part of the heat of vaporization is added, the resulting mixture of water and dry vapor will have a quality  $x$  and the entropy will be

$$s = s_f + \frac{xh_{fg}}{T} = s_f + xs_{fg} = s_g - (1 - x)s_{fg} \quad (8:6)$$

It is again evident from a study of the relative values of  $s_f$  and  $s_g$  that the entropy of the liquid portion of the mixture cannot be neglected.

*Example C.* From the tables, calculate the entropy of a pound of steam at a pressure of 150 p.s.i. abs. and a quality of 0.90.

*Solution.*

$$s = s_g - (1 - x)s_{fg} = 1.5694 - 0.1(1.0556) = 1.4638$$

**8:11. Internal Energy of Saturated Steam.** The internal energy of the saturated liquid  $u_f$ , the change of internal energy during vaporization  $u_{fg}$ , and the total internal energy of the dry saturated vapor  $u_g$  are presented in Table 2 only. The internal energy of saturated steam of quality  $x$  may be calculated as before, as

$$u = (1 - x)u_f + xu_g = u_f + xu_{fg} = u_g - (1 - x)u_{fg} \quad (8:7)$$

An alternate method of calculation is based on a rearrangement of Eq. (2:7), defining enthalpy, into the form

$$u = h - APv \quad (8:8)$$

The latter equation has the advantage of being more general in its application.

*Example.* Calculate the internal energy of a pound of steam at 150 p.s.i. abs. and 90 per cent quality.

*Solution.* The total enthalpy for steam in this condition was found to be 1107.7 B.t.u. per pound in part (b) of the example of Art. 8:9. The volume has been calculated as 2.714 cu. ft. in Example B of Art. 8:8. Then

$$APv = \frac{144 \times 150 \times 2.714}{778} = 75.3 \text{ B.t.u. per pound}$$

and the internal energy is

$$u = h - APv = 1107.7 - 75.3 = 1032.4 \text{ B.t.u. per pound}$$

If the use of Eq. (8:7) is preferred,

$$u = u_g - (1 - x)u_{fg} = 1110.5 - 0.1(780.5) = 1032.4 \text{ B.t.u. per pound}$$

**8:12. Superheated Steam.** The properties of superheated steam are tabulated in Table 3. In this condition the pressure is not determined by the temperature as for saturated steam; instead, the two may be



selected independently. The temperature, however, must always be higher than the saturation temperature at the stated pressure. The tabulated values for the various combinations of pressure and temperature are the total volume  $v$ , the total enthalpy  $h$ , and the total entropy  $s$ .

The total volume is the sum of the volume of the dry saturated vapor and the increase of volume which accompanies the superheating process.

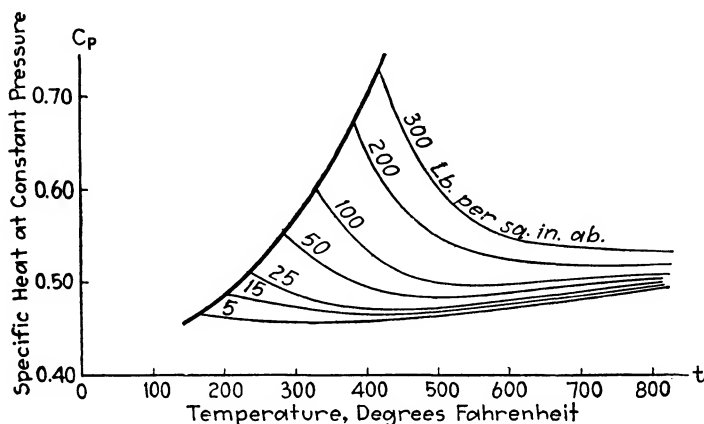


FIG. 8:3.—True specific heat of superheated steam.

The total enthalpy is the sum of the enthalpy of the dry saturated vapor and the enthalpy of superheat. The increase of enthalpy during superheating is

$$h - h_g = C_{ps}(t - t_{sat}) \quad (8:9)$$

in which  $h$  is the total enthalpy of the superheated steam and  $t$  is its temperature,  $C_{ps}$  is the mean specific heat at constant pressure of superheated steam at the stated pressure, and  $t_{sat}$  is the saturation temperature corresponding to the stated pressure. The specific heat of superheated steam is a variable and depends not only upon the temperature but also upon the pressure. A *mean*, or average, value of  $C_{ps}$  may be calculated for a specific process by an application of Eq. (8:9), using values of the enthalpy taken from the tables for superheated steam. The *true* specific heat is the *instantaneous* value for a stated pressure and temperature instead of the average value over a range of temperature. Values of the true specific heat of superheated steam are charted in Fig. 8:3. The values from this chart cannot be employed in Eq. (8:9) in which the mean specific heat must be used, but the use of this equation in finding the total enthalpy of superheated steam is unnecessary if a complete

tabulation of the properties of the superheated vapor such as Table 3 of the Keenan and Keyes tables is available.

Figure 8:3 illustrates the tendency of the specific heat to vary less with the pressure as the degree of superheat increases and the vapor approaches the condition of a perfect gas.

The entropy  $s$ , as tabulated in the superheat tables, is the sum of the entropy of the dry saturated steam and the entropy of superheat. The entropy of superheat is

$$s_{\text{sup}} = s - s_g = \int_{T_{\text{sat}}}^{T_{\text{sup}}} \frac{dQ}{T} = C_{ps} \int_{T_{\text{sat}}}^{T_{\text{sup}}} \frac{dT}{T} = C_{ps} \log_e \frac{T_{\text{sup}}}{T_{\text{sat}}} \quad (8:10)$$

in which  $s$  is the entropy of the superheated steam,  $T_{\text{sup}}$  its absolute temperature, and  $T_{\text{sat}}$  the absolute saturation temperature at the stated pressure. Since  $C_{ps}$  is a variable, the entropy of superheat can be accurately calculated by this method only by treating it as such in the integration process; Eq. (8:10) is therefore an approximation. The tabulated values of the total entropy of superheated steam take this variation of specific heat into consideration.

The internal energy of superheated steam can be calculated by applying Eq. (8:8).

*Example.* What are the number of degrees of superheat, the volume, enthalpy, entropy, and internal energy of steam at 150 p.s.i. abs. and 500°F.? What is the mean specific heat of the superheated steam for this range? Calculate the entropy of superheat, considering the mean specific heat as a constant, and compare with the value obtained from the steam tables.

*Solution.* The saturation temperature of steam at a pressure of 150 p.s.i. abs. is 358.42°F. The steam is therefore superheated 141.58°.

From Table 3 the total volume, total enthalpy, and total entropy may be read directly as

$$\begin{aligned} v &= 3.681 \text{ cu. ft. per pound} \\ h &= 1274.1 \text{ B.t.u. per pound} \\ s &= 1.6599 \\ AP_v &= \frac{144 \times 150 \times 3.681}{778} = 102.2 \text{ B.t.u. per pound} \end{aligned}$$

and the internal energy

$$u = h - AP_v = 1274.1 - 102.2 = 1171.9 \text{ B.t.u. per pound}$$

The heat of superheat is the difference between the enthalpy of the superheated steam and that of dry saturated steam at the same pressure, or

$$1274.1 - 1194.1 = 80.0 \text{ B.t.u. per pound}$$

and the mean specific heat

$$C_{ps} = \frac{80.0}{141.58} = 0.565$$

The entropy of superheat, as calculated, is

$$s - s_g = C_p \log_e \frac{T_{\text{sup}}}{T_{\text{sat}}} = 0.565 \log_e \frac{500 + 459.7}{358.42 + 459.7} = 0.0901$$

This compares with a value from the tables of

$$1.6599 - 1.5694 = 0.0905$$

**8:13. Properties of the Compressed Liquid.** When a liquid has a temperature below the saturation temperature equivalent to its pressure, it has (Art. 8:3) been variously termed a *subcooled*, a *superpressure*, or a *compressed* liquid. Although it has long been realized that the volume of water could actually be reduced slightly by placing it under pressure, lack of reliable quantitative data has made the calculation of the properties of the compressed fluid, as they are affected by this compressibility, impractical. The errors involved in considering water to be incompressible are not great except at the higher temperatures and pressures that are becoming characteristic of the steam power plant of recent design, and it has been the custom to accept these errors and carry out the calculation on this simplified basis.

Thus, the specific volume of the superpressure liquid would be considered the same as that of the saturated liquid *at the same temperature*. On the same basis, the entropy and internal energy can also be obtained as the tabulated values for the saturated liquid based on the *temperature alone*.

The  $APv$  term is larger for the superpressure than for the saturated liquid because of the increased pressure, and the enthalpy may be calculated as

$$h = u + APv = u_f + APv_f$$

or, since  $h_f = u_f + AP_{\text{sat}}v_f$ ,

$$h = h_f + A(P - P_{\text{sat}})v_f \quad (8:11)$$

When greater accuracy is required and it is desired to take into account the actual compressibility of water, Table 4 of the Keenan and Keyes unabridged tables may be used. Temperatures are keyed along the top of this table, pressures along the left side. Critical examination of this table will indicate that the degree of compressibility is not large at the lower temperatures but increases rapidly above 500°F. and that above this temperature the reduction in specific volume, together with the reduction in internal energy, may more than compensate for the superposed pressure resulting in enthalpies *less* than those associated with the saturated liquid. For all calculations involving the properties of com-

pressed water at these higher temperatures or when maximum accuracy is desired, Table 4 should be employed.

*Example A.* Calculate the specific volume, entropy, enthalpy, and internal energy of a pound of water at 100°F. and 400 p.s.i. abs., (a) assuming incompressibility and (b) taking compressibility into account.

*Solution.*

$$(a) \quad v = v_f = 0.01613 \text{ cu. ft.}$$

$$P_{\text{sat}} = 0.9492 \text{ p.s.i. abs.}$$

$$h = h_f + A(P - P_{\text{sat}})v_f = 67.97 + \frac{1}{778}(400 - 0.9492)0.01613 = 69.16 \text{ B.t.u.}$$

$$s = s_f = 0.1295$$

$$u = u_f = h - APv = 69.16 - \frac{1}{778}(400)0.01613 = 67.97 \text{ B.t.u.}$$

(b) From Table 4, Keenan and Keyes unabridged tables,

$$v - v_f = -0.000021 \text{ cu. ft.}; \quad h - h_f = +1.09 \text{ B.t.u.}; \quad s - s_f = -0.00016$$

$$v = 0.016132 - 0.000021 = 0.016111 \text{ cu. ft.}$$

$$h = 67.97 + 1.09 = 69.06 \text{ B.t.u.}$$

$$s = 0.12948 - 0.00016 = 0.12932$$

$$u = h - APv = 69.06 - \frac{1}{778}(400)0.01611 = 67.87 \text{ B.t.u.}$$

*Example B.* Calculate the specific volume, enthalpy, entropy, and internal energy of a pound of water at 600°F. and 3,000 p.s.i. abs., (a) assuming incompressibility and (b) taking compressibility into account.

*Solution.*

$$(a) \quad v = v_f = 0.0236 \text{ cu. ft.}$$

$$P_{\text{sat}} = 1542.9 \text{ p.s.i. abs.}$$

$$h = h_f + A(P - P_{\text{sat}})v_f = 617.0 + \frac{1}{778}(3000 - 1542.9)0.0236 = 623.4 \text{ B.t.u.}$$

$$s = s_f = 0.8131$$

$$u = u_f = h - APv = 623.4 - \frac{1}{778}(3000)0.0236 = 610.3 \text{ B.t.u.}$$

(b) From Table 4, Keenan and Keyes unabridged tables,

$$v - v_f = -0.000879 \text{ cu. ft.}; \quad h - h_f = -6.9 \text{ B.t.u.}; \quad s - s_f = -0.0124$$

$$v = 0.023629 - 0.000879 = 0.022750 \text{ cu. ft.}$$

$$h = 617.0 - 6.9 = 610.1 \text{ B.t.u.}$$

$$s = 0.8131 - 0.0124 = 0.8007$$

$$u = h - APv = 610.1 - \frac{1}{778}(3000)0.0227 = 597.5 \text{ B.t.u.}$$

**8:14. Properties of Other Vapors.** Abbreviated tables of the properties of other vapors of thermodynamic interest will be found in Appendix I. The methods which apply to steam and which have been set forth in Eqs. (8:1) to (8:11) can also be applied to all other vapors. The pressure-temperature relationships of various vapors are contrasted in Fig. 8:4.

The zero point from which enthalpy and entropy are calculated is lowered from 32 to  $-40^\circ\text{F.}$  for most vapors which are principally of interest because of their use in connection with the production of tem-

peratures below that of the atmosphere (as "refrigerants"). Examples are the tables of the properties of ammonia, sulphur dioxide, and F-12 Freon to be found in the Appendix. Others (see the table of the properties of carbon dioxide) retain the same zero point as the steam tables but show values for temperatures below 32°F. In this case, the enthalpies and entropies of the saturated liquid are recorded as negative quantities at temperatures below 32°F.

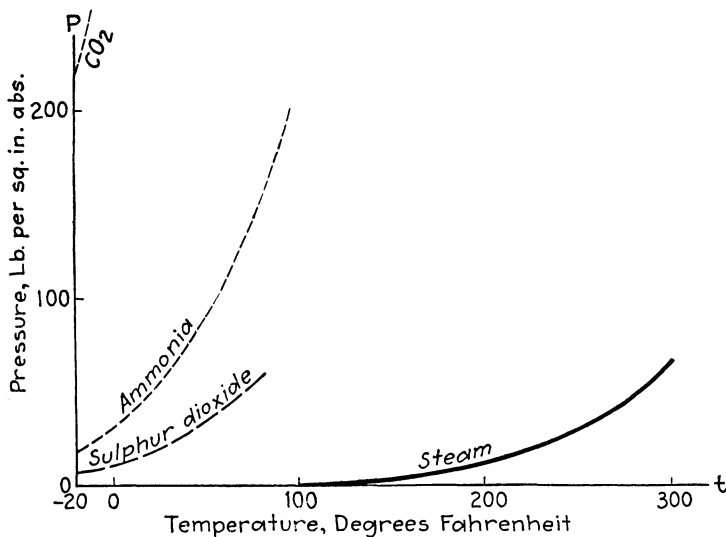


Fig. 8:4.—Pressure-temperature relationship of various saturated vapors.

**8:15. Properties of the Solid.** It is not often necessary to deal with the properties of the solid in engineering thermodynamics. Exceptions include solid water (ice) and solid carbon dioxide ("dry ice") which are principally of importance since they can be manufactured at a central plant and then transported to any location where a need for a refrigerating effect exists; they may be said, therefore, to give portability to refrigeration.

Although it may leave the ice plant at a somewhat lower temperature, the temperature of ice is usually 32°F. by the time it has reached the location where refrigeration is desired. As it changes into water at the same temperature (melts), the latent heat of fusion (see Art. 1:11) will be absorbed from surrounding systems. The latent heat of fusion is approximately 143 B.t.u. per pound of ice melted, and the specific heat of ice is about 0.5. The temperature at which fusion takes place, like the vaporization temperature, is a function of the pressure, at first

decreasing very slowly as the pressure increases. The average rate of this decrease is about  $0.0015^{\circ}\text{F.}$  per pound per square inch until the pressure reaches about 30,000 p.s.i.; above this pressure, the fusion temperature rises with increased pressure. This variation is of greater interest to the physicist than in engineering thermodynamics, and it is customary to give the melting temperature of pure ice as  $32^{\circ}\text{F.}$  without qualification. Mixtures of water with certain other substances such as salt in solution (brine), alcohol and glycerine will solidify at temperatures lower than  $32^{\circ}\text{F.}$ , the fusion temperature varying with the ratio of the mixture.

It will be observed that, at atmospheric pressure, ice may exist in equilibrium with water but not with steam. If the pressure is reduced to 0.088 p.s.i. abs., on the other hand, all three may coexist in equilibrium. Below this pressure, the solid may coexist in equilibrium with the vapor but not with the liquid. This pressure of 0.088 p.s.i. abs. and temperature of  $32^{\circ}\text{F.}$  is the so-called *triple point* for water, ice, and steam. For carbon dioxide, the triple point is located at a pressure of approximately 75 p.s.i. abs. and an approximate temperature of  $-70^{\circ}\text{F.}$  At atmospheric pressure, the temperature of dry ice is approximately  $-110^{\circ}\text{F.}$ ; and since the pressure is below that at the triple point, it will change directly from the solid into the vapor form. This process is called *sublimation* and requires the absorption of the sum of the latent heats of fusion and of evaporation. At atmospheric pressure, this total is about 246 B.t.u. per pound. The total refrigerating effect will include the heat absorbed by the carbon dioxide gas during any increase in temperature which takes place after sublimation and before it leaves the space to be cooled; the calculation of this additional refrigerating effect may be based on the use of a specific heat of the gas of approximately 0.2.

### Problems

1. In each of the cases below, the properties as stated are those of 1 lb. of  $\text{H}_2\text{O}$  and refer to the tabulated values in the Keenan and Keyes tables. For each case, state the condition (superheated or saturated steam, saturated or compressed liquid), and whether the missing properties can be determined based on the given data. Barometric pressure is standard.

- |   |  |
|---|--|
| (a) $p = 150$ p.s.i. abs., $t = 300^{\circ}\text{F.}$   | (b) $p = 30$ p.s.i. gage, $t = 270^{\circ}\text{F.}$ |
| (c) $p = 1,363$ p.s.i. gage, $t = 585^{\circ}\text{F.}$ | (d) $p = 65$ p.s.i. gage, $t = 300^{\circ}\text{F.}$ |
| (e) $p = 200$ p.s.i. abs., $v = 0.01750$ cu. ft.        | (f) $p = 75$ p.s.i. gage, $v = 5$ cu. ft.            |
| (g) $p = 130$ p.s.i. abs., $v = 1.5$ cu. ft.            | (h) $p = 150$ p.s.i. abs., $h = 1,200$ B.t.u.        |
| (i) $p = 100$ p.s.i. gage, $h = 1,189$ B.t.u.           | (j) $p = 40$ p.s.i. abs., $s = 1.6$                  |
| (k) $p = 20$ p.s.i. gage, $s = 1.7$                     | (l) $p = 50$ p.s.i. abs., $u = 275$ B.t.u.           |
| (m) $t = 350^{\circ}\text{F.}$ , $h = 350$ B.t.u.       | (n) $t = 20^{\circ}\text{F.}$ , $h = 500$ B.t.u.     |
| (o) $t = 500^{\circ}\text{F.}$ , $s = 1.4$              | (p) $t = 400^{\circ}\text{F.}$ , $u = 1,100$ B.t.u.  |

2. What is the temperature of dry and saturated steam at a pressure of 120 p.s.i. abs.? Determine its specific volume, enthalpy, entropy, and internal energy.

3. Steam at a pressure of 175 p.s.i. gage has a quality of 100 per cent. What is its temperature? Find its volume, enthalpy, entropy, and internal energy per pound.

4. Dry and saturated steam has a temperature of 420°F. What is its pressure? Tabulate its volume, enthalpy, entropy, and internal energy per pound.

5. Two pounds of dry and saturated steam is confined in a tank. A gage on the tank indicates a pressure of 145 p.s.i. Barometric pressure is 29 in. Hg. What is the temperature of the steam in the tank? Calculate its volume, enthalpy, entropy, and internal energy.

6. What is the temperature of saturated water at a pressure of 110 p.s.i. abs.? List its volume, enthalpy, entropy, and internal energy per pound.

7. Under what pressure is saturated water at 350°F. confined? Determine its volume, enthalpy, entropy, and internal energy per pound.

8. What is the temperature of a pound of steam at a pressure of 180 p.s.i. abs. if its quality is 98 per cent? Calculate its volume, enthalpy, entropy, and internal energy.

9. Three pounds of steam has a quality of 90 per cent and a temperature of 360°F. Calculate its pressure, volume, enthalpy, entropy, and internal energy.

10. A pressure gage on a closed tank which has an internal volume of 10 cu. ft. and contains 3 lb. of steam records a pressure of 105 p.s.i. Barometer is standard. What is the quality or superheat of the steam in the tank? Its temperature? Its total enthalpy, entropy, and internal energy?

11. The enthalpy of a pound of steam at a pressure of 400 p.s.i. abs. is 900 B.t.u. What is its quality or superheat? Its temperature, volume, entropy, and internal energy?

12. The entropy of 3 lb. of steam at a temperature of 150°F. is 5.00. What is its quality or superheat? Its pressure? Its total volume, enthalpy, and internal energy?

13. The internal energy of 2 lb. of steam at 170 p.s.i. abs. is 1600 B.t.u. What is its quality or superheat? Its temperature? Its total volume, enthalpy, and entropy?

14. A pound of steam has a pressure of 220 p.s.i. abs. and a temperature of 490°F. What is its quality or superheat? Obtain its volume, enthalpy, entropy, and internal energy.

15. The volume of 3 lb. of steam at a pressure of 90 p.s.i. abs. is 18 cu. ft. What is its quality or superheat? Its temperature? Its total enthalpy, entropy, and internal energy?

16. The entropy of 2 lb. of steam at a pressure of 161 p.s.i. abs. is 3.4. What is its quality or superheat? Its temperature? Its volume, enthalpy, and internal energy?

17. The enthalpy of 5 lb. of steam at a pressure of 130 p.s.i. abs. is 6250 B.t.u. What is its temperature? Its volume, entropy, and internal energy?

18. A pound of steam at 318 p.s.i. abs. is superheated 160°F. What is its temperature? Its volume, enthalpy, entropy, and internal energy?

19. A pound of steam at a temperature of 400°F. has a superheat of 110°F. What is its pressure? Its volume, enthalpy, entropy, and internal energy?

20. Three pounds of steam has a temperature of 600°F. and an enthalpy of 3900 B.t.u. What is its pressure? Its volume, entropy, and internal energy?

21. Two pounds of steam at 500°F. has an entropy of 3.0. What is its pressure? Its volume, enthalpy, and internal energy?

22. Three pounds of steam at 550°F. has a volume of 4.5 cu. ft. What is its pressure? Its enthalpy, entropy, and internal energy?

23. What is the mean specific heat at constant pressure of superheated steam at a pressure of 400 p.s.i. abs. and a temperature of 600°F.? What is its instantaneous specific heat at constant pressure for that condition? Making use of the mean specific heat as a constant, calculate the entropy of superheat for the given condition. Compare with the value as obtained from the steam tables, and explain any difference.

24. Calculate approximately, using steam-table values over the interval 580 to 620°F., the value of the ratio  $dh/ds$  for steam at 600°F. if the pressure is constant at 400 p.s.i. abs. Note that for a constant pressure process  $\Delta Q = \Delta H$ . Applying Eq. (5:28), what should be the value of  $dh/ds$  in terms of  $T$ ? Check and compare.

25. Assuming the specific heat to be constant at its average value during the superheating process, calculate the entropy of superheat for superheated steam at a pressure of 300 p.s.i. abs., temperature of 700°F. Compare with the value obtained from the steam tables. Why is the latter value larger?

26. Calculate the specific volume, enthalpy, entropy, and internal energy of water at 100°F. and 800 p.s.i. abs., (a) based on assumed incompressibility and (b) taking compressibility into account. Compare results under (a) and (b), and discuss the importance of the differences which develop.

27. Calculate the specific volume, enthalpy, entropy, and internal energy of water at 600°F. and 3,500 p.s.i. abs., (a) based on assumed incompressibility and (b) taking compressibility into account. Compare results under (a) and (b), and discuss the importance of the differences which develop. Do your conclusions differ from those obtained from a study of the result of Prob. 26?

28. Taking compressibility into account, determine the specific volume, enthalpy, entropy, and internal energy of water at 325°F. and 1,200 p.s.i. abs.

29. Calculate the volume, enthalpy, entropy, and internal energy of a pound of ammonia at 10°F. and having a quality of 15 per cent. What is its pressure?

30. Calculate the volume, enthalpy, entropy, and internal energy per pound of ammonia at a pressure of 153 p.s.i. abs. and a temperature of 172°F.

31. Ammonia at a pressure of 176 p.s.i. abs. has a specific entropy of 1.32. What is its temperature? Its volume, enthalpy, and internal energy per pound?

32. A pound of sulphur dioxide at a temperature of 10°F. has an enthalpy of 150 B.t.u. What is its pressure? Its volume, entropy, and internal energy?

33. Determine the volume, enthalpy, entropy, and internal energy per pound of sulphur dioxide if the pressure is 65 p.s.i. abs. and the temperature 155°F.

34. Calculate the temperature and the specific volume, enthalpy, entropy, and internal energy of carbon dioxide vapor at a pressure of 300 p.s.i. abs. and having a quality of 95 per cent.

35. Compare the pressures, specific volumes, enthalpies, and entropies of steam, sulphur dioxide, Freon, ammonia, and carbon dioxide for the dry saturated vapor at a temperature of 40°F.

36. Construct a chart similar to Fig. 8:1 for carbon dioxide.

37. Construct a chart similar to Fig. 8:2 for carbon dioxide.

38. Construct a chart similar to Fig. 8:2 for ammonia based on the values given in Table 4 of the Appendix. The critical temperature, pressure, and specific volume of ammonia are 270°F., 1,640 p.s.i. abs., and 0.068 cu. ft. Using these values, show the approximate curve for the range not covered by Table 4.



## CHAPTER IX

### FRICTIONLESS NONFLOW PROCESSES FOR VAPORS

**9:1. The Temperature-entropy Diagram for Vapors.** The characteristics of a diagram for which absolute temperatures are the ordinates and entropies the abscissas have already been noted in connection with the study of gases and gas cycles. The application of this type of chart, called the temperature-entropy diagram, is even more helpful in the study of vapor processes, for reasons which will be indicated in this chapter. In Appendix II (Plate I) is shown a temperature-entropy diagram for steam. On this chart are shown the liquid and dry-steam lines, meeting at the critical temperature, and lines of constant pressure, constant volume, constant enthalpy, constant internal energy, constant quality, and constant superheat. The chart as shown does not extend below an absolute temperature of  $460^{\circ}$  ( $0^{\circ}\text{F.}$ ) since all ordinary steam conditions fall above that temperature. If, as shown in Fig. 9:1, the diagram is extended to absolute zero, the heat required to change the condition of the steam from state  $A$  to state  $B$  is represented by the area between the reversible state path  $AB$  and the  $S$  axis. This property of the  $TS$  diagram has already been discussed in Chap. V and is of particular value in the study of the thermodynamics of vapors.

The enthalpy of the steam at point  $A$  is equal, it will be remembered, to the heat necessary to raise it from water at  $32^{\circ}\text{F.}$  to state  $A$  at constant pressure. The state path for this constant-pressure process is represented very closely by the line  $CDA$ , the portion  $CD$  corresponding to the heating of the water to saturation temperature  $T_A$  and the portion  $DA$  corresponding to the partial evaporation to the quality  $x_A$ . It will be noted that line  $CD$  is actually only a close approximation of a constant-pressure state path, since the liquid line of the  $TS$  diagram, which it follows, is really the locus of the entropies of the *saturated* liquid and, since the saturation pressure increases with the temperature, represents a path along which the pressure is continually changing. The true constant-pressure path between  $C$  and  $D$  would represent the condition of a superpressure liquid; but if the liquid is considered incompressible, it has been shown in Chap. VIII that the superpressure liquid will have the same entropy as the saturated liquid at the same temperature, and

therefore there will be no displacement of line  $CD$ ; furthermore, even if compressibility is taken into account, the difference between the entropy of the saturated and that of the compressed liquid is relatively small, and the shift in the position of line  $CD$  will be correspondingly slight. Thus, line  $CD$  may be used with negligible error to represent a line of constant pressure.

Making use of the important property of the  $TS$  diagram which is referred to above, the area  $OCDD_1$  under the curve  $CD$  will represent

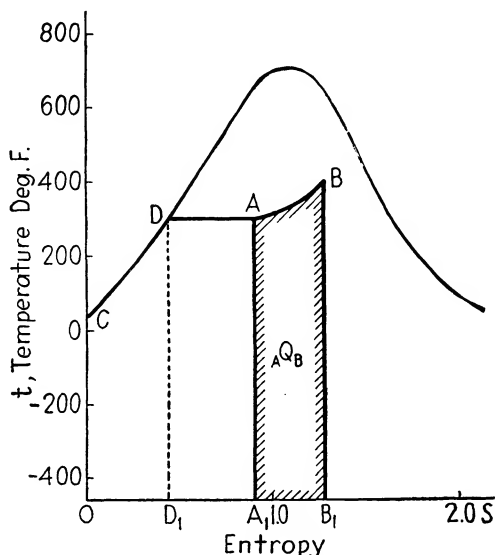


FIG. 9:1.—Application of  $TS$  diagram to vapors.

the heat flow required to raise the liquid from  $32^{\circ}\text{F.}$  to the saturation temperature  $T_A$ ; since the heat flow during a constant-pressure process is measured as the difference between the initial and final enthalpies and since the enthalpy at  $32^{\circ}\text{F.}$  is arbitrarily zero, this area will represent the enthalpy of the saturated liquid. The corresponding area in Fig. 9:2 is  $OABB_1$  and has been appropriately labeled. In Fig. 9:2, line  $BC$  is the constant-pressure path followed during vaporization, and a parallel line of reasoning will indicate that the area  $B_1BCC_1$  will equal the increase of enthalpy during vaporization  $h_{fg}$ . Similarly, when the vapor is superheated at constant pressure along the state path  $CD$ , the area  $C_1CDD_1$  represents the enthalpy of superheat.

The preceding analysis should not lead the reader to the conclusion that areas as developed on the  $TS$  diagram represent changes of enthalpy.



form of the complete Mollier diagram for steam, indicating by the dotted border lines the area covered by the chart in the Appendix. Most of the processes for which this type of diagram is particularly useful involve only the higher qualities or conditions in the superheat region. It will be observed that the critical point is an inflection point on the continuous curve which represents the condition of both the saturated liquid and the dry saturated vapor.

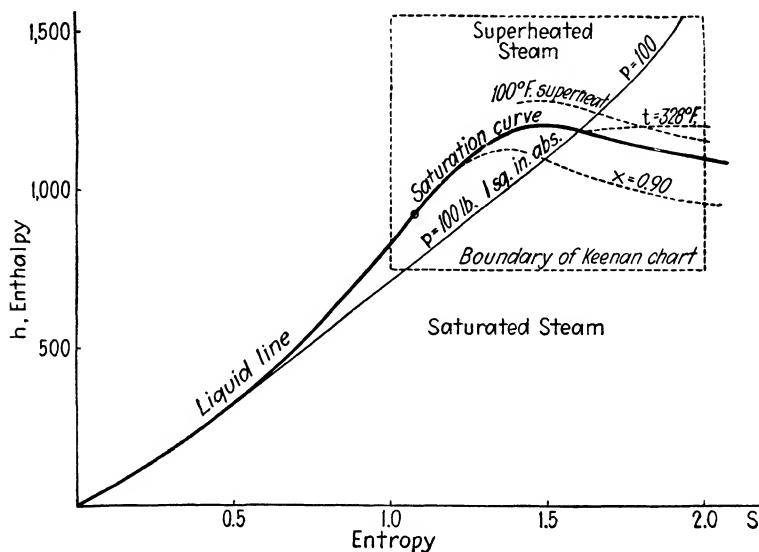


FIG. 9:3.—Mollier diagram for steam.

The slope of any line on the Mollier diagram may be denoted as  $dh/ds$ . For a constant-pressure path,  $dh = dQ$ ; and since  $dQ = T dS$  for a reversible process,  $dh/ds = T$ ; therefore, if a reversible constant-pressure process is plotted on the Mollier diagram, the slope of the line which represents it will be equal to the absolute temperature. In the saturated vapor region (beneath and between the liquid and saturation curves), evaporation at constant pressure proceeds also at constant temperature; lines of constant pressure are consequently straight lines (lines of constant slope) in this space. Moreover, the slope at any point along the liquid line may also be shown to be  $T$  since the pressure changes only infinitesimally in response to the addition of an infinitesimal amount of heat. Thus the slope of the constant-pressure line is equal to that of the liquid line at the point where they join; the line of constant pressure therefore leaves the liquid line as a tangent to that curve. For super-

heated steam (above the saturation curve), the temperature increases with the enthalpy along lines of constant pressure, and the slope of these lines will become continuously greater at higher superheats, as will be observed by referring to the Mollier diagram.

Lines of constant temperature are designated on the Mollier diagram only in the superheated-vapor region; however, in the saturated-vapor region, pressure and temperature are interdependent, and a line of con-

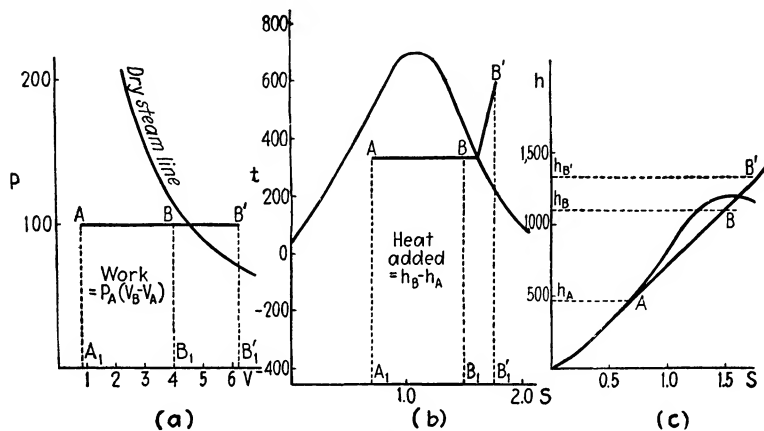


FIG. 9:4.—Vapor changes at constant pressure.

stant pressure is also the locus of conditions for which the temperature is constant.

**9:3. Vapor Changes at Constant Pressure.** For heat added to a vapor at constant pressure, the changes which take place have been detailed in Chap. VIII. These changes are illustrated in Fig. 9:4 on  $Pv$ ,  $Ts$ , and  $hs$  coordinates. Two end points,  $B$  and  $B'$ , are shown, the first in the saturated and the second in the superheat region.

If the nonflow energy equation is applied to the process,

$${}_A Q_B = u_B - u_A + AP_A(v_B - v_A)$$

but  $P_A = P_B$ , and therefore

$${}_A Q_B = u_B - u_A + AP_B v_B - AP_A v_A = h_B - h_A \quad (9:1)$$

The heat added at constant pressure is therefore equal to the change of enthalpy. This applies to processes of this kind through all the states of the vapor, whether liquid, saturated, or superheated.

*Example.* A pound of steam at 80 p.s.i. abs. and 20 per cent quality is heated at constant pressure until it becomes dry and saturated. (a) How much heat is

required? (b) If the process had been continued until the final temperature was 400°F., how many B.t.u. would be necessary?

*Solution.* (a) The initial enthalpy of the steam is

$$h_A = h_f + xh_{fg} = 282.02 + 0.2(901.1) = 462.2 \text{ B.t.u. per pound}$$

The final enthalpy is that of dry saturated steam at 80 p.s.i. abs. or 1183.1 B.t.u. per pound. The heat added is

$${}_AQ_B = 1183.1 - 462.2 = 720.9 \text{ B.t.u. per pound}$$

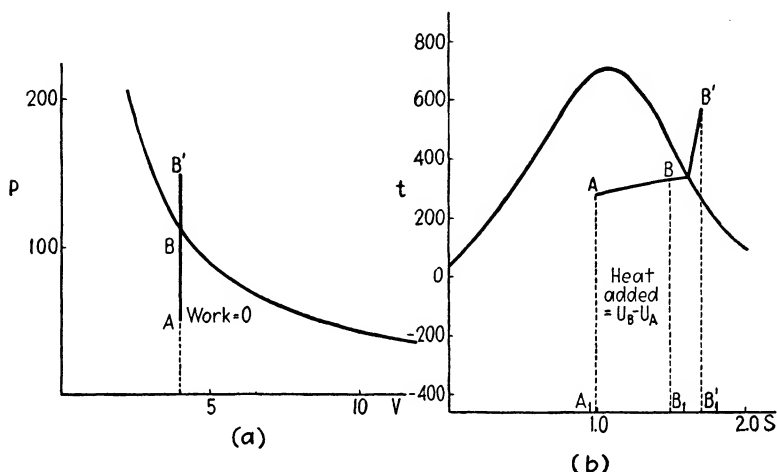


FIG. 9:5.—Vapor changes at constant volume.

(b) For this change the steam in its final condition is superheated to a temperature of 400°F. at a pressure of 80 p.s.i. abs. From Table 3, the enthalpy in the final condition is 1230.7 B.t.u. per pound, and

$${}_AQ_B = 1230.7 - 462.2 = 768.5 \text{ B.t.u. per pound}$$

**9:4. Vapor changes at constant volume** are illustrated in Fig. 9:5. Since there is no change of volume,  $dv$  is zero and no external work is performed. Applying the nonflow energy equation,

$${}_AQ_B = u_B - u_A + A \int P dv = u_B - u_A \quad (9:2)$$

Therefore, the heat required for a constant-volume process is equal to the difference in the internal energy of the steam at the beginning and at the end of the change. This, also, is evidently true for all constant-volume processes, no matter what the state of aggregation of the vapor may be.

*Example.* A pound of steam at 80 p.s.i. abs. and 20 per cent quality is heated at constant volume until the pressure becomes 300 p.s.i. abs. (a) How much heat

is required? (b) If the process had been continued until the pressure became 500 p.s.i. abs., how many B.t.u. would have been necessary?

*Solution.* (a) The volume of the pound of steam is

$$v = x_A v_{gA} = 0.2 \times 5.472 = 1.094 \text{ cu. ft. per pound}$$

The final quality is

$$x_B = \frac{v}{v_{gB}} = \frac{1.094}{1.5433} = 0.709$$

$$u_A = h_A - AP_A v = 462.2 - \frac{144 \times 80 \times 1.094}{778} = 446.0 \text{ B.t.u. per pound}$$

$$u_B = h_B - AP_B v_B = 393.84 + 0.709(809.0) - \frac{144 \times 300 \times 1.094}{778} = 906.7 \text{ B.t.u. per pound}$$

$${}_AQ_B = u_B - u_A = 906.7 - 446.0 = 460.7 \text{ B.t.u. per pound}$$

(b) Dry saturated steam at 500 p.s.i. abs. has a specific volume less than 1.094 cu. ft. per pound, and therefore the final condition for part (b) must lie in the superheat region. With reference to Table 3, steam at 500 p.s.i. abs. and with a specific volume of 1.094 cu. ft. will have a temperature of 558.6°F. and an enthalpy of 1272.5 B.t.u. per pound.

$$u_B = 1272.5 - \frac{144 \times 500 \times 1.094}{778} = 1171.2 \text{ B.t.u. per pound}$$

$${}_AQ_B = 1171.2 - 446.0 = 725.2 \text{ B.t.u.}$$

**9:5. Isothermal (Constant-temperature) Processes for Vapors.** In the saturated-vapor region, constant-temperature lines are also lines of constant pressure. If expansion is carried past the dry-vapor line at constant temperature, the pressure will gradually decrease somewhat as it decreases for a gas during an isothermal expansion. This type of change is illustrated in Fig. 9:6. If the change is confined to the saturated-vapor region, the work performed may easily be calculated, but the value of  $n$  for the curve in the superheat region is not unity as for a perfect gas but is decidedly a variable, depending on both the temperature and the pressure throughout the change. However, the heat added is equal to the area under the state path as plotted on the  $TS$  diagram, or

$${}_AQ_B = T_A(s_B - s_A) \quad (9:3)$$

and, by applying the nonflow energy equation,

$${}_AQ_B = T_A(s_B - s_A) = u_B - u_A + A({}_AW_B)$$

and

$$A({}_AW_B) = T_A(s_B - s_A) - (u_B - u_A) \quad (9:4)$$

**Example.** A pound of steam at 80 p.s.i. abs. and 20 per cent quality is heated isothermally until it becomes dry and saturated. (a) How many B.t.u. are required, and how much external work is performed? (b) If the process is continued until the volume becomes 11.24 cu. ft., what is the total amount of heat added and external work performed?

**Solution.** (a) For this change, which takes place entirely within the saturated-steam region, the isothermal change is also a change at constant pressure. With

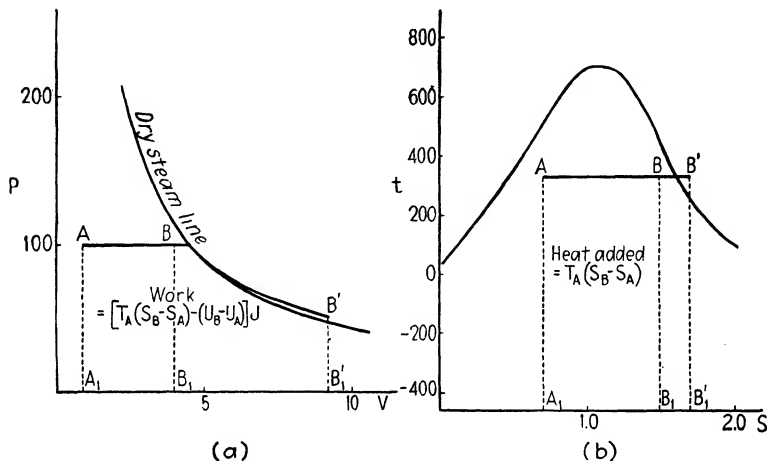


FIG. 9:6.—Isothermal changes for a vapor.

reference to the example of Art. 9:3, the heat added is 720.9 B.t.u., and the external work performed is

$${}_A W_B = P_A(v_B - v_A) = 144 \times 80 \times (5.472 - 1.094) = 50,400 \text{ ft.-lb.}$$

Applying the method suggested in Eqs. (9:3) and (9:4),

$$s_A = s_{fA} + x_A s_{fgA} = 0.4531 + 0.2 \times 1.1676 = 0.6866$$

$$s_B = 1.6207$$

$${}_A Q_B = T_A(s_B - s_A) = (312.03 + 459.6)(1.6207 - 0.6866) = 720.9 \text{ B.t.u.}$$

$$u_A = 446.0 \text{ B.t.u. (as calculated in example of Art. 9:4)}$$

$$u_B = 1102.1 \text{ B.t.u.}$$

$${}_A W_B = J[{}_A Q_B - (u_B - u_A)] = 778[720.9 - (1,102.1 - 446.0)] = 50,400 \text{ ft.-lb.}$$

(b) From Table 3, steam at 312.03°F. (the saturation temperature for a pressure of 80 p.s.i. abs. and therefore the temperature at which the process takes place) and a volume of 11.24 cu. ft. per pound has the following properties:

$$p_B = 40 \text{ p.s.i. abs.}; s_B = 1.7073; h_B = 1192.9 \text{ B.t.u.}$$

$$u_B = h_B - AP_{BvB} = 1192.9 - \frac{144 \times 40 \times 11.24}{778} = 1109.7 \text{ B.t.u. per pound}$$

The heat added during the process is

$${}_A Q_B = T_A(s_B - s_A) = 771.73(1.7073 - 0.6866) = 788.0 \text{ B.t.u.}$$



and

$${}_A W_B = J[{}_A Q_B - (u_B - u_A)] = 778[788 - (1109.7 - 446.0)] = 96,700 \text{ ft.-lb.}$$

It will be noted that this is a constant-pressure change for only a part of the process.

**9:6. The isentropic** (frictionless adiabatic) change for a vapor is illustrated in Fig. 9:7. For this process, for vapors as for gases, the entropy will remain constant. The exponent of the curve which represents this type of change on  $PV$  coordinates no longer has a constant

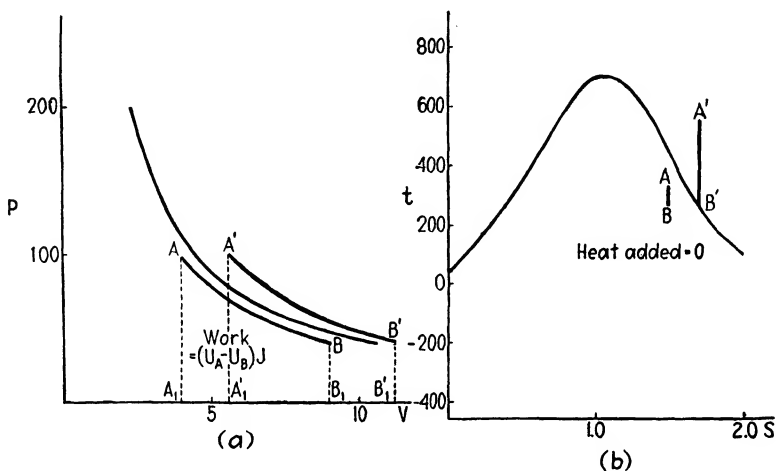


FIG. 9:7.—Isentropic changes for a vapor.

value  $k$  as it has for a perfect gas, however, and therefore the amount of external work performed cannot be calculated directly. Applying the nonflow energy equation,

$${}_A Q_B = 0 = u_B - u_A + A({}_A W_B)$$

and

$${}_A W_B = J(u_A - u_B) \quad (9:5)$$

If the initial state at  $A$  is known and some one fact about the condition at  $B$  (such as pressure or quality) is established, the final state may be definitely located by applying the requirement that  $s_A = s_B$ . The Mollier diagram lends itself readily to the graphical solution of this type of problem.

*Example.* A pound of steam at 200 p.s.i. abs. and quality of 90 per cent is expanded isentropically to a pressure of 15 p.s.i. abs. What is the final quality, and how much external work is performed?

*Solution.*

$$s_B = s_A = s_{fA} + x_A s_{fgA} = 0.5435 + 0.9 \times 1.0018 = 1.4451$$

$$x_B = \frac{s_B - s_{fB}}{s_{fgB}} = \frac{1.4451 - 0.3135}{1.4415} = 0.785$$

This final quality may also be obtained by vertical projection on the Mollier diagram.

$$\begin{aligned} u_A &= h_A - AP_{AvA} = 1198.4 - 0.1 \times 843.0 - \frac{144 \times 200 \times 0.9 \times 2.288}{778} \\ &= 1037.8 \text{ B.t.u. per pound} \end{aligned}$$

$$\begin{aligned} u_B &= h_B - AP_{BvB} = 1150.8 - 0.215 \times 969.7 - \frac{144 \times 15 \times 0.785 \times 26.29}{778} \\ &= 885.0 \text{ B.t.u. per pound} \end{aligned}$$

$$AW_B = J(u_A - u_B) = 778(1037.8 - 885.0) = 118,900 \text{ ft.-lb.}$$

If extreme accuracy is not required, the enthalpies  $h_A$  and  $h_B$  need not be calculated but may be read directly from the Mollier diagram.

### Problems

1. (a) Plot the liquid and dry vapor lines for ammonia on a  $TS$  diagram based on data from Table 4 of the Appendix. Note that the upper portion of these curves cannot be plotted from the data of this table. (b) The critical temperature of ammonia is  $270^\circ\text{F}$ . Assuming an average specific heat of liquid ammonia over the range  $-40$  to  $270^\circ\text{F}$ . to be 1.3, estimate  $s_f$  at the critical point, plot this critical point on your diagram, and draw in as dashed lines on the diagram your conception of the approximate form of the missing portion of the curve. (c) Using Tables 4 and 5, show the location of a line of constant pressure of 200 p.s.i. abs. on this diagram in both the saturated and superheat regions.

2. Plot the liquid and dry-vapor lines for carbon dioxide on a  $TS$  diagram based on data from Table 8 of the Appendix. Estimate the specific heat of the liquid near the critical temperature, and use it to supply the needed data for plotting the critical point on your diagram. Plot the line of constant pressure of 300 p.s.i. abs. in the saturated region of your diagram. Assuming a constant specific heat at constant pressure for carbon dioxide gas (superheated vapor) of 0.22, plot this constant-pressure line in the superheat region.

3. Sketch a  $TS$  diagram for steam, showing the liquid and dry-vapor lines. On this diagram locate points representing each of the following conditions. The stated properties refer to unit weight. Assume incompressibility of water.

- |   |   |
|---|---|
| (a) $t = 300^\circ\text{F.}, s = 1.2$                     | (b) $t = 300^\circ\text{F.}, x = 0.40$                      |
| (c) $t = 300^\circ\text{F.}, p = 100 \text{ p.s.i. abs.}$ | (d) $t = 300^\circ\text{F.}, p = 14.7 \text{ p.s.i. abs.}$  |
| (e) $p = 200 \text{ p.s.i. abs.}, s = 1.0$                | (f) $p = 200 \text{ p.s.i. abs.}, x = 0.90$                 |
| (g) $p = 200 \text{ p.s.i. abs.}, s = 0.4$                | (h) $p = 200 \text{ p.s.i. abs.}, h = 1,250 \text{ B.t.u.}$ |
| (i) $p = 200 \text{ p.s.i. abs.}, u = 400 \text{ B.t.u.}$ | (j) $p = 200 \text{ p.s.i. abs.}, v = 1 \text{ cu. ft.}$    |
| (k) $v = 1 \text{ cu. ft.}, s = 1.6$                      | (l) $v = 1 \text{ cu. ft.}, x = 0.30$                       |

4. Assume that it is possible for saturated water at  $400^\circ\text{F}$ . to undergo a reversible process which will change its condition to dry and saturated steam at 67 p.s.i. abs. and that this process is represented by a straight line on a  $TS$  diagram. Illustrate this process, and calculate the heat flow per pound which accompanies it.

5. Same as Prob. 1, but plot a Mollier diagram for ammonia based on the same data.

6. Repeat Prob. 2, using a Mollier instead of a  $TS$  diagram.
7. Locate points (a) to (l) of Prob. 3 on a Mollier diagram for steam.
8. Calculate, from steam-table data, (a) the slope  $\Delta h / \Delta s$  of the liquid line on the Mollier diagram at a temperature of  $212^\circ\text{F}$ ., using the interval 208 to  $216^\circ\text{F}$ . for the purpose; (b) the slope of the line of constant pressure of 14.7 p.s.i. abs. in the saturated region; (c) the slope of the line of constant pressure of 14.7 p.s.i. abs. when the temperature of the steam is  $300^\circ\text{F}$ . What is the significance of the numerical values of these slopes?
9. In an idealized steam-power-plant cycle, water enters the boiler at  $212^\circ\text{F}$ . and leaves as dry and saturated steam at 200 p.s.i. abs. At the latter condition, it enters the engine and expands at constant entropy to leave this unit and enter the condenser at a pressure of 14.7 p.s.i. abs. It leaves the condenser as a saturated liquid. The boiler and condenser processes take place at constant pressure. Assume the liquid to be incompressible. Show the processes which take place in each of the three units mentioned above on  $TS$  and Mollier diagrams. Do these processes taken together form a closed cycle on both diagrams? Explain. What necessary unit in the power plant has been omitted, and how is its action represented on each diagram? At what condition (quality or superheat) does the steam leave the engine and enter the condenser? Show on the  $TS$  diagram the area which represents the heat added per pound in the boiler; the heat removed in the condenser.
10. A pound of steam at 2 p.s.i. abs. and 90 per cent quality is condensed at constant pressure. Illustrate this process on  $TS$  and Mollier diagrams. Calculate the heat flow, external work, and change of internal energy during the process.
11. A pound of water at  $200^\circ\text{F}$ . is converted into dry and saturated steam. During the process, the pressure is constant at 400 p.s.i. abs. (a) Based on the approximation that water is incompressible, show the process on  $TS$  and Mollier diagrams, and calculate the heat flow, external work, and change of internal energy during the process. (b) If the compressibility of water is taken into account, show changes necessary in diagrams and in calculated quantities.
12. A pound of water at  $500^\circ\text{F}$ . is converted into superheated steam at 1,500 p.s.i. abs. and  $800^\circ\text{F}$ . The process takes place at constant pressure. (a) Based on the approximation that water is incompressible, show the process on  $TS$  and Mollier diagrams, and calculate the heat flow, external work, and change of internal energy during the process. (b) If the compressibility of water is taken into account, show changes necessary in diagrams and in calculated quantities.
13. A pound of steam at 300 p.s.i. abs. and 98 per cent quality enters a constant-pressure process at the end of which it has attained a temperature of  $700^\circ\text{F}$ . Show this process on  $TS$  and Mollier diagrams. Calculate the heat flow, external work, and change of internal energy during the process. The process carried out in the superheater of the steam power plant is a constant-pressure flow rather than a non-flow process. Assuming that the velocities at entrance to and exit from the superheater are equal, show that the heat flow will be the same as calculated above.
14. A pound of steam at 5 p.s.i. abs., quality 0.85, is condensed to saturated water at constant pressure. Show this process on  $TS$  and Mollier diagrams. Calculate the heat flow, external work, and the change of internal energy. The condenser of a steam power plant carries out this process as a constant-pressure flow process. Assuming the velocities of the steam at entrance to and the water leaving the condenser are equal, show that the heat flow will be the same as calculated for the non-flow process.

15. The process which takes place in the boiler of the steam power plant is essentially a constant-pressure flow process. Assuming that the difference between the velocities of the entering feed water and the steam at exit from the boiler is negligible, compare with the nonflow process which connects the equivalent initial and final states and show that (a) the heat flow per pound is the same in both cases and (b) the net flow energy in the flow process is equal to the external work which accompanies the nonflow process.

16. A pound of ammonia at  $0^{\circ}\text{F}$ . and quality of 0.20 receives heat at constant pressure until dry and saturated. Show the process on  $TS$  and Mollier diagrams. Calculate the heat flow, the external work, and the change of internal energy.

17. A pound of steam at 5 p.s.i. abs., quality 0.85, is confined within a closed rigid container. Heat is abstracted from the steam until its quality is reduced to 0.05. What is its final pressure? How much heat must be removed? Show this process on a  $TS$  diagram.

18. A pound of dry and saturated steam is confined in a closed tank at a pressure of 90 p.s.i. abs. The pressure gradually decreases to 20 p.s.i. abs. What is the final condition (quality or superheat)? Show this process on a  $TS$  diagram. Calculate the heat flow, the external work, and the change of internal energy during the process.

19. A pound of dry and saturated steam at 50 p.s.i. abs. is confined in a closed tank. Heat is applied until the pressure becomes 80 p.s.i. abs. What is the final condition (quality or superheat)? Show this process on a  $TS$  diagram. Calculate the heat flow, the external work, and the change of internal energy during the process.

20. A pound of saturated water at  $350^{\circ}\text{F}$ . is converted into dry and saturated steam. During the process the temperature is constant. Show this process on  $TS$  and Mollier diagrams. Calculate the heat flow, the external work, and the change of internal energy.

21. A pound of saturated water at  $400^{\circ}\text{F}$ . is converted into steam at a pressure of 100 p.s.i. abs. During the process the temperature is constant. Show this process on  $TS$  and Mollier diagrams, and calculate the heat flow, the external work, and the change of internal energy which accompanies it.

22. A closed rigid container holds dry and saturated steam at the critical temperature. Heat is abstracted until the temperature decreases to  $100^{\circ}\text{F}$ . (a) Find the condition of the steam (quality or superheat) at the final condition. (b) If the original contents had been saturated water at the critical temperature, how would your answer have been affected? (c) How much heat would it have been necessary to abstract in (a) and (b) per pound of  $\text{H}_2\text{O}$ ?

23. A pound of steam at 400 p.s.i. abs., quality of 0.99, expands adiabatically and without friction to a pressure of 5 p.s.i. abs. Locate this process on  $TS$  and Mollier diagrams. What are the final quality and enthalpy? Calculate the heat flow, the external work, and the change of internal energy.

24. A pound of steam at 300 p.s.i. abs. and  $600^{\circ}\text{F}$ . expands isentropically to a final pressure of 5 p.s.i. abs. Locate this process on  $TS$  and Mollier diagrams. Determine the final quality and enthalpy. Calculate the heat flow, external work, and change of internal energy.

25. Show that, for the adiabatic flow process which connects conditions 1 and 2, the heat equivalent of the external work performed per pound is  $h_1 - h_2$  if the original and final velocities are equal.

26. Construct a theoretical indicator card (a  $Pv$  diagram) for the adiabatic and frictionless expansion of 1 lb. of steam in a steam-engine cylinder having neither

clearance nor compression. Show (a) the flow of steam into the cylinder at pressure  $P_1$  until the entire pound is contained within the cylinder (condition 1), (b) the adiabatic and frictionless expansion of the charge to exhaust pressure (condition 2) which takes place with both admission and exhaust valves closed, (c) the rejection of *all* the charge at exhaust pressure  $P_2$ , and (d) the rise in pressure which takes place as the exhaust valve closes and the admission valve opens (assume this takes place at zero cylinder volume). By algebraic addition of areas on this diagram, show that the external work of the cycle is  $h_1 - h_2$  B.t.u., per pound of steam in the cylinder during the time both valves are closed.

27. Dry and saturated ammonia at  $0^\circ\text{F.}$  is compressed isentropically to a final pressure of 180 p.s.i. abs. Locate the process on a  $TS$  diagram. What is the final condition (quality or superheat, temperature, specific enthalpy)? Calculate the heat flow, external work, and change of internal energy per pound during the process.

28. A pound of saturated ammonia liquid at 180 p.s.i. abs. undergoes a process at the end of which its temperature is  $0^\circ\text{F.}$  and its enthalpy is the same as at the original condition. (a) What is its final condition (quality or superheat, pressure)? (b) If this is a nonflow adiabatic process (not frictionless), how much external work accompanies it? (c) If a flow process, the initial and final velocities being negligible, what is the external work?

29. From data taken during a test of a steam engine, it is determined that 0.3 lb. of steam is present in the cylinder during the expansion. Two points on this expansion curve are marked on the indicator card. Measurements taken from the card, based on the piston displacement and clearance volume of the cylinder, the scale of the indicator spring, and the barometric pressure, show that the pressure at the earlier of these points is 120 p.s.i. abs. and the volume 1 cu. ft. The corresponding values at the second point are 60 p.s.i. abs. and 1.8 cu. ft. Measurement of area under the expansion curve indicates that the steam performs 10,000 ft.-lb. of external work on the piston in its expansion between these two stations. (a) What is the condition of the steam (quality or superheat) at each of these two points? (b) How much heat flow takes place between the steam and the cylinder walls, head, and piston during this interval, and in what direction does it flow?

## CHAPTER X

### STEADY-FLOW PROCESSES FOR GASES AND VAPORS

**10:1. Fluid Flow.** The nonflow process has been considered individually for gases (Chap. IV) and for vapors (Chap. IX). There is little advantage to be gained from a separate discussion as applied to the flow condition, and therefore both phases of fluid flow will be developed along parallel lines in this chapter. The term "fluid" will be understood as having a general application to liquids, gases, and vapors.

It will be remembered that thermodynamically the flow condition applies when a fluid is passing continuously and steadily through a channel and the energy balance is based upon the conditions at two or more locations along the channel. A knowledge of the principles which govern the flow of fluids is necessary for the design of nozzles, for their expansion, and for the design of pipe lines for their transmission.

**10:2. Flow Assumptions.** In the study of flow conditions as applied to the gas or vapor, it is customary to make the following assumptions:

1. That the channel is completely filled by the fluid.
2. That the weight passing any section  $A$  in unit time is equal to the weight passing any other section  $B$  of the channel in the same time interval.
3. That the individual particles of the fluid follow nonintersecting paths, *i.e.*, that the flow is without turbulence.

The general equations of fluid flow will be developed on the basis of these assumptions and their accuracy tested by comparison with equations derived from experimental results.

**10:3. The Flow Equation.** The first and second assumptions make possible the statement of the first relationship,

$$aw = Mv \quad (10:1)$$

in which  $a$  is the cross-sectional area of the channel in square feet, measured perpendicular to the direction of flow,

$w$  is the velocity of flow in feet per second,

$M$  is the weight of fluid flowing past that section in pounds per second,

and

$v$  is the specific volume of the fluid at the given section in cubic feet per pound.

The units of measurement suggested refer to those usually employed in the English system of units; any other consistent set of units may be used. Each side of the equation represents the total volume of fluid passing the given section in unit time. If  $w$  and  $v$  are understood as *average* values throughout the section, this equation will hold as accurately in practice as in theory.

The third assumption is not quite true in practice; actually, some interference will exist between the particles in flow. The effect of such interference is to reduce the amount of energy which can be transformed into velocity (kinetic energy) at any section. This is a frictional effect; the properly designed channel will have a minimum of friction.

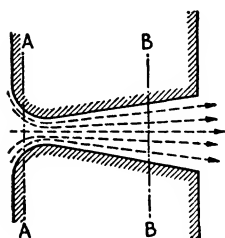


FIG. 10:1.—Fluid flow.

If it is assumed that no external work is performed by the fluid as it passes through the channel, the application of the flow equation [Eq. (2:6)] to an element of mass of the fluid justifies the statement that the total energy at any section A, as in Fig. 10:1, plus the heat added between the sections is equal to the total energy of this element of the mass at the second section B.

The total energy at any section is equal to the sum of the kinetic energy, the internal energy, the flow energy, and the potential mechanical energy due to the elevation of the channel. Of these, it is possible to omit the last when the equation is written for a gas or vapor for the reasons explained in Chap. II. The following expression may be written as the relationship which exists, in terms of energy per pound, between any two sections A and B:

$$\frac{Aw_A^2}{2g} + u_A + AP_Av_A + {}_AQ_B = \frac{Aw_B^2}{2g} + u_B + AP_Bv_B \quad (10:2)$$

in which  ${}_AQ_B$  represents the heat entering through the channel walls between sections A and B per pound of fluid flowing.

**10:4. Velocity of Flow.** If the flow is adiabatic, even if not frictionless,  ${}_AQ_B$  is zero and

$$\frac{Aw_B^2}{2g} - \frac{Aw_A^2}{2g} = (u_A + AP_Av_A) - (u_B + AP_Bv_B) = h_A - h_B \quad (10:3)$$

If, as is often the case in practical applications of this equation, the section A is assumed at a point in the reservoir from which the flow takes place,  $w_A$  is practically zero. For this condition, Eq. (10:3) may

be written (representing the initial condition in the reservoir by the subscript 1) in the shortened form

$$\frac{Aw_B^2}{2g} = h_1 - h_B \quad (10:4)$$

If the velocity  $w_2$  at exit from the nozzle is desired, the second section  $B$  will, of course, be taken at exit from the nozzle and Eq. (10:4) becomes

$$\frac{Aw_2^2}{2g} = h_1 - h_2 \quad (10:5)$$

**10:5. Application to Gas Flow.** Since the change of enthalpy of a gas is a function of the temperature change [see Eq. (4:28)], Eq. (10:5) as applied to gas flow becomes

$$\frac{Aw_2^2}{2g} = C_p(T_1 - T_2) \quad (10:6)$$

It will be noted that the foregoing equation assumes a constant value for the specific heat.

The temperature  $T_2$  at exit from the nozzle, for a process which is adiabatic and frictionless, can be found by the use of the equations of Chap. IV by using  $k$  as the exponent of the expansion curve and the known pressure ratio at inlet and exhaust. If internal friction exists, the effect will be to increase  $T_2$  and so decrease the velocity below its ideal or maximum value. With the proper adjustment of  $T_2$  to the actual temperature at exit, Eq. (10:6) therefore applies for a process involving friction.

Since, for unit weight,  $T = Pv/R$  and since  $C_p = kAR/(k - 1)$ , if the initial velocity is inappreciable and the flow is adiabatic and frictionless, the terminal velocity is [from Eq. (10:6)]

$$\begin{aligned} w_2 &= \left[ \frac{2g}{A} \times C_p \times (T_1 - T_2) \right]^{\frac{1}{2}} \\ &= \left[ \frac{2g}{A} \times \frac{k}{k-1} \times AR \times \frac{(P_1v_1 - P_2v_2)}{R} \right]^{\frac{1}{2}} \\ &= \left[ 2g \times \frac{k}{k-1} \times (P_1v_1 - P_2v_2) \right]^{\frac{1}{2}} \\ &= \left[ 2g \times \frac{k}{k-1} \times RT_1 \left( 1 - \frac{T_2}{T_1} \right) \right]^{\frac{1}{2}} \\ &= \left\{ 2g \times \frac{k}{k-1} \times RT_1 \left[ 1 - \left( \frac{p_2}{p_1} \right)^{\frac{k-1}{k}} \right] \right\}^{\frac{1}{2}} \end{aligned} \quad (10:7)$$



*Example A.* A nozzle expands air adiabatically from an initial pressure of 60 p.s.i. gage and temperature of 80°F. to the atmosphere. Calculate the theoretical velocity at the exit.

*Solution.*

$$p_1 = 60 + 14.7 = 74.7 \text{ p.s.i. abs.}$$

$$p_2 = 14.7 \text{ p.s.i. abs.}$$

The theoretical flow will be based on an initial velocity of zero and upon adiabatic flow without friction; then

$$\begin{aligned} w_2 &= \left\{ 2g \times \frac{k}{k-1} \times RT_1 \left[ 1 - \left( \frac{p_2}{p_1} \right)^{\frac{k-1}{k}} \right] \right\}^{\frac{1}{2}} \\ &= \left\{ 64.4 \times \frac{1.405}{0.405} \times 53.3 \times 539.7 \left[ 1 - \left( \frac{14.7}{74.7} \right)^{0.288} \right] \right\}^{\frac{1}{2}} \\ &= 1,550 \text{ ft. per second} \end{aligned}$$

Or, since

$$T_2 = T_1 \left( \frac{p_2}{p_1} \right)^{\frac{k-1}{k}} = 539.7 \left( \frac{14.7}{74.7} \right)^{0.288} = 338^\circ\text{F. abs.}$$

$$\begin{aligned} w_2 &= [2g \times 778 \times C_p \times (T_1 - T_2)]^{\frac{1}{2}} = 223.8[C_p(T_1 - T_2)]^{\frac{1}{2}} \\ &= 223.8[0.2375(539.7 - 338)]^{\frac{1}{2}} = 1,550 \text{ ft. per second} \end{aligned}$$

In Fig. 10:1 the nozzle has been represented as first converging and, ultimately, diverging (increasing in cross-sectional area). The narrowest section of the nozzle is called the throat; at the throat, the walls of the nozzle are parallel, and two successive cross sections will have the same area. If Eq. (10:1) is arranged in the form  $a = Mv/w$ , it will be evident that, of two successive sections across the nozzle passage, the downstream section should have the smaller area if  $w$  has increased at a *rate* relatively more rapid than the *rate* of increase of  $v$  between the two sections; if the opposite condition holds, the downstream section should have the larger area. At the throat, these two sections have equal area, and it may be shown that  $dv/v = dw/w$ . This relationship may be employed to determine the pressure at the throat. Experimental results have shown that friction develops in a nozzle almost entirely after the throat has been passed; the flow will therefore be assumed as frictionless as well as adiabatic. If the unknown pressure and specific volume of the fluid at any nozzle section are designated as  $P$  and  $v$ , we may write

$$Pv^k = P_1v_1^k \quad \text{or} \quad v = P_1^{\frac{1}{k}}v_1 P^{-\frac{1}{k}}$$

from which

$$dv = -\frac{1}{k} P_1^{\frac{1}{k}}v_1 P^{-\frac{k+1}{k}} dP$$

and

$$\frac{dv}{v} = \left( -\frac{1}{k} P_1^{\frac{1}{k}}v_1 P^{-\frac{k+1}{k}} \right) \left( P_1^{-\frac{1}{k}}v_1^{-1} P^{\frac{1}{k}} \right) dP = -\frac{dP}{kP} \quad (10:8)$$

Also,

$$w^2 = 2g \left( \frac{k}{k-1} \right) P_1 v_1 \left[ 1 - \left( \frac{P}{P_1} \right)^{\frac{k-1}{k}} \right] = 2g \left( \frac{k}{k-1} \right) P_1 v_1 - 2g \left( \frac{k}{k-1} \right) P_1^{\frac{1}{k}} v_1 P^{\frac{k-1}{k}}$$

from which

$$d(w^2) = 2w dw = - \left( \frac{k-1}{k} \right) 2g \left( \frac{k}{k-1} \right) P_1^{\frac{1}{k}} v_1 P^{-\frac{1}{k}} dP = - 2g P_1^{\frac{1}{k}} v_1 P^{-\frac{1}{k}} dP$$

and

$$\begin{aligned} \frac{dw}{w} &= \frac{2w dw}{2w^2} = \frac{- 2g P_1^{\frac{1}{k}} v_1 P^{-\frac{1}{k}} dP}{4g [k/(k-1)] P_1 v_1 \left[ 1 - (P/P_1)^{\frac{k-1}{k}} \right]} \\ &= \frac{- P^{-\frac{1}{k}} dP}{2 [k/(k-1)] \left[ P_1^{\frac{k-1}{k}} - P^{\frac{k-1}{k}} \right]} \end{aligned} \quad (10:9)$$

At the throat,

$$\frac{dv}{v} = \frac{dw}{w} \quad (10:10)$$

and, designating the pressure at the throat as  $P_0$ ,

$$- \frac{dP}{kP_0} = \frac{- P_0^{-\frac{1}{k}} dP}{2 [k/(k-1)] P_1^{\frac{k-1}{k}} - 2 [k/(k-1)] P_0^{\frac{k-1}{k}}} \quad [\text{from Eqs. (10:8) and (10:9)}]$$

from which

$$2 \left( \frac{k}{k-1} \right) P_1^{\frac{k-1}{k}} - 2 \left( \frac{k}{k-1} \right) P_0^{\frac{k-1}{k}} = k P_0^{\frac{k-1}{k}}$$

or

$$\begin{aligned} \left( k + \frac{2k}{k-1} \right) P_0^{\frac{k-1}{k}} &= \frac{2k}{k-1} P_1^{\frac{k-1}{k}} \\ \left( \frac{P_0}{P_1} \right)^{\frac{k-1}{k}} &= \frac{2k (k-1)}{k (k+1)/(k-1)} = \frac{2}{k+1} \end{aligned}$$

and

$$\frac{P_0}{P_1} = \left( \frac{2}{k+1} \right)^{\frac{k}{k-1}} \quad (10:11)$$

If the value of  $k$  for monatomic gases (1.67) is substituted in Eq. (10:11), the ratio of throat to initial pressure will be found to be about 0.49. For air and the diatomic gases, the ratio is 0.53; for more complicated molecules, higher ratios will result. This ratio will be referred to as the *critical-pressure ratio* for gas flow.

In the flow of a gas through a nozzle which is shaped as in Fig. 10:2, the pressure at the smallest section of the channel will be equal to the pressure on the exhaust side of the nozzle (the final pressure  $P_2$ ) if that pressure is higher than the critical throat pressure as calculated by applying Eq. (10:11). In that case, this section of smallest area would be the last section which would play any part in the expansion of the

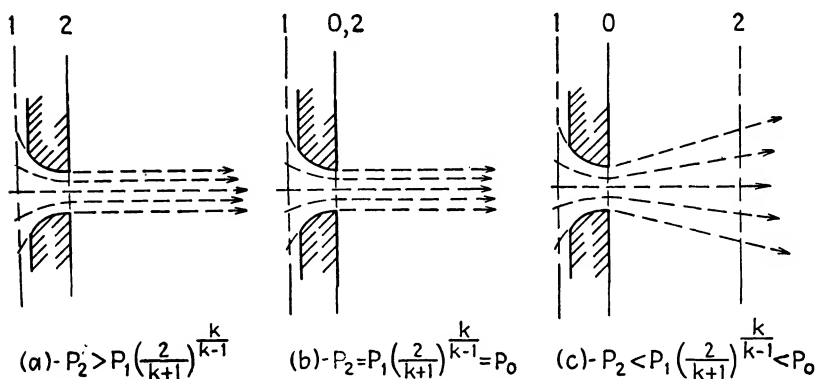


FIG. 10:2.—Flow through a gas nozzle for various values of the ratio  $P_2/P_1$ .

fluid and would therefore be the exit section. Accordingly, as indicated in Fig. 10:2a, conditions at that point are designated by the subscript 2. If the nozzle is circular in cross section, the jet will issue from this section in cylindrical form, as shown. If, as in Fig. 10:2b, the exit pressure  $P_2$  is equal to the critical throat pressure, the flow will again issue in the form of a cylinder from the smallest section of the nozzle. This section may now, however, be properly called the throat, and the subscript 0 can be applied to designate conditions at that point. The mass rate of flow will be somewhat greater than for case *a* because of the larger drop in pressure and temperature. Lastly, as the exhaust pressure becomes less than the critical throat pressure, the pressure *at the throat* will remain constant at  $P_0$ , the pressure drop from  $P_0$  to  $P_2$  taking place after that point has been passed. In this part of the expansion,  $dv/v > dw/w$ , and the jet expands in the exhaust space as shown in Fig. 10:2c. The mass rate of flow through the nozzle will be determined by the flow through its smallest section and therefore by the drop in pressure between the en-

trance and this section. As the final pressure  $P_2$  decreases, the pressure at this smallest section will decrease with it until  $P_2$  drops below the critical throat pressure; any further decrease of  $P_2$  will not change the pressure at the throat, and consequently the mass rate of flow through the nozzle will reach a maximum as  $P_2$  becomes equal to  $P_0$  and will thereafter remain constant at this maximum for any further decrease of the exhaust pressure. When  $P_2 < P_0$ , the nozzle is called an *expanding nozzle*. To reduce frictional effects, expanding nozzles should be designed with walls which continue beyond the throat section, diverging until the exit area is sufficient to meet the requirements of the flowing jet as it reaches the final pressure  $P_2$ .

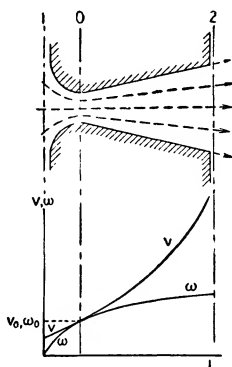


FIG. 10:3.—Variation of specific volume and velocity in flow through an expanding nozzle.

In Fig. 10:3, the variation of specific volume and of velocity of the fluid during flow through an expanding nozzle is plotted against the distance traversed along the nozzle axis. Examination of the equation which connects the pressure and the specific volume will indicate that the specific volume will increase at first slowly and then more and more rapidly with respect to a unit decrease of pressure. The velocity, starting from zero, will increase at first very rapidly and later at a decreasing rate with respect to a unit pressure drop, as the fluid passes through the nozzle; this may be confirmed by returning to Eq. (10:7). If the scales to which  $v$  and  $w$  are measured on the graph are adjusted to values such that the same distance measures both  $v_0$  and  $w_0$ , the two curves will be tangent at the throat of the nozzle, as shown in Fig. 10:3.

The velocity at the throat, in terms of the properties of the gas at that point, may be developed as follows [from Eq. (10:7)]:

$$w_0^2 = 2g \left( \frac{k}{k-1} \right) RT_1 \left( 1 - \frac{T_0}{T_1} \right)$$

From Eqs. (4:24) and (10:11),

$$T_1 = T_0 \left( \frac{P_1}{P_0} \right)^{\frac{k-1}{k}} = T_0 \left[ \left( \frac{k+1}{2} \right)^{\frac{k}{k-1}} \right]^{\frac{k-1}{k}} = T_0 \left( \frac{k+1}{2} \right)$$

and  $\frac{T_0}{T_1} = \frac{2}{k+1}$

Also,

$$RT_0 = P_0 v_0$$

and, substituting in the equation for  $w_0^2$ ,

$$\begin{aligned} w_0^2 &= 2g \left( \frac{k}{k-1} \right) P_0 v_0 \left( \frac{k+1}{2} \right) \left[ 1 - \frac{2}{k+1} \right] \\ &= 2g \left( \frac{k}{k-1} \right) P_0 v_0 \left( \frac{k+1}{2} \right) \left( \frac{k-1}{k+1} \right) = kg P_0 v_0 \end{aligned}$$

or

$$w_0 = (kg P_0 v_0)^{\frac{1}{2}} \quad (10:12)$$

Referring to texts in the field of physics, it is found that this expression is given as the velocity of sound in a gas at the pressure  $P_0$  and having the specific volume  $v_0$ . Thus the velocity at the throat of an expanding nozzle will have attained the velocity of sound in that gas under the conditions which exist at that point. The velocity downstream from the throat in an expanding nozzle has been shown to be greater than  $w_0$  and will exceed the velocity of sound in the medium.

Sound is propagated in a gas or vapor by alternate bands of compressed and rarefied fluid, and it is not surprising to note that, under certain conditions of operation, pressure (or "shock") waves will develop in the diverging section of the nozzle, usually near the throat. A discussion of the special conditions under which these shock waves may be expected to develop and the effects which they produce is left to more advanced texts in thermodynamics and fluid mechanics.

Nozzles are classified as converging or diverging according to whether the area at exit is the smallest area of the nozzle or is larger than the throat area, as illustrated in *a* and *b* of Fig. 10:4. Converging nozzles are also called orifices, a term that includes the sharp-edged opening shown in *c* of Fig. 10:4.

As has been shown, when the pressure in the discharge reservoir is continuously reduced, with  $P_1$  held constant, the pressure at the exit of a converging nozzle or orifice  $P_2$  will also decrease, remaining equal to the discharge pressure until that pressure reaches the critical pressure  $P_0(0.53P_1$  for air flow); thereafter the pressure at the nozzle exit will

remain constant, and expansion will continue beyond the exit section of the nozzle, the fluid forming its own channel between walls formed of the fluid itself. The velocity will continue to increase until the pressure has dropped to the discharge pressure. Thus, a converging nozzle may become an expanding nozzle; this condition is not desirable, however, since the frictional effects set up will be greater than if the expansion takes place entirely in a smooth-walled channel. Therefore, the design of a nozzle that is to develop a maximum kinetic energy of the discharge jet should be based carefully upon the known inlet and exhaust (exit) pressures as well as the required discharge rate and should be designed

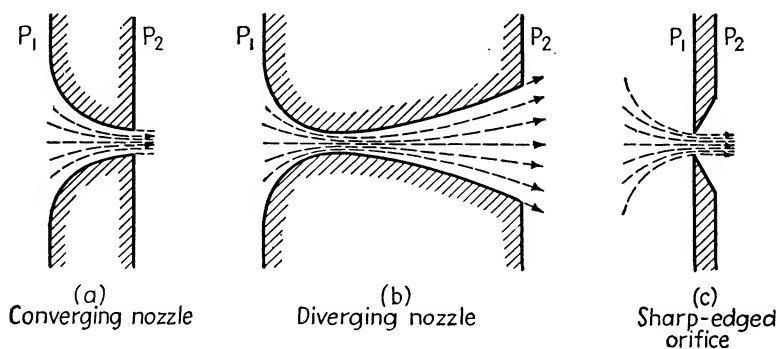


FIG. 10:4.—Nozzles and orifices.

as a converging nozzle if the discharge pressure is equal to or above the critical, as a diverging nozzle if below the critical pressure.

Orifices with sharp edges often do not permit the same discharge rates as converging nozzles with rounded entrances having the same minimum cross-sectional area, since the jet will converge to smaller cross section at some point beyond the opening, as shown in Fig. 10:4. The amount of convergence will depend upon the area of the opening, its shape, and the pressure ratio  $P_2/P_1$  as well as the density of the flowing fluid. Orifices are often used for the measurement of rates of fluid flow, and this measurement is based upon the excess of upstream over downstream pressure. So-called *discharge coefficients* are used to reflect the effect of convergence of the jet, frictional, and other effects; these discharge coefficients are determined by calibration of the orifice.

*Example B.* If the area at the throat of the nozzle in Example A of Art. 10:5 is 0.5 sq. in., what weight of air will be discharged per hour?

*Solution.* The pressure at the throat of the nozzle is

$$p_0 = p_1 \times 0.53 = 74.7 \times 0.53 = 39.6 \text{ p.s.i. abs.}$$

The velocity at the throat is

$$w_0 = \left\{ 64.4 \times \frac{1.405}{0.405} \times 53.3 \times 539.7 \times [1 - (0.53)^{0.288}] \right\}^{\frac{1}{2}} = 1,040 \text{ ft. per second}$$

$$T_0 = T_1 \left( \frac{P_0}{P_1} \right)^{\frac{k-1}{k}} = 539.7(0.53)^{0.288} = 450^\circ\text{F. abs.}$$

$$v_0 = \frac{RT_0}{P_0} = \frac{53.3 \times 450}{144 \times 39.6} = 4.21 \text{ cu. ft. per pound}$$

and

$$M = \frac{a_0 w_0}{v_0} = \frac{0.5 \times 1,040}{144 \times 4.21} = 0.86 \text{ lb. per second or 3,100 lb. per hour}$$

**10:6. Fliegner's Equations for Air Flow.** The derivation of Eqs. (10:7) to (10:12) has been based on frictionless adiabatic flow. Based upon his experiments, Fliegner deduced the following empirical equations for the flow of air:

When  $p_1$  is greater than  $2p_2$ ,

$$M = 0.53 \frac{a_0 p_1}{(T_1)^{\frac{1}{4}}} \quad (10:13)$$

When  $p_1$  is less than  $2p_2$ ,

$$M = 1.06 a_0 \left[ \frac{p_2(p_1 - p_2)}{T_1} \right]^{\frac{1}{2}} \quad (10:14)$$

in which  $M$  is the weight of air flowing in pounds per second,  $a_0$  is the area of the throat of the nozzle in square inches, and the pressures are expressed in pounds per square inch absolute.

When the proper values for air are substituted in the equations developed in Art. 10:3 to 10:5 for adiabatic and frictionless flow of a gas, the maximum rate of flow will agree exactly with the result obtained by the use of Eq. (10:13). When the flow is less than the maximum (when  $P_2 > 0.53P_1$ ), the flow as calculated by Fliegner's equation [Eq. (10:14)] will be about 1 to 5 per cent less than when frictionless flow is assumed. This corresponds to a discharge coefficient of 0.99 to 0.95.

**10:7. Design of an Air Nozzle.** In the design of an expanding air nozzle for a stated discharge rate, it is necessary to calculate the areas at the throat  $a_0$  and at the mouth  $a_2$ . The entrance area is not calculated, the only requirement being that the entrance to the nozzle shall be well rounded to avoid contraction of the jet; nor is the length of the nozzle capable of thermodynamic calculation, this feature of the design usually being based on empirical formulas. If  $p_2$  is greater than the critical pressure (the pressure for maximum discharge), only one area ( $a_2$ ) is

calculated since the nozzle is nonexpanding. The method of calculation is indicated in the example below.

*Example.* Design a nozzle to expand 3,000 lb. of air per hour under the following conditions:  $p_1 = 160$  p.s.i. abs.;  $p_2 = 16$  p.s.i. abs.;  $t_1 = 70^\circ\text{F}$ . (a) Assuming adiabatic and frictionless flow. (b) Assuming that internal friction reduces the final kinetic energy by one-tenth.

*Solution.* (a) The calculation at the throat is

$$p_0 = 0.53p_1 = 0.53 \times 160 = 84.8 \text{ p.s.i. abs.}$$

$$T_0 = T_1 \left( \frac{p_0}{p_1} \right)^{\frac{k-1}{k}} = 529.7(0.53)^{0.288} = 441^\circ\text{F. abs.}$$

$$w_0 = 223.8[C_p(T_1 - T_0)]^{\frac{1}{2}} = 223.8[0.2375(529.7 - 441)]^{\frac{1}{2}} = 1,027 \text{ ft. per second}$$

$$v_0 = \frac{RT_0}{P_0} = \frac{53.3 \times 441}{144 \times 84.8} = 1.93 \text{ cu. ft. per pound}$$

$$a_0 = \frac{Mv_0}{w_0} = \frac{3,000 \times 1.93}{3,600 \times 1,027} = 0.00157 \text{ sq. ft. or } 0.226 \text{ sq. in.}$$

At the mouth of the nozzle the conditions are

$$T_2 = T_1 \left( \frac{p_2}{p_1} \right)^{\frac{k-1}{k}} = 529.7 \left( \frac{16}{160} \right)^{0.288} = 273^\circ\text{F. abs.}$$

$$w_2 = 223.8[0.2375(529.7 - 273)]^{\frac{1}{2}} = 1,750 \text{ ft. per second}$$

$$v_2 = \frac{53.3 \times 273}{144 \times 16} = 6.32 \text{ cu. ft. per pound}$$

$$a_2 = \frac{Mv_2}{w_2} = \frac{3,000 \times 6.32}{3,600 \times 1,750} = 0.00301 \text{ sq. ft. or } 0.434 \text{ sq. in.}$$

(b) It will be assumed that friction occurs only in the expanding portion of the nozzle, between throat and mouth. The area at the throat will therefore be the same as for part (a). Since the kinetic energy is proportional to the square of the velocity and therefore to the drop of temperature of the air [see Eq. (10:6)], the drop of temperature through the nozzle will be nine-tenths as great as if there were no friction; therefore

$$529.7 - T_2 = 0.9(529.7 - 273) \quad \text{and} \quad T_2 = 298.6^\circ\text{F. abs.}$$

$$w_2 = 223.8[0.2375(529.7 - 298.6)]^{\frac{1}{2}} = 1,655 \text{ ft. per second}$$

$$v_2 = \frac{53.3 \times 298.6}{144 \times 16} = 6.92 \text{ cu. ft. per pound}$$

$$a_2 = \frac{3,000 \times 6.92}{3,600 \times 1,655} = 0.00348 \text{ sq. ft. or } 0.502 \text{ sq. in.}$$

**10:8. Available Energy for Adiabatic and Frictionless Flow.** The maximum kinetic energy per pound which can be developed by the jet of gas in adiabatic flow is that for the frictionless condition. Assuming the initial velocity is zero, its amount is

$$\frac{w_2^2}{2g} = JC_p(T_1 - T_2)$$



or, since

$$C_p = kAR/(k - 1) \text{ and } RT = Pv,$$

$$\frac{w_2^2}{2g} = \frac{k}{k - 1} (P_1v_1 - P_2v_2)$$

This expression is the mathematical statement for the area between the adiabatic curve of expansion (1-2 in Fig. 10:5) and the  $P$  axis, represented by area 4123 in Fig. 10:5, as can be shown by an integration of this area [also cf. Eq. (7:3)].

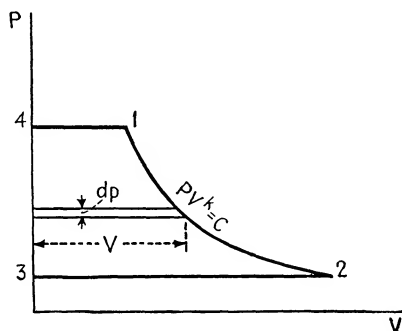


FIG. 10:5.—Available energy for adiabatic flow.

**10:9. Application to Vapor Flow.** The flow equations in their general form [Eqs. (10:1) to (10:5)] apply to the flow of vapors as well as gases. For vapors, however, the velocity cannot be stated in terms of the initial and final temperatures since the enthalpy of a vapor is not a function of the temperature alone. The values of the enthalpy at the initial condition and at some second location in the channel may be obtained from the vapor tables, or, more readily, from the Mollier diagram, if the characteristics of the expansion are known. Equation (10:5) may be altered by the substitution of the values of the constants  $A$  and  $g$  and suitable transposition to the more convenient form

$$w_2 = 223.8(h_1 - h_2)^{\frac{1}{2}} \quad (10:15)$$

The mass rate of flow through vapor nozzles is again evidently dependent upon the area of the smallest section or throat of the nozzle. For maximum flow,  $dv/v_0 = dw/w_0$  at this point, as for gas flow. Equation (10:11) cannot be applied to find the critical pressure ratio  $P_0/P_1$ , however, since  $k$  cannot be assigned a constant value in the case of a vapor. But the flow remains practically adiabatic and frictionless until after the throat has been passed; and, based on the tables of properties of the

specific vapor and isentropic flow, a pressure may be located by trial-and-error methods at which the rate of change of specific volume equals the rate of change of velocity; comparison of this pressure with the initial pressure will determine the critical-pressure ratio *for the assumed conditions*. A large number of such calculations are, of course, necessary before any conclusions may be drawn as to a general value of this ratio. For steam, the critical ratios are very nearly as below:

$$\text{For steam initially saturated,} \quad \frac{p_0}{p_1} = 0.58$$

$$\text{For steam initially superheated,} \quad \frac{p_0}{p_1} = 0.55$$

**10:10. Design of Steam Nozzles.** If the ratio of the final (exhaust) pressure to the initial pressure is greater than that given in the listing above, the nozzle becomes merely an orifice and thermodynamically its design consists in calculating the area of throat required for the desired weight of flow. For this case, the initial condition is located on the Mollier diagram, and a projection is made vertically (at constant entropy) to the final pressure at exhaust from the nozzle, reading the corresponding initial and final enthalpies and the final quality or superheat. From these data the velocity in the orifice may be calculated by Eq. (10:15) and the specific volume of the steam in the throat or orifice computed from the steam tables. Substitution in Eq. (10:1) will permit the calculation of the required area for the desired weight of flow or the weight of flow through a given orifice area.

If the ratio of final to initial pressure is less than the critical ratio, the nozzle will be designed as an expanding nozzle and two areas, the area at the throat and the area at the mouth of the nozzle, must again be calculated. The pressure in the throat of the nozzle is found by applying the critical ratio, and the corresponding enthalpy and quality or superheat of the steam in the throat of the nozzle are read from the Mollier diagram, the area of the throat being calculated as outlined above. For the calculation of the area of the mouth of the nozzle, the adiabatic is continued to the final pressure  $p_2$  and the enthalpy and quality or superheat obtained for that condition. The velocity at the mouth is calculated on the basis of the drop in enthalpy from the initial condition  $h_1 - h_2$ . The method of using the Mollier diagram as a basis for the design computations is indicated in Fig. 10:6, in which the theoretical expansion is represented by the line  $ABC$ .

*Example.* Design a nozzle to expand 3,000 lb. of steam per hour from 200 p.s.i. abs., 540°F., to a final pressure of 2 p.s.i. abs.

*Solution.* The saturation temperature at the initial pressure is 381.8°F. In its initial condition the steam is therefore superheated 158.2°, and  $p_0/p_1$  is 0.55.

$$p_0 = 0.55 \times 200 = 110 \text{ p.s.i. abs.}$$

From the Mollier diagram, the following data may be read:

	Pressure	Condition	Enthalpy
Initial.....	200	540°F.	1,290.5
At throat.....	110	410°F.	1,232
At mouth.....	2	$x = 0.844$	956

The velocities at throat and mouth may now be calculated.

$$w_0 = 223.8(h_1 - h_0)^{\frac{1}{2}} = 223.8(1,290.5 - 1,232)^{\frac{1}{2}} = 1,710 \text{ ft. per second.}$$

$$w_2 = 223.8(h_1 - h_2)^{\frac{1}{2}} = 223.8(1,290.5 - 956)^{\frac{1}{2}} = 4,090 \text{ ft. per second.}$$

The specific volume of the steam in the throat of the nozzle may be read from the tables for superheated steam. The specific volumes at throat and mouth may now be found.

$$v_0 = 4.532 \text{ cu. ft. per pound; } v_2 = 0.844 \times 173.73 = 147 \text{ cu. ft. per pound}$$

The required areas at the throat and the mouth are

$$a_0 = \frac{Mv_0}{w_0} = \frac{3,000 \times 4.532}{3,600 \times 1,710} = 0.00221 \text{ sq. ft. or } 0.318 \text{ sq. in.}$$

$$a_2 = \frac{Mv_2}{w_2} = \frac{3,000 \times 147}{3,600 \times 4,090} = 0.0299 \text{ sq. ft. or } 4.31 \text{ sq. in.}$$

If the nozzle is round in cross section, these areas would correspond to diameters at throat and mouth as follows:

$$D_0 = \left( \frac{4a_0}{\pi} \right)^{\frac{1}{2}} = \left( \frac{4 \times 0.318}{3.1416} \right)^{\frac{1}{2}} = 0.636 \text{ in.}$$

$$D_2 = \left( \frac{4a_2}{\pi} \right)^{\frac{1}{2}} = \left( \frac{4 \times 4.31}{3.1416} \right)^{\frac{1}{2}} = 2.34 \text{ in.}$$

**10:11. The Effect of Friction in Nozzles.** Although heat may be prevented from entering or leaving the nozzle from or to an outside system during the flow, the final condition of the steam in practice is not such as would be expected as the result of an adiabatic expansion. Owing to friction between the flowing vapor and the sides of the channel and to internal friction between the particles of vapor caused by flow eddies, the final quality will be increased above the ideal value, representing the conversion of some of the available energy back into the form of heat. This effect is given the name of *reheat* and, in the actual nozzle, usually amounts to 8 to 15 per cent of the total available energy  $h_1 - h_2$ . The amount of reheat is expressed as a decimal fraction of the entire available energy and is denoted by the symbol  $y$ , called the *reheat fraction*. Thus, if the total available energy per pound,  $h_1 - h_2$ , is 100 B.t.u. and the

reheat is 10 per cent,  $y$  has a value of 0.10 and the net available energy is  $(1 - y)100$  or 90 B.t.u. In practice, all this reheating effect has been shown to take place between the throat of the nozzle and its mouth, and the calculation of throat area is therefore not affected by taking reheat into consideration.

The method of applying a given or assumed value of the reheat fraction to the design of a nozzle is indicated in Fig. 10:6. The expansion follows the line  $ABC'$ , which must fall to the right of  $BC$  since a decrease

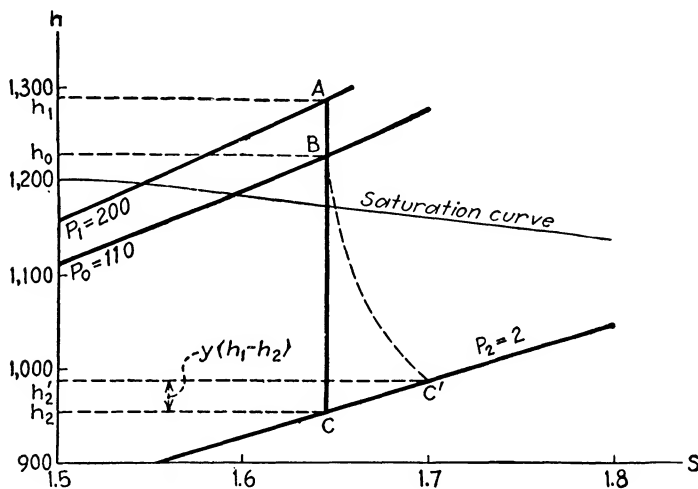


FIG. 10:6.—Design of expanding nozzle.

of availability is represented.  $C'$  lies on the same constant-pressure line (representing the final pressure  $p_2$ ) as  $C$  but is a vertical distance  $y(h_1 - h_2)$  above it. The area at the mouth of the nozzle will depend upon the velocity at that point,

$$w_2 = 223.8[(h_1 - h_2)(1 - y)]^{\frac{1}{2}} \quad (10:16)$$

and the specific volume, which will be somewhat greater than for the theoretical case since  $x_2'$  is greater than  $x_2$ . It is evident that the combined effects of reduced velocity and increased specific volume at the exit will require a larger exit area than for frictionless flow if expansion is to be completed within the walls of the nozzle.

*Example.* In the example of Art. 10:10, redesign the nozzle on the basis of a reheat fraction of 0.10.

*Solution.* The design of the throat of the nozzle is not changed since all the reheat is assumed to take place between the throat and the mouth. The total decrease

of enthalpy between the initial condition and the mouth of the nozzle based on frictionless flow is 334.5 B.t.u. One-tenth of this amount, or 33.4 B.t.u., is used to reheat the steam in its final condition. The final condition of the steam as it leaves the nozzle is therefore

$$p_2 = 2 \text{ p.s.i. abs.}$$

$$h_2 = 956 + 33.4 = 989.4 \text{ B.t.u. per pound}$$

On the Mollier diagram, it is observed that these conditions correspond to a final quality of 0.876. The corrected velocity and specific volume at the mouth of the nozzle may now be computed.

$$w_2 = 223.8(1,290.5 - 989.4)^{1/2} = 3,880 \text{ ft. per second}$$

$$v_2 = 0.876 \times 173.73 = 152 \text{ cu. ft. per pound}$$

and the area at the mouth, as corrected, is

$$a_2 = \frac{Mv_2}{w_2} = \frac{3,000 \times 152}{3,600 \times 3,880} = 0.0327 \text{ sq. ft. or 4.71 sq. in.}$$

**10:12. The Throttling Process.** If a gas or vapor is permitted to expand through an orifice or nozzle into a section of the channel large enough so that the velocity again becomes inappreciable, without performing external work or exchanging heat with any external system, the kinetic mechanical energy developed in the orifice will be entirely consumed by internal friction. This type of expansion, which is called *throttling*, is illustrated by the expansion which takes place in a throttling calorimeter. This device is used for determining the quality of steam. The steam sample, drawn from the line, is caused to expand through a small orifice to a lower pressure, usually atmospheric. The total energy per pound above the orifice is

$$u_A + AP_A v_A + \frac{Aw_A^2}{2g}$$

and, below the orifice,

$$u_B + AP_B v_B + \frac{Aw_B^2}{2g}$$

Ultimately, all the kinetic energy developed in the orifice is reconverted to thermal agitation by internal eddies and, consequently, is returned to the steam in the form of thermal energy. Since  $w_A$  and  $w_B$  are both small and practically equal to each other and since, by proper insulation, heat is prevented from entering or leaving the system, the following equation may be written:

$$u_A + AP_A v_A = u_B + AP_B v_B$$

or

$$h_A = h_B \quad (10:17)$$

If the original quality of the steam in the line is not too low, it will become superheated in the calorimeter and  $h_B$  can then be readily determined from the tables for superheated steam by a comparison of its temperature and pressure. The original quality  $x_A$  (the quality of the steam in the line) can then be calculated as follows:

$$\begin{aligned} h_B &= h_A = h_{fA} + x_A h_{fgA} \\ x_A &= \frac{h_B - h_{fA}}{h_{fgA}} \end{aligned} \quad (10:18)$$

The Mollier diagram can again be used to good advantage for a graphical solution of throttling-calorimeter problems since on it lines of constant enthalpy are horizontal.

The reader will observe that, although the initial and final enthalpies are equal, the throttling process cannot be considered as taking place at constant enthalpy. It should rather be treated as an adiabatic flow with total reheat.

Reference to the Mollier diagram will indicate that the throttling calorimeter has a useful range of qualities only between 100 and 95 per cent. Furthermore, at the higher range of pressures, it suffers a rapid decrease in this effective range; above about 1,500 p.s.i. abs., it cannot be used unless exhaust is to a region below atmospheric pressure. For lower quality (or higher pressure) steam, the percentage of moisture content can be determined by mechanical separation of steam and moisture as in the *separating calorimeter*; in this device, the greater density of the water is utilized to bring about the separation.

*Example.* The pressure of the steam in a throttling calorimeter is atmospheric, and its temperature is 260°F. If the steam-line pressure is 200 p.s.i. abs., what is the quality of the steam in the line?

*Solution.* From Table 3, the enthalpy  $h_B$  of steam at 14.7 p.s.i. abs. and 260°F. is 1173.8 B.t.u. per pound. The enthalpy of the liquid at the pressure of the steam in the line may be read from Table 2 as 355.36 B.t.u. per pound and the enthalpy of vaporization as 843.0 B.t.u. per pound. Substituting these values in Eq. (10:18),

$$x_A = \frac{1173.8 - 355.36}{843.0} = 0.971$$

The same result can be obtained graphically by locating the point on the Mollier diagram which represents the condition of the superheated steam in the calorimeter and projecting horizontally to the left (at constant enthalpy) from this point until the line which represents a pressure of 200 p.s.i. abs. is intersected.

**10:13. Unstable Conditions in Steam Flow.** The time required for the passage of an individual particle of vapor through a nozzle is so short (of the order of 1/10,000 sec.) and the pressure and volume changes

which occur during that passage are so considerable that it is not surprising that, in practice, conditions of nonequilibrium are often found to exist. When steam which is moderately superheated expands isentropically, calculations based on steam-table values indicate that the  $Pv$  relationship which represents a condition of equilibrium is approximately expressed as  $Pv^{1.3} = \text{a constant}$ . If superheated steam were treated as a perfect gas, the value of 1.3 could therefore be assigned to  $k$ ; furthermore, substitution of this value in Eq. (10:11) will yield a critical pressure ratio  $P_0/P_1$  which agrees very closely with that suggested in Art. 10:9 as applying to superheated steam. For saturated steam the value of  $k$ ,

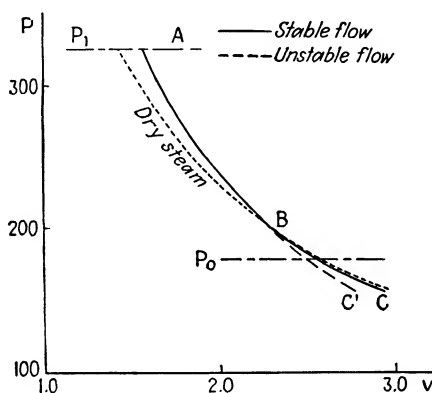


FIG. 10:7.—Unstable steam flow.

similarly determined, is about 1.13. It is of interest to note that, if this value is substituted in Eq. (10:11), the ratio  $P_0/P_1$  obtained is 0.575, again conforming very nearly to the ratio as stated in Art. 10:9.

In Fig. 10:7, the ideal adiabatic expansion of steam initially superheated is represented on a  $Pv$  diagram by the line  $ABC$ . This path assumes that the steam is at all times in a state of thermal equilibrium. The dotted line represents the condition of dry and saturated steam (corresponding to the saturation line on the Mollier diagram) so that, as the condition of the steam changes from  $B$  to  $C$ , increasing percentages of water content are accumulating. The abrupt change of direction at point  $B$ , where the expansion curve crosses into the saturated-steam region, is quite noticeable.

It has been demonstrated by methods beyond the scope of this text that water particles of smaller diameter than about  $1/10,000$  in. cannot exist in thermal equilibrium with the surrounding vapor. It has further been shown that condensation takes place about certain nuclei, such as

small dust particles. Taking into consideration the extreme rapidity with which these pressure-volume changes take place, it is not surprising that experiments show that the condensation in steam as it flows through a nozzle is delayed considerably beyond the point which would be characteristic of a condition of thermal equilibrium.

The effect of this delay in the formation of condensate particles is to cause the steam to continue to expand as a dry vapor below point *B* of Fig. 10:7. Assuming that the exponent of the curve continues to be 1.3 below *B*, the lower portion of the curve may be plotted as *BC'*. It will be observed that the expansion along *BC'* is, relative to the pressure drop, less rapid than along *BC* and that the density of the vapor at the throat of the nozzle is correspondingly greater. A reference to Eq. (10:1) will indicate that, if the velocity at the throat is not greatly different in the two cases, the same cross-sectional area of throat will permit the flow of steam at a higher mass rate if the expansion curve *BC'* is followed. Thus, in many cases when steam which has, initially, a moderate degree of superheat flows through an expanding nozzle, experiments indicate that the rate of flow will exceed the theoretical by 2 or 3 per cent.

Again assuming that the change of condition represented by *BC'* follows closely that characteristic of a gas, it may be shown that the rate of decrease of temperature with respect to pressure is higher along that curve than along *BC*. This creates an ever-increasing thermal "tension" which ultimately upsets the delicate balance which exists in the unstable expansion and causes a more or less rapid shift to the stable expansion curve, a shift which is evidenced experimentally by the sudden accumulation of a dense cloud of moisture particles (fog) at a point well beyond that at which this phenomenon would have occurred if thermal equilibrium had been continuously maintained. Little is known of the state path followed during this transition or of the temperature differential necessary to provide the tension to cause it; the latter seems to be quite variable. However, assuming that equilibrium is not restored until after the throat of the nozzle has been passed, it is possible to predict the effect of unstable flow on the mass flow rate of a nozzle under specified conditions.

Steam in the unstable condition described in the foregoing paragraphs is said to be *supersaturated*. A similar condition of instability, of less interest to the engineer, is the cooling of water below its freezing temperature without immediately causing the formation of ice; in both cases the thermal tension eventually brings about a rapid adjustment to a condition of equilibrium.



*Example.* Superheated steam confined in a reservoir at a pressure of 325 p.s.i. abs. and a temperature of 480°F. is allowed to expand to atmospheric pressure through a divergent nozzle with a throat area of 0.1 sq. in. Calculate the mass rate of discharge, (a) based on the assumption that thermal equilibrium is maintained and (b) assuming that the steam becomes supersaturated. (c) At the throat, what temperature differential is acting to restore the steam to a condition of equilibrium?

*Solution.*

$$(a) p_0 = (0.55)(325) = 179 \text{ p.s.i. abs.}$$

From the Mollier diagram,

$$h_1 = 1242 \text{ B.t.u.}; \quad h_0 = 1188 \text{ B.t.u.}; \quad x_0 = 0.99$$

$$w_0 = 223.8(1242 - 1188)^{\frac{1}{2}} = 1,640 \text{ ft. per second}$$

$$v_0 = (0.99)(2.545) = 2.52 \text{ cu. ft. per pound}$$

$$M = \frac{av}{v} = \frac{(0.1)(1,640)}{(144)(2.52)} = 0.452 \text{ lb. per second}$$

(b) Expansion as a gas with  $k = 1.3$  will be assumed, so that  $p_1 v_1^{1.3} = p_0 v_0^{1.3}$ . From the steam tables,  $v_1 = 1.5712$  cu. ft. per pound, and

$$v_0 = v_1 \left( \frac{p_1}{p_0} \right)^{\frac{1}{1.3}} = 1.5712 \left( \frac{325}{179} \right)^{\frac{1}{1.3}} = 2.48 \text{ cu. ft. per pound}$$

Using a convenient form of Eq. (10:7),

$$\begin{aligned} w_0 &= \left[ 2g \left( \frac{k}{k-1} \right) (P_1 v_1 - P_0 v_0) \right]^{\frac{1}{2}} \\ &= \left[ (64.4) \left( \frac{1.3}{0.3} \right) (144)(325 \times 1.5712 - 179 \times 2.48) \right]^{\frac{1}{2}} = 1,640 \text{ ft. per second} \end{aligned}$$

$$M = \frac{(0.1)(1,640)}{(144)(2.48)} = 0.460 \text{ lb. per second}$$

Comparing the results of (a) and (b), it will be observed that supersaturated flow accounts for a mass rate of flow about 2 per cent above the theoretical.

(c) For supersaturated flow,

$$T_0 = T_1 \left( \frac{P_0}{P_1} \right)^{\frac{k-1}{k}} = 939.7(0.55)^{\frac{0.3}{1.3}} = 818^\circ \text{ F. abs. or } 358^\circ \text{ F.}$$

For the stable flow, the temperature at the throat would have been the saturation-temperature equivalent of the pressure of 179 p.s.i. abs. at that point, or 373°F. The supersaturated steam may therefore be said to be *subcooled* by 15° as it passes the throat of the nozzle.

### Problems

1. A fluid having a density of 0.5 lb. per cubic foot flows through a channel having a cross-sectional area of 0.75 sq. in. with a velocity of 500 ft. per second. What is the mass rate of flow in pounds per second?

2. Fifty cubic feet of water flows per minute through a circular orifice with a diameter of 1 in. What is its velocity (in feet per second) in the orifice?

3. A circular opening permits the passage of 2,400 lb. of steam per hour. If the velocity is 2,000 ft. per second and the steam has a density of 0.25 lb. per cubic foot in the opening, what is its diameter in inches?

4. A fluid flows through a closed channel. As it progresses from a given upstream section of the channel to a certain downstream section, its specific enthalpy decreases by 150 B.t.u. per pound. If the velocity at the upstream section is 600 ft. per second and the flow is assumed to be adiabatic (not necessarily frictionless), what is the velocity at the downstream section?

5. Air flows through a closed channel. At an upstream section, the temperature of the air is 200°F., and the velocity is 400 ft. per second. At a later section, the temperature has decreased to 100°F. Assuming that no heat has entered or left the air through the walls of the channel between the two sections, what is the velocity of the air at the downstream section? Is your answer based on the further assumption that the flow was not retarded by friction?

6. As a fluid flows adiabatically through a closed channel, the velocity at an upstream section is 1,000 ft. per second. At a downstream section of the channel, the velocity is 2,500 ft. per second. What is the change of enthalpy per pound of fluid between the two sections?

7. A fluid flows from a large reservoir, in which a pressure of 150 p.s.i. abs. and an enthalpy of 1,250 B.t.u. per pound are maintained, through an orifice placed in the wall of the reservoir. In the orifice, a velocity of 1,750 ft. per second is attained. What is the specific enthalpy of the steam in the orifice, assuming the flow to be adiabatic?

8. Calculate the exit velocity from an air nozzle, based on adiabatic flow, if the temperature in the supply reservoir is 300°F. and at the exit is 75°F.

9. Assuming adiabatic and frictionless flow, calculate the temperature of the air at a point in an air nozzle where the pressure is 20 p.s.i. abs. if the supply-reservoir temperature and pressure are 100°F. and 50 p.s.i. abs., respectively.

10. An incompressible liquid is confined in a tank. A horizontal nozzle is located in the wall of the tank at a level where the pressure inside the tank is 150 p.s.i. abs., and the liquid issues to the atmosphere through this nozzle. Assuming that the temperature and internal energy of the liquid are not changed owing to the decrease in pressure which takes place in the nozzle and that the flow is adiabatic and frictionless, calculate the velocity of the issuing jet, (a) if the density of the liquid is 62 lb. per cubic foot and (b) if the density is 30 lb. per cubic foot. Assume the velocity in the tank to be negligible.

11. Write Eq. (10:3) in the special form which would apply to the flow of an incompressible fluid under the assumptions of Prob. 10.

12. Carbon dioxide, confined in a tank at a pressure of 200 p.s.i. gage and a temperature of 150°F., is allowed to escape into the atmosphere through an orifice having a cross-sectional area of 0.3 sq. in. What is the pressure in the orifice? Assuming a discharge coefficient of 0.98, what weight is discharged per second?

13. Helium, a monatomic gas with a molecular weight of 4, escapes to the atmosphere through an orifice in the wall of a tank in which it is confined at a pressure of 300 p.s.i. abs. and a temperature of 200°F. What is the pressure in the orifice? Assuming no jet contraction and no friction, what weight of helium is discharged per second per square inch of orifice area?

14. The air in a tank has a pressure of 100 p.s.i. gage and a temperature of 200°F. It expands to the atmosphere through a nozzle. What are the theoretical velocities and temperatures at the throat and exit of this nozzle? What weight of air is discharged per second if the throat area is 0.25 sq. in.?

15. Assuming the velocity of the air entering the nozzle to be negligible, plot the

velocity attained, the specific volume of the air, and the required cross-sectional area of the nozzle as ordinates against the pressure as abscissas for the frictionless flow of air at a rate of 1 lb. per second from an initial pressure of 100 p.s.i. abs., temperature of 140°F., to a final pressure of 14.7 p.s.i. abs.

16. Using theoretical equations and assuming adiabatic and frictionless flow, calculate the weight of air discharged per hour from a nozzle having a throat area of 0.3 sq. in. when the pressure and temperature in the reservoir from which the flow issues are 150 p.s.i. gage and 200°F., respectively, and the pressure at exit is atmospheric. Also calculate the discharge by Fliegner's equation, and compare the predicted actual with the theoretical flow. What coefficient of discharge is indicated?

17. The pressure and temperature of the air in a reservoir are, respectively, 36 in. Hg abs. and 70°F. The air flows into the atmosphere ( $p = 14.7$  p.s.i. abs.) through a nozzle with a minimum cross-sectional area of 0.5 sq. in. Calculate the theoretical flow in pounds per minute, based on frictionless and adiabatic expansion in the nozzle. Predict the actual flow, based on Fliegner's equation. Compare the results and determine the discharge coefficient.

18. A converging nozzle is placed in the wall of a tank containing helium, a monatomic gas, at a pressure of 200 p.s.i. abs. and temperature of 120°F. The pressure in the space surrounding the tank may be controlled. What is the pressure at the narrowest section of the nozzle when the exhaust pressure is (a) 140 p.s.i. abs.? (b) 120 p.s.i. abs.? (c) 100 p.s.i. abs.? (d) 80 p.s.i. abs.? (e) 60 p.s.i. abs.?

19. A converging nozzle with a minimum cross-sectional area of 0.4 sq. in. is placed in the wall of a tank containing air at 100 p.s.i. abs. and 180°F. Calculate the weight of air discharged per second when the exhaust pressure is (a) 80 p.s.i. abs.; (b) 60 p.s.i. abs.; (c) 53 p.s.i. abs.; (d) 40 p.s.i. abs.; (e) 20 p.s.i. abs. Plot the weight discharged as ordinates against the ratio of exhaust pressure to initial pressure as abscissas.

20. A converging nozzle with a minimum cross-sectional area of 0.20 sq. in. is placed in the wall of a tank containing air at 120 p.s.i. abs. and a temperature of 140°F. Flow takes place into the atmosphere at a barometric pressure of 30 in. Hg. (a) What is the pressure at the nozzle exit? (b) What weight of air is discharged per second? (c) How would your answers to (a) and (b) have been affected if the pressure outside the tank were 30 p.s.i. abs.?

21. Design a round nozzle (give throat and exit diameters) to expand 3,000 lb. of air per hour from an initial pressure (at negligible velocity) of 140 p.s.i. abs. and temperature of 170°F. to a final pressure of 15 p.s.i. abs., based on (a) frictionless adiabatic flow and (b) the assumption that friction dissipates 10 per cent of the kinetic energy theoretically attainable.

22. Design a nozzle to expand 2,400 lb. of air per hour from its initial pressure of 60 p.s.i. abs. and temperature of 90°F. in a reservoir to a final pressure of 14.7 p.s.i. abs., (a) based on adiabatic and frictionless flow and (b) based on adiabatic flow but assuming that friction accounts for a reduction of 6 per cent in the final velocity below that theoretically possible.

23. Design a nozzle to expand 1,500 lb. of air per hour from an initial (reservoir) pressure of 60 p.s.i. abs. and temperature of 80°F. to a final pressure of 45 p.s.i. abs.

24. Steam confined in a tank at a pressure of 150 p.s.i. abs. and a temperature of 480°F. flows through a converging nozzle into the atmosphere. The barometer is 29 in. Hg. What is the pressure in the nozzle at the exit? What is the condition of the steam in the exit?

25. Steam is supplied to a diverging nozzle at 200 p.s.i. abs. and 460°F. and with negligible velocity. The exit pressure is 20 p.s.i. abs. What is the pressure at the throat of the nozzle? What is the quality or superheat at the throat and at the exit, assuming frictionless flow?

26. Steam enters a nozzle dry and saturated at 100 p.s.i. abs. and with negligible velocity. Assuming frictionless adiabatic flow, calculate the minimum nozzle areas required for a flow rate of 1 lb. per second at sections where the pressure is, respectively, 85, 70, 60, 58, 55, 50, 40, and 20 p.s.i. abs. Plot the area as ordinates against the pressure ratios  $P/P_1$  as abscissas. What pressure corresponds to minimum nozzle area? Plot the specific volume and velocity against the same abscissas.

27. Steam enters a nozzle at 100 p.s.i. abs. and 500°F. and with negligible velocity. Assuming isentropic flow, calculate the minimum nozzle areas required for a flow rate of 1 lb. per second at sections where the pressure is, respectively, 85, 70, 60, 58, 55, 50, 40, and 20 p.s.i. abs. Plot the areas as ordinates against the pressure ratio  $P/P_1$  as abscissas. What pressure corresponds to minimum nozzle area? Plot the specific volume and the velocity against the same abscissas.

28. Napier's equation is an empirical relation giving the approximate maximum mass rate of flow of saturated steam through a nozzle. It is stated as  $M = a_0 p / 70$  ( $M$  in pounds per second,  $a_0$  in square inches,  $p$  in pounds per square inch absolute). Using this equation, calculate the mass rate of flow through a nozzle with a throat area of 0.5 sq. in., the steam initially being dry and saturated at a pressure of 200 p.s.i. abs. Calculate the theoretical discharge, and compare.

29. A pipe line carries steam at 180 p.s.i. abs., temperature of 500°F. A hole in the pipe 0.1 in. in diameter permits steam to escape to the atmosphere. What weight of steam will be lost per hour?

30. Design a nozzle to expand 3,000 lb. of steam per hour from an initial pressure of 90 p.s.i. abs., quality of 0.98, to a final pressure of 75 p.s.i. abs.

31. Calculate the throat and exit areas of a nozzle which is required to expand 2,400 lb. of dry and saturated steam per hour from an initial pressure of 150 p.s.i. abs. to a final pressure of 16 p.s.i. abs. Assume (a) frictionless flow and (b) a reheat fraction of 0.08.

32. Calculate the throat and exit areas of a nozzle designed to expand 2,700 lb. of steam per hour from an initial pressure of 240 p.s.i. abs., temperature of 500°F., to a final pressure of 18 p.s.i. abs., assuming (a) frictionless flow and (b) a reheat fraction of 0.10.

33. Calculate the exit area of a nozzle designed to expand 3,200 lb. of steam per hour from an initial pressure of 180 p.s.i. abs., temperature of 400°F., to a final pressure of 10 p.s.i. abs., assuming (a) frictionless flow, (b) that the final kinetic energy of the jet is 10 per cent below the theoretical, (c) that the final velocity of the jet is 5 per cent below the theoretical, and (d) that the reheat fraction is 0.10.

34. The pressure in a throttling calorimeter is atmospheric (barometer 29.92 in Hg), and the temperature is 270°F. The pressure in the steam line is 200 p.s.i. gage. What is the quality of the steam in the line?

35. The steam in a calorimeter is superheated 30°F. at atmospheric pressure (standard). What is its quality in the steam line where the pressure is 125 p.s.i. gage?

36. What is the lowest quality that can be determined by the use of a throttling calorimeter when the calorimeter pressure is 14.7 p.s.i. abs., if a minimum of 10° of superheat is required in the calorimeter to establish the fact that the steam is super-

heated at that point and if the steam-line pressure is (a) 300 p.s.i. abs. and (b) 2,000 p.s.i. abs.?

37. Saturated water at a pressure of 40 p.s.i. gage is contained in a water heater. A faucet is opened, permitting escape to the atmosphere. Regarding this as a throttling process, what is the proportion of steam in the mixture as it escapes to the atmosphere (a) by weight (quality) and (b) by volume?

38. Saturated liquid ammonia at a temperature of 100°F. is throttled through the expansion valve of a refrigerating plant to a pressure of 30 p.s.i. abs. What is its temperature below the valve? Its quality?

39. Steam enters a nozzle with negligible velocity at a pressure of 200 p.s.i. abs., temperature of 400°F. The throat area of the nozzle is 0.3 sq. in. Calculate the discharge rate, (a) based on the assumption that thermal equilibrium is maintained and (b) assuming that the steam becomes supersaturated. (c) What is the discharge coefficient in the latter case?

## CHAPTER XI

### VAPOR CYCLES

**11:1. The Carnot cycle** for a vapor, as for a gas, measures the maximum efficiency which can be attained. Bounded by two isothermals and two adiabatics, its appearance is unchanged on  $TS$  coordinates. However, if the cycle is carried out entirely within the saturated-vapor region,

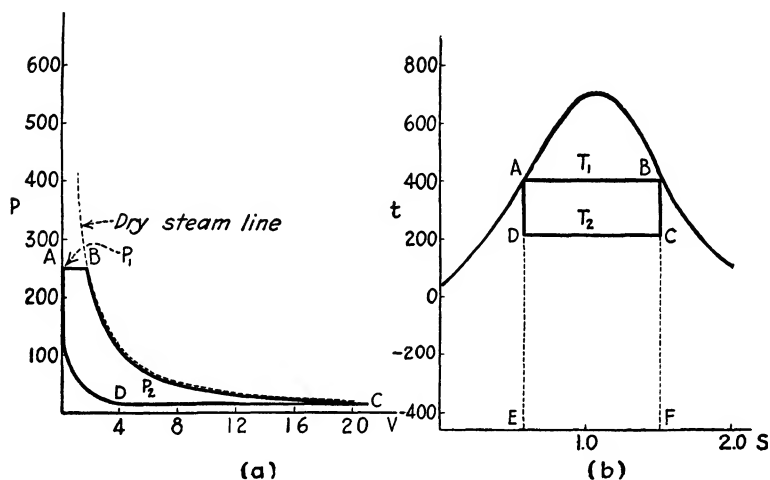


FIG. 11:1.—Carnot cycle for vapors.

where a constant-temperature line is also a line of constant pressure, the upper and lower lines of the cycle, as plotted on  $PV$  coordinates, become horizontal lines as shown in Fig. 11:1. The efficiency remains unchanged and may still be expressed by the relationship

$$e = \frac{T_s - T_R}{T_s} \quad (11:1)$$

In the practical application of this cycle, the upper isothermal may be carried out as the evaporation of water at constant pressure within a boiler; the adiabatic operation  $BC$  is also possible practically through adiabatic expansion in a steam engine or turbine, and the rejection of heat at constant temperature from  $C$  to  $D$  may be duplicated by the

action of the condenser in the practical installation. The adiabatic line  $DA$  is more difficult of practical attainment. In order to follow out in practice the complete Carnot cycle, it would be necessary that the engine cylinder serve also as boiler and condenser. The practical losses which would result would more than balance any theoretical gain in efficiency.

*Example.* Calculate the efficiency and steam consumption per horsepower-hour of an ideal Carnot steam engine when the steam at cutoff is 98 per cent dry at a pressure of 250 p.s.i. abs., the cylinder contains water at the beginning of the expansion stroke, and the exhaust pressure is 15 p.s.i. abs.

*Solution.* The heat added per pound of steam during the isothermal expansion is

$$Q_S = h_B - h_A = 0.98 \times 825.1 = 808.6 \text{ B.t.u. per pound}$$

The efficiency of the cycle is

$$e = \frac{T_1 - T_2}{T_1} = \frac{400.95 - 213.03}{400.95 + 459.7} = 0.218$$

The useful work performed per pound of steam supplied is

$$AW = e \times Q_S = 0.218 \times 808.6 = 176.2 \text{ B.t.u.}$$

Since a horsepower-hour is the equivalent of 2545 B.t.u. of useful work, the steam consumption per horsepower-hour is

$$\frac{2,545}{AW} = \frac{2,545}{176.2} = 14.45 \text{ lb. per horsepower-hour}$$

In the study of gas cycles, the thermodynamic system passed through its various processes while enclosed in a cylinder, and it was noted that the cycle was a machine cycle only, since the weight of fluid within the cylinder did not remain constant throughout the cycle, and the plotted cycle was therefore not a true record of the condition of the fluid. In dealing with the practical vapor cycle, we find this single piece of apparatus replaced by at least four. In addition to the prime mover itself, a boiler for the addition of heat, a condenser for its removal, and a pump to raise the pressure of the feed water so that it may reenter the boiler must be provided. Even in the so-called "noncondensing" plant, condensation takes place, in this case due to the action of the atmosphere. The power cycle of a vapor should therefore not be regarded as a record of the changes that occur within the engine or turbine alone; it must be understood that only a small part of the cycle, the adiabatic expansion, is carried out in the prime mover and the balance in other apparatus through which the vapor passes before completing its cycle. Thus the vapor-power cycle is not to be considered a machine cycle; it is, in general, a *cycle of the condition of the fluid* as it moves through the complete power plant. There are exceptions to this statement which apply

to certain parts of the power-plant cycle and which will be discussed later.

**11:2. Prime Movers.** The prime mover in a vapor power plant may, as has been indicated above, be either an engine or a turbine. In the former, illustrated in Fig. 11:2, steam is admitted to a cylinder in which it operates to impart reciprocating motion to a piston; this reciprocating motion is converted into rotary motion through the action of a piston rod, crosshead, connecting rod, and crank. Referring to the illustration, the steam chest receives steam directly from the boiler or superheater and therefore contains steam at full boiler pressure while the engine is in operation. The valves at the upper ends of the cylinder operate alternately to admit and shut off the flow of live steam to the two sides of the piston; those at the lower ends alternately open and release the charge, so that it may flow into the exhaust line and thus to the condenser, and close, bringing to an end the escape of expanded steam from the cylinder. This cylinder, as is common in steam-engine design, is double-acting, *i.e.*, as steam is admitted and expanded on one side of the piston, it is exhausted from the opposite end of the cylinder. This approximately doubles the capacity of a cylinder of given size. The type of engine cylinder illustrated is only one of many forms in which the steam engine is designed and built.

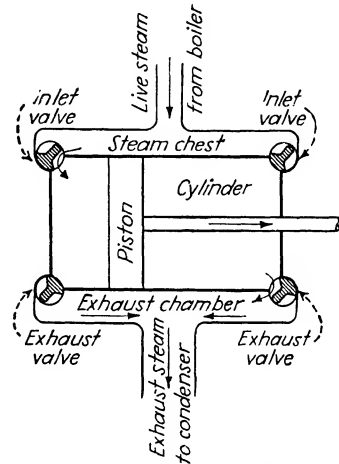


FIG. 11:2.—The steam engine.

The admission of live steam takes place through only a part of the power stroke of the expanding steam engine; after the supply has been cut off by valve action, the charge continues to expand until the piston reaches the end of its stroke. It is *fully* expanded only if the pressure in the cylinder has dropped to exhaust pressure at that point. Expansion in the practical steam engine is seldom carried to this point, as the size of cylinder required for a given capacity would be excessive. Thus, when the exhaust valve opens at the end of the power stroke, a rapid drop in pressure takes place as the partly expanded charge begins to escape into the exhaust passage. This is a feature that has already been discussed with respect to the air engine in Chap. VII.

In the steam-turbine prime mover, the steam is expanded in nozzles,



its available thermal energy thus being converted into kinetic energy of the steam jet as it emerges from these nozzles. This kinetic energy is then removed by allowing the jets to impinge on properly designed

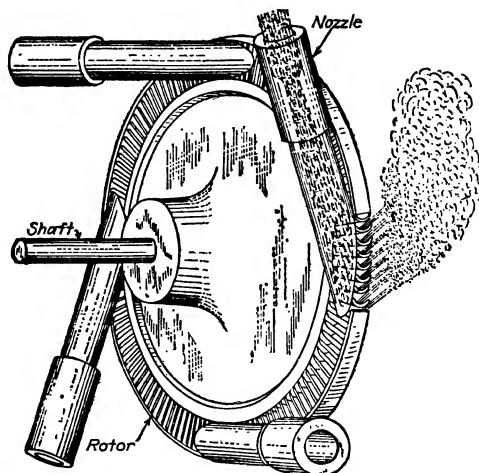


FIG. 11:3.—De Laval steam turbine.

blades, set in a rotor disk which is keyed to the turbine shaft, as illustrated in Fig. 11:3. This represents a very simple type of the turbine,

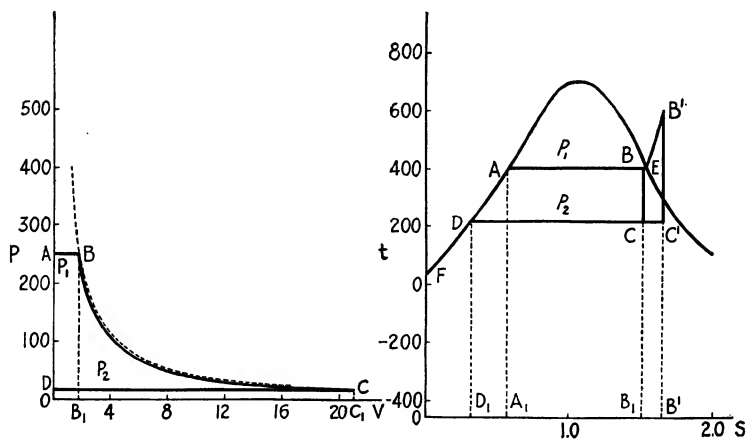


FIG. 11:4.—Rankine cycle.

which is built in a multitude of forms. It is not the purpose of this text to do more than indicate the basic principles involved, and it will be sufficient to note that in the turbine the development of shaft power

results from a double process, the conversion of available thermal energy into kinetic energy in the jet and the removal of this kinetic energy as it does work on moving blades. Complete expansion to exhaust pressure is entirely practical in the turbine, which considerably affects its application to the various cycles discussed on succeeding pages of this chapter.

**11:3. The Rankine Cycle.** If the line  $CD$  in Fig. 11:1, which represents the partial condensation of the exhaust steam as it leaves the engine, is continued until condensation is complete, the Rankine cycle is formed. Figure 11:4 represents this cycle on  $PV$  and  $TS$  coordinates,

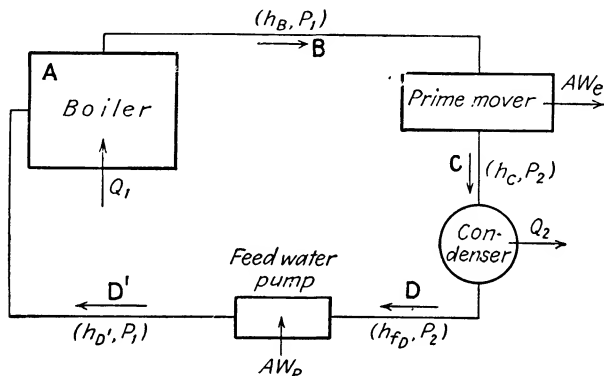


FIG. 11:5.—The Rankine-cycle circuit.

and Fig. 11:5 illustrates the apparatus through which the fluid passes in completing the cycle. With reference to the latter figure, the live steam at boiler pressure  $p_1$  and enthalpy  $h_B$  enters the prime mover (engine or turbine) and is there expanded isentropically to exhaust pressure  $p_2$ . As the result of this adiabatic expansion, shaft work  $W_e$  is developed and leaves the system, the enthalpy decreasing to  $h_C$ . The amount of shaft work so developed is represented by the area of the  $PV$  diagram of Fig. 11:4 and can be calculated by referring to that figure.

$$W_e = \text{area } OABB_1 + \text{area } B_1BCC_1 - \text{area } ODCC_1$$

But, since area  $OABB_1 = P_B v_B$ , area  $B_1BCC_1 = J(u_B - u_C)$  (see Art. 9:6), and area  $ODCC_1 = P_C v_C$ , therefore,

$$AW_e = AP_B v_B + (u_B - u_C) - AP_C v_C = h_B - h_C$$

The same result can be obtained by applying the energy equation for steady flow to a section in the steam-supply line just before the live steam enters the prime mover and to a section in the exhaust line immediately after it has left it. Thus

$$u_B + AP_B v_B + \frac{Aw_B^2}{2g} + {}_BQ_C = u_C + AP_C v_C + \frac{Aw_C^2}{2g} + A({}_B W_C)$$

Since  $w_B$  and  $w_C$  are relatively small and approximately equal,  ${}_BQ_C$  is zero, and  $A({}_B W_C) = AW_e$ , this equation may be simplified to the form

$$AW_e = h_B - h_C$$

In the condenser, the exhaust steam in condensation into liquid form at constant pressure rejects heat  $Q_2$  equivalent to the difference between

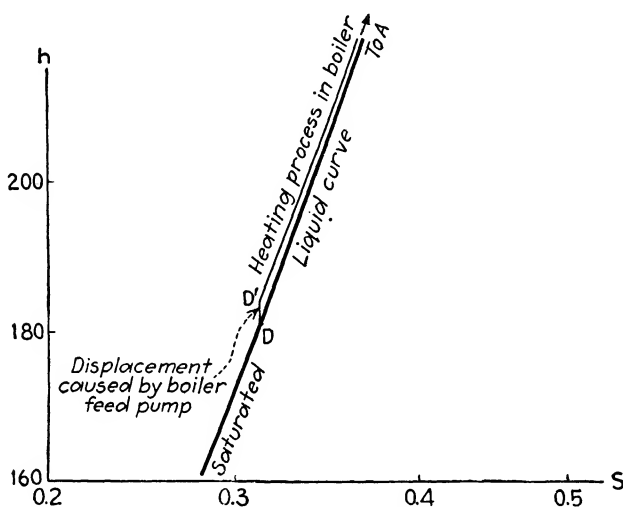


Fig. 11:6.—Effect of boiler-feed pump on condensate condition.

the initial and final enthalpies, or

$$Q_2 = h_C - h_{f_D} = h_C - h_{f_2}$$

in which the subscript 2 refers to the exhaust pressure.

The function of the feed-water pump is to raise the pressure of the condensate isentropically so that it may enter the boiler against the pressure  $p_1$ . The heat equivalent of the amount of work required is

$$AW_p = A(P_1 - P_2)v_{f_2}$$

The work performed by the feed-water pump is reflected in increased energy content of the flowing fluid leaving that unit, and the enthalpy per pound of fluid as it leaves the pump is therefore  $h_{f_2} + A(P_1 - P_2)v_{f_2}$ . The displacement on the Mollier diagram is illustrated in Fig. 11:6. If the compressibility of water is taken into account, it is clear that this isentropic compression will also result in a slight rise in tempera-

ture so that a similar (though relatively much smaller) effect will be produced on  $TS$  coordinates.

The boiler increases the enthalpy of the fluid to its original value  $h_B$  by first heating it to the saturation temperature corresponding to the pressure of the live steam and then evaporating it to condition B. This process takes place at constant pressure, and the amount of heat  $Q_1$  entering each pound of steam in the boiler is therefore the difference between the final and initial enthalpies, or

$$Q_1 = h_B - [h_{f_2} + A(P_1 - P_2)v_{f_2}]$$

The process of heating the condensate to saturation temperature, which is a part of the function of the boiler, may be readily illustrated on coordinates of enthalpy and entropy. Figure 11:6 shows an enlarged illustration of a portion of the cycle transferred to these coordinates. The point  $D$  lies on the saturated liquid line and represents the condition of the condensate as it enters the pump. At  $D'$  the action of the pump has increased the enthalpy but has left the entropy unchanged;  $D'$  therefore represents the condition of the superpressure liquid as it enters the boiler. As it is heated in the boiler to the saturation temperature corresponding to boiler pressure, its state path gradually nears the saturated liquid curve until it joins it at point  $A$ , where evaporation begins.

The total amount of energy (work and heat) entering the system during the circuit must evidently balance the amount leaving in the course of the cycle, or

$$Q_1 + AW_p = AW_e + Q_2$$

or, substituting the values of each item as outlined above,

$$h_B - [h_{f_2} + A(P_1 - P_2)v_{f_2}] + A(P_1 - P_2)v_{f_2} = h_B - h_C + h_C - h_{f_2}$$

That such a balance does exist is readily apparent.

The efficiency of the cycle may now be calculated as the ratio of the *net* amount of work delivered to the heat added per pound of fluid, or

$$e_R = \frac{\text{net work}}{JQ_1} = \frac{AW_e - AW_p}{Q_1} = \frac{(h_B - h_C) - A(P_1 - P_2)v_{f_2}}{h_B - [h_{f_2} + A(P_1 - P_2)v_{f_2}]} \quad (11:2)$$

Since the heat equivalent of the pump work is very small in comparison with the work of the prime mover and with the amount of heat added in the boiler unless the range of pressure between  $p_1$  and  $p_2$  is very great, it is sometimes neglected and Eq. (11:2) is shortened to the following form:

$$e_R = \frac{h_B - h_C}{h_B - h_{f_2}} \text{ (approximately)}$$

The gross convertible energy per pound based on the Rankine cycle is the numerator of the foregoing expression,  $h_B - h_C$ , or the difference of enthalpy at the beginning and the end of the adiabatic expansion. The Mollier diagram may be used to advantage in the determination of the gross convertible energy since the condition at  $C$  may be located by vertical projection from the point representing the condition of the steam as it leaves the boiler (point  $B$ ) and the enthalpies at  $B$  and  $C$  may be read at the side of the chart. The enthalpy of the liquid as it leaves the condenser  $h_{f_2}$  is based upon condenser temperature (the saturation temperature corresponding to condenser pressure) and is read from the tables.

If the steam is superheated as it leaves the boiler, the cycle will be represented by the diagram  $AEB'C'D$  in Fig. 11:4 and the same method of analysis used above will develop a similar expression for the efficiency.

$$e_R = \frac{(h_B' - h_C') - A(P_1 - P_2)v_{f_2}}{h_B' - [h_{f_2} + A(P_1 - P_2)v_{f_2}]} \quad (11:3)$$

The method of solution by the use of the Mollier diagram remains unchanged.

It will be noted that the Rankine cycle is entirely a cycle of the condition of the working fluid. Its efficiency is less than that of a Carnot cycle which operates between the same (constant) limiting temperatures of source and refrigerator; it is therefore an irreversible cycle under these conditions. Examination will show that, for a Rankine cycle utilizing saturated steam, all processes are reversible except that connecting  $D'$  with  $A$  and representing the heating of the feed water to boiler temperature within the boiler. During this process, heat flows across a finite temperature differential.

*Example.* Steam enters a Rankine engine at 250 p.s.i. abs. and 98 per cent quality. Exhaust is to 15 p.s.i. abs. What are the efficiency of the cycle and the steam consumption per horsepower-hour?

*Solution.* Using the Mollier diagram for the determination of the values of  $h_B$  and  $h_C$  and the tables for  $h_{f_2}$  and  $v_{f_2}$ ,  $h_B = 1185$  B.t.u.;  $h_C = 984$  B.t.u.;  $h_{f_2} = 181$  B.t.u.;  $v_{f_2} = 0.01672$  cu. ft.; and

$$e_R = \frac{1,185 - 984 - \frac{144(250 - 15)0.01672}{778}}{1,185 - 181 - \frac{144(250 - 15)0.01672}{778}} = \frac{201 - 0.73}{1,004 - 0.73} = 0.1995$$

The steam consumption is  $2,545/200.27 = 12.7$  lb. per horsepower-hour. If the pump work is neglected, the efficiency becomes

$$e_R = \frac{1,185 - 984}{1,185 - 181} = 0.200$$

As will be noted, the difference is small and may well be neglected.

**11:4. The Pump Cycle.** In early engines a cycle was employed which did not include the adiabatic expansion represented by  $BC$  in Fig. 11:4. The efficiency of such units as work producers is low, and they are used in modern practice only as emergency equipment or when a use exists for the exhaust steam. An example of such a use is the steam cylinder used

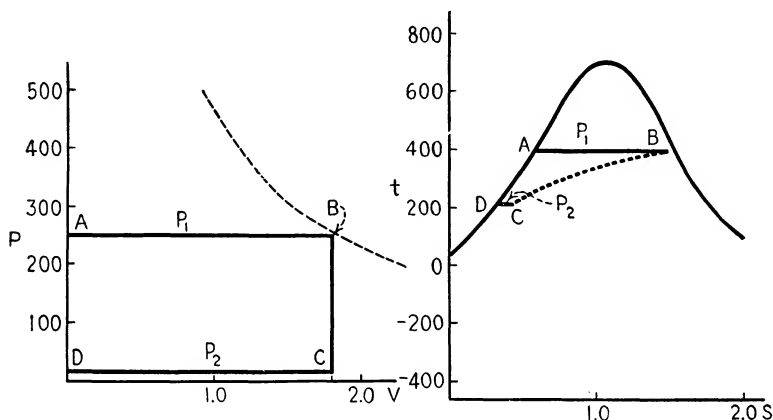


Fig. 11:7.—Pump cycle.

to drive a liquid pump of the boiler-feed type, and the cycle has therefore come to be known as the *pump cycle*. The arrangement of apparatus is essentially the same as shown in Fig. 11:5, with the substitution of the nonexpanding unit for the engine or turbine. The gross amount of work delivered per pound of steam entering the cylinder is the area of the  $PV$  diagram of Fig. 11:7, or

$$W = (P_1 - P_2)v_B$$

This  $PV$  diagram as replotted on  $TS$  coordinates is shown in the right-hand section of Fig. 11:7. The line  $BC$  does not in this case represent the true condition of the steam and has been therefore shown as a dotted line. The  $TS$  diagram may be interpreted in this case only as representing what might have taken place in the cylinder if, when point  $B$  had been reached, condensation of the charge had proceeded within that unit until exhaust pressure had been reached. It is therefore comparable with the machine cycles discussed in Chap. VI with the exception that, in this case, the reservation need be applied only to the process  $BC$ , the balance of the cycle being a true record of the fluid state path.

Point  $C$  as located on the diagrams of Fig. 11:7 does not represent the condition of the steam as it enters the condenser. In order to establish what this condition is, the steady-flow energy equation may be applied between a section in the line through which steam is supplied to the prime mover and a section in the exhaust pipe which carries steam away from that unit. Assuming that the velocities at the two sections are equal and that there is no heat flow,

$$h_C = h_B - AW_e$$

Since the pressure at this point is  $P_2$ , two independent properties of the steam at point  $C$  are known and its condition may be determined.

The efficiency of the cycle is again the ratio of the heat equivalent of the net work to the heat added per pound of fluid, or

$$e = \frac{A(P_1 - P_2)v_B - A(P_1 - P_2)v_{f_2}}{h_B - [h_{f_2} + A(P_1 - P_2)v_{f_2}]} = \frac{A(P_1 - P_2)(v_B - v_{f_2})}{h_B - h_{f_2} - A(P_1 - P_2)v_{f_2}} \quad (11:4)$$

*Example.* What are the efficiency and the steam consumption of a pump cycle using steam under the conditions of the example of Art. 11:3? What is the condition of the steam as it leaves the pump cylinder?

*Solution.*

$$v_B = 0.98 \times 1.8438 = 1.807 \text{ cu. ft. per pound}$$

The other values needed have already been tabulated in the preceding example.

$$e = \frac{\frac{144(250 - 15)(1.807 - 0.01672)}{778}}{1,185 - 181 - \frac{144(250 - 15)0.01672}{778}} = \frac{77.8}{1,003.3} = 0.0775$$

The steam consumption is  $2,545/77.8 = 32.7$  lb. per horsepower-hour.

The work performed by the pump is

$$W_e = (144)(250 - 15)(1.807) = 61,100 \text{ ft.-lb. or } 78.5 \text{ B.t.u.}$$

The enthalpy of the steam as it leaves the pump is

$$h_C = h_B - AW_e = 1185 - 78.5 = 1106.5 \text{ B.t.u. per pound}$$

At 15 p.s.i. abs., this corresponds to a quality of 95.4 per cent.

**11:5. Rankine Cycle with Incomplete Expansion.** If a turbine is used as the prime mover, the expansion line ( $BC$  in Fig. 11:4) is usually continued down to the final, or exhaust, pressure. On the other hand, complete expansion is impractical for the steam engine, and expansion is continued only to some intermediate pressure  $P_i$ . When this pressure is reached at the end of the stroke of the engine, the pressure drops at constant volume to the exhaust pressure, as shown in Fig. 11:8. The complete cycle is seen to be a combination of the Rankine cycle  $ABCF$

and the pump cycle  $FCDE$ . The line  $CD$  has been shown as a dotted line on the  $TS$  diagram for the reasons outlined in the preceding article. The heat equivalent of the gross work represented by the first area has been shown to be  $h_B - h_C$ , that corresponding to the second is  $A(P_1 - P_2)v_C$ , the heat equivalent of the work of the feed-water pump is  $A(P_1 - P_2)v_{f_2}$ , and the heat added in the boiler is, as before,

$$h_B - h_{f_2} - A(P_1 - P_2)v_{f_2}$$

The efficiency of the cycle is therefore

$$e = \frac{A(W_e - W_p)}{Q_1} = \frac{[h_B - h_C + A(P_1 - P_2)v_C] - A(P_1 - P_2)v_{f_2}}{h_B - [h_{f_2} + A(P_1 - P_2)v_{f_2}]} \quad (11:5)$$

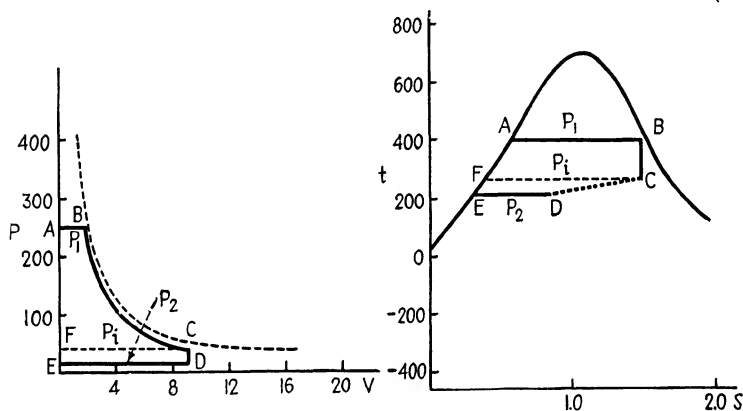


FIG. 11:8.—Rankine cycle with incomplete expansion.

*Example.* In the example of Art. 11:3, assume the same conditions except that adiabatic expansion ends at a pressure of 40 p.s.i. abs. Exhaust pressure remains at 15 p.s.i. abs. What is the efficiency of the cycle?

*Solution.* From the Mollier diagram,  $h_C = 1047$  B.t.u.,  $x_C = 0.868$ . The volume of dry steam at the intermediate pressure of 40 p.s.i. abs. may be read from the steam tables as 10.498 cu. ft. per pound. Therefore,

$$v_C = 0.868 \times 10.498 = 9.11 \text{ cu. ft. per pound}$$

$$e = \frac{1,185 - 1,047 + \frac{144(40 - 15)9.11}{778} - 0.73}{1,185 - 181 - 0.73} = 0.179$$

**11:6. Engine and Turbine Losses.** Although the engine and the turbine differ entirely with respect to their principle of operation, they are alike in the amount of mechanical energy that they can theoretically develop under parallel conditions of steam supply and exhaust pressure. The choice between these two types of prime mover must therefore be



based upon practical considerations such as cost, maintenance, speed, space available, and the percentage of ideal performance that can be developed; the last-named factor will depend upon practical losses over and above those (such as the heat rejected in the condenser) associated with the theoretical cycle.

The principal practical losses associated with engine operation are *wire drawing*, *clearance* and *compression*, *incomplete expansion*, *mechanical friction*, and *initial condensation*. Both the inlet and the exhaust valves of the engine are intermittently closed and opened; as they open and close, they throttle the passage provided for the flow of steam. This causes a throttling action which, although it does not lower the energy content of the steam, does result in a pressure and temperature drop and thus a decrease in the availability of that energy.

All steam engines must have clearance; steam is compressed in this space, acting as a cushion for the piston as it nears the end of its stroke. As in the air compressor and the air engine, these provisions do not cause a loss except as they require larger cylinders for a given output and consequently entail higher frictional losses. Incomplete expansion and its effects have been considered in Art. 11:5, and the larger mechanical friction losses that are associated with the engine are due to the necessity for converting reciprocating into rotary motion.

Initial condensation is an effect produced by the fluctuation of temperature of the metal cylinder as the charge is successively admitted, expanded, and exhausted. As the live steam enters the cylinder, it comes in contact with the walls of the ports and cylinder, which have just been cooled by the exhaust steam during the preceding stroke. The fresh charge of live steam must give up heat to raise the temperature of these walls and thus will suffer a reduction in quality or superheat. As the cutoff is shortened, this initial condensation effect will increase. When dry and saturated steam is supplied by the boiler, the quality at cutoff is likely to be between 0.50 and 0.80. Initial condensation is the most important of the practical losses associated with the vapor-engine prime mover.

Since the temperature, pressure, and condition of the steam remain practically constant at any specified point in the flow of steam through a turbine, the losses due to fluctuation of temperature and consequent undesirable heat exchange between the steam and cylinder walls, which are so costly in the operation of the steam engine, may be almost entirely eliminated. Thus the initial condensation effect will disappear. The pressure at entrance to the expanding nozzles in the turbine will be slightly less than boiler pressure owing to the differential of pressure

required to maintain flow, but this effect (corresponding to *wire drawing*, in the engine) is much less pronounced than in the case of the engine, where it is necessary to open and close these passages periodically. Also, the turbine has no effect which is directly comparable with that of *clearance loss* in the engine.

On the other hand, certain losses are present in the turbine which either do not appear or are relatively unimportant in engine operation. The more important of these are discussed below.

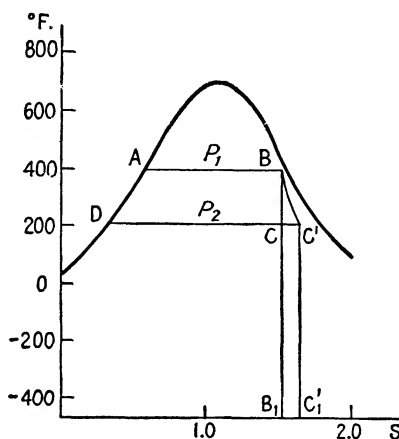


FIG. 11:9.—Rankine cycle showing effect of internal friction.

1. A part of the steam which is directed upon the blades by the nozzles or vanes will escape through the annular space between the perimeter of the outer blade circle and the turbine casing without doing work on the blading. This is called *leakage loss*.

2. It is not possible to remove all the kinetic energy from the steam jet in the blading. Sufficient velocity must remain to enable the steam to clear the blading and pass out through the exhaust passages. This effect, which may be termed the *residual velocity loss*, has the effect of reducing the net convertible energy.

3. Internal friction increases rapidly with the velocity of flow. In the engine, the velocities are relatively low and the *internal-friction loss* is small; but the velocities in the turbine nozzles are so high that this effect, which has been discussed in Chap. X, becomes quite important. In Fig. 11:9, this effect is illustrated by the divergence of the line  $BC'$  from the vertical (isentropic) of the ideal Rankine cycle. The basic Rankine cycle is illustrated by the area  $ABCD$ , and the area  $ABC'D$  shows the

change in the cycle caused by internal friction. At first thought, it would seem that, since the enclosed area is larger, the performance is improved; but it will be remembered that the enclosed area represents shaft work only if no friction is present. As a matter of fact, the area under the state line  $BC'$  (area  $B_1BC'C_1'$ ) represents work which has been reconverted into heat because of friction, and this area must be subtracted from the gross area of the diagram ( $ABCD$ ) in order to obtain the shaft work. Since these two areas contain the common area  $BC'C$ , it will be evident that the shaft work is less than for the ideal Rankine cycle by the work equivalent of the heat area  $B_1CC'C_1'$ . Further, since the area representing the heat equivalent of shaft work for the Rankine cycle (area  $ABCD$ ) is  $h_B - h_C$  (see Art. 11:3) and since area  $B_1CC'C_1'$  equals  $h_{C'} - h_C$ , the heat equivalent of shaft work available with friction,

$$\begin{aligned} AW &= \text{area } ABCD - \text{area } B_1CC'C_1' \\ &= (h_B - h_C) - (h_{C'} - h_C) = h_B - h_{C'} \end{aligned} \quad (11:6)$$

It will be understood that in Fig. 11:9 no attempt has been made to show the effect of the other losses which have been mentioned as important in turbine operation. However, a final analysis of the combined effect of all losses can be obtained as the result of an economy test. In general, the aggregate of the effects of the practical losses associated with either type of prime mover cannot be said to be less than the total of those associated with the other. Refinements of design are, in general, directed toward a reduction of these practical losses.

The expansion of the steam in a turbine is frequently carried out in a number of stages. Although this is often done primarily with the idea of reducing the velocity of the blading, it will also lessen the frictional losses to a considerable extent since the maximum steam velocities in each stage are reduced. When staging is employed, all or part of the steam may be withdrawn from the turbine at the end of any stage. This possibility has led to the development of the reheat and the regenerative cycles.

**11:7. The Reheat Cycle.** In practice, the moisture which accumulates in the steam as a result of the adiabatic expansion in the nozzles of a steam turbine is a cause of considerable difficulty in operation and maintenance. Carried at extremely high velocities with the entering steam jet, particles of moisture so formed impinge upon the blading, causing rapid erosion. In order to prevent the moisture content from becoming too great, superheated steam is frequently used for turbines. The *reheat cycle* provides for removing all the steam from the turbine at some intermediate stage before the moisture content has become too large,

reheating it at constant pressure, and returning it to the turbine for further expansion in the later stages. Figure 11:10 illustrates this cycle. The first stages of the turbine expand the steam from boiler pressure  $p_1$  to some intermediate pressure  $p_i$ . At this point the steam is removed from the turbine and reheated along the constant-pressure path  $CcD$ ; from this condition it is expanded in the low-pressure stages to the condenser pressure  $p_2$ .

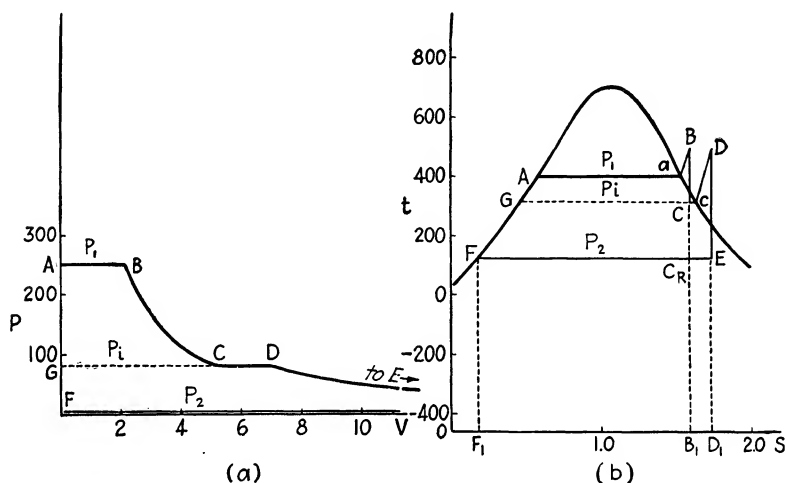


FIG. 11:10.—The reheat vapor cycle.

The efficiency of the reheat cycle is calculated as follows: The total heat added per pound of steam is the sum of the heat added in the boiler and the heat added in the reheating process. The heat added in the boiler has been shown to be

$$h_B - [h_{f_2} + A(P_1 - P_2)v_{f_2}]$$

and, since reheating takes place at constant pressure, the heat added during the reheating process is  $h_D - h_C$ . Therefore,

$$Q_1 = h_B - [h_{f_2} + A(P_1 - P_2)v_{f_2}] + (h_D - h_C)$$

The gross work done in the early stages of the turbine before reheating can be shown to be  $h_B - h_C$  by applying the method of Art. 11:3 to this part of the cycle, and the work performed after reheating, similarly, is  $h_D - h_E$ . From the sum of these two quantities must be deducted the thermal equivalent of the work of the feed-water pump to obtain the net shaft work. Therefore

$$e = \frac{A(\text{net shaft work})}{Q_1} = \frac{h_B - h_C + h_D - h_E - A(P_1 - P_2)v_{f_2}}{h_B - h_{f_2} + h_D - h_C - A(P_1 - P_2)v_{f_2}} \quad (11:7)$$

The theoretical efficiency of the reheat cycle will not differ greatly from the efficiency of the Rankine cycle produced by completing the expansion in a single step (from  $B$  to  $C_R$  in Fig. 11:10). The advantage in the use of this cycle is practical rather than theoretical and is based upon the lower moisture content of the exhaust steam. In the solution of problems based on the reheat cycle, the Mollier diagram will again be found very useful.

*Example.* What is the theoretical efficiency of the reheat cycle which takes steam from a boiler at 250 p.s.i. abs. and 500°F., expands it to 80 p.s.i. abs., reheats to 500°F., and exhausts at a pressure of 2 p.s.i. abs.? What is the final quality of the steam as it leaves the turbine? If the initial expansion were continued to exhaust pressure, what would be the efficiency of the resulting Rankine cycle? What would be the final quality for this case?

*Solution.* From the Mollier diagram,

$$h_B = 1,263; \quad h_C = 1,163; \quad h_D = 1,281; \quad h_E = 1,008$$

From the steam tables,

$$h_{f_2} = 94 \text{ B.t.u.}; \quad v_{f_2} = 0.01623 \text{ cu. ft. per pound}$$

The heat equivalent of the pump work is

$$A(P_1 - P_2)v_{f_2} = \frac{144(250 - 2)0.01623}{778} = 0.7 \text{ B.t.u. per pound}$$

Based on Eq. (11:7), the efficiency is

$$e = \frac{1,263 - 1,163 + 1,281 - 1,008 - 0.7}{1,263 - 94 + 1,281 - 1,163 - 0.7} = 0.289$$

The final quality  $x_E$ , as read from the Mollier diagram, is 0.894.

For the Rankine cycle without reheat, from the Mollier diagram, the quality at  $C_R$  is 0.814, and the enthalpy at the same point is 926 B.t.u. The efficiency for this cycle is

$$e_R = \frac{1,263 - 926 - 0.7}{1,263 - 94 - 0.7} = 0.288$$

**11:8. The regenerative vapor cycle** is illustrated diagrammatically for a three-heater installation in Fig. 11:11. The steam entering the turbine at  $A$  is partly expanded to successively lower intermediate pressures at  $B$ ,  $C$ , and  $D$ , finally entering the condenser at  $E$ . At each of the intermediate stations  $B$ ,  $C$ , and  $D$ , a part of the steam is bled to a corresponding open heater. Flow to these heaters is induced by the condensation of steam within them, and in each the bled steam comes into contact with the feed water which is returning to the boiler and raises it to the satu-

ration temperature corresponding to the pressure within the heater. The weight of feed water leaving each heater is the sum of the weight of feed water entering the heater and the weight of steam condensed in the heater. Feed-water pumps are used between the condenser and the low-pressure heater, between successive heaters, and between the high-pressure heater and the boiler.

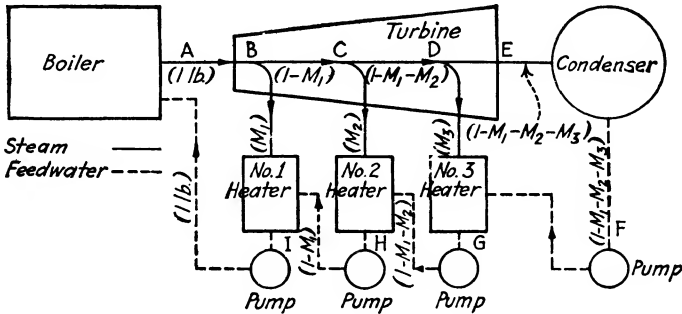


FIG. 11:11.—Diagram of the regenerative cycle.

The weight of steam bled at B to No. 1 heater per pound of steam entering the turbine will be designated as  $M_1$ , at C by  $M_2$ , etc. The weight of steam expanding from A to B is therefore 1; from B to C,  $1 - M_1$ ; from C to D,  $1 - M_1 - M_2$ ; and from D to E,  $1 - M_1 - M_2 - M_3$ . The heat equivalent of the total work performed per pound of steam entering the turbine is therefore

$$AW = 1(h_A - h_B) + (1 - M_1)(h_B - h_C) + (1 - M_1 - M_2)(h_C - h_D) + (1 - M_1 - M_2 - M_3)(h_D - h_E) \quad (11:8)$$

The weight bled to No. 1 heater,  $M_1$ , may be calculated on the basis that the heat given up by the bled steam is equal to the heat gained by the feed water in its passage through the heater, or

$$M_1(h_B - h_{fB}) = (1 - M_1)(h_{fB} - h_{fC})$$

and

$$M_1 = \frac{h_{fB} - h_{fC}}{h_B - h_{fC}} \quad (11:9)$$

The weight bled to the second heater may be calculated by a similar process, as follows:

$$M_2(h_C - h_{fC}) = (1 - M_1 - M_2)(h_{fC} - h_{fD})$$

from which

$$M_2 = \frac{(1 - M_1)(h_{fC} - h_{fD})}{h_C - h_{fD}} \quad (11:10)$$

The exchange of heat within the third heater is

$$M_3(h_D - h_{fD}) = (1 - M_1 - M_2 - M_3)(h_{fD} - h_{fE})$$

and

$$M_3 = \frac{(1 - M_1 - M_2)(h_{fD} - h_{fE})^*}{h_D - h_{fE}} \quad (11:11)$$

The same method of calculation may be employed for any number of heaters which may be used; in practice, the limit is about four heaters since, although the theoretical efficiency of the cycle will increase with a greater number of heaters, the practical disadvantages of greater complexity and greater fixed charges will offset the decreasing theoretical gain.

The gross convertible energy per pound of steam entering the turbine has been expressed in Eq. (11:8). With pump work neglected, the heat supplied per pound is  $h_A - h_{fB}$ , and the efficiency of the cycle is

$$e = \frac{AW}{Q_1} = \frac{h_A - h_B + (1 - M_1)(h_B - h_C) + (1 - M_1 - M_2)(h_C - h_D) + (1 - M_1 - M_2 - M_3)(h_D - h_E)}{h_A - h_{fB}} \quad (11:12)$$

Although the equation as written applies to a cycle including three heaters, the method of applying it to a smaller or larger number is obvious.

It will be noted that the effect of pump work has been neglected in the foregoing development for the sake of simplification. If it is included, the effect on the numerical value of the efficiency will be slight but considerable extra work will be involved. For instance, in the calculation of the weight  $M_1$  bled to the first heater, a correction must be added to the term  $h_{fC}$  in the numerator since the feed water as it enters the heater is a superpressure liquid and its enthalpy is greater than that of saturated liquid by the amount of energy added through the action of the feed-water pump, or  $A(P_B - P_C)v_{fC}$  B.t.u. per pound. The corrected expression for the weight bled to No. 1 heater will then become

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\* By substitution of the previously determined values of  $M_1, M_2$ , etc., in succeeding equations of the series, Eqs. (11:10) and (11:11) may be changed to the following forms:

$$M_2 = \left( \frac{h_B - h_{fB}}{h_B - h_{fC}} \right) \left( \frac{h_{fC} - h_{fD}}{h_C - h_{fD}} \right)$$

$$M_3 = \left( \frac{h_B - h_{fB}}{h_B - h_{fC}} \right) \left( \frac{h_C - h_{fC}}{h_C - h_{fD}} \right) \left( \frac{h_{fD} - h_{fE}}{h_D - h_{fE}} \right), \text{ etc.}$$

$$M_1 = \frac{h_{f_B} - h_{f_C} - A(P_B - P_C)v_{f_C}}{h_B - h_{f_C}}$$

and, similarly,

$$M_2 = \frac{(1 - M_1)[h_{f_C} - h_{f_D} - A(P_C - P_D)v_{f_D}]}{h_C - h_{f_D}}$$

$$M_3 = \frac{(1 - M_1 - M_2)[h_{f_D} - h_{f_E} - A(P_D - P_E)v_{f_E}]}{h_D - h_{f_E}}, \text{ etc.}$$

Having obtained these corrected values of  $M_1$ ,  $M_2$ ,  $M_3$ , etc., they may be substituted in Eq. (11:8) to obtain the gross convertible energy of the cycle. A deduction must be made from this gross value to allow for pump work as in the analysis of previous cycles. The amount of this deduction is evidently the heat equivalent of the sum of the work expended on all the pumps.

Heat equivalent of total pump work =  $A(P_A - P_B)v_{f_B}$

$$+ A(1 - M_1)(P_B - P_C)v_{f_C} + A(1 - M_1 - M_2)(P_C - P_D)v_{f_D} \\ + A(1 - M_1 - M_2 - M_3)(P_D - P_E)v_{f_E} \text{ (three-heater cycle)}$$

The value of this expression is deducted from the numerator of the efficiency equation [Eq. (11:12)]. A deduction must also be made from the denominator to allow for the increased enthalpy of the superpressure liquid entering the boiler. This deduction is evidently the heat equivalent of the work expended in the pump between No. 1 heater and the boiler, or  $A(P_A - P_B)v_{f_B}$ .

*Example A.* Steam at 400 p.s.i. abs. and 600°F. enters a turbine operating on the regenerative cycle. The turbine is bled at 100, 35, and 10 p.s.i. abs. Condenser pressure is 2 p.s.i. abs. Neglecting pump work, calculate the weight of steam bled to each heater per pound of steam entering the turbine, the efficiency, and the steam consumption of the cycle. What are the efficiency and steam consumption of the equivalent Rankine cycle without regeneration?

*Solution.* The initial enthalpy of the steam at entrance to the turbine and the enthalpies at the bleeding points and at entrance to the condenser may be read from the Mollier diagram as follows:

$$h_A = 1,307; \quad h_B = 1,177; \quad h_C = 1,097; \quad h_D = 1,014; \quad h_E = 923$$

As read from the steam tables, the enthalpy of the feed water at the saturation temperatures corresponding to the pressures within the heaters and the condenser are

$$h_{f_B} = 298.4; \quad h_{f_C} = 227.9; \quad h_{f_D} = 161.2; \quad h_{f_E} = 94.0$$

The weights bled to the heaters are



$$M_1 = \frac{h_{fB} - h_{fC}}{h_B - h_{fC}} = \frac{298.4 - 227.9}{1,177 - 227.9} = 0.0743 \text{ lb.}$$

$$M_2 = \frac{(1 - M_1)(h_{fC} - h_{fD})}{h_C - h_{fD}} = \frac{(1 - 0.0743)(227.9 - 161.2)}{1,097 - 161.2} = 0.0713 \text{ lb.}$$

$$M_3 = \frac{(1 - M_1 - M_2)(h_{fD} - h_{fE})}{h_D - h_{fE}} = \frac{(1 - 0.0743 - 0.0713)(161.2 - 94.0)}{1,014 - 94.0} \\ = 0.0624 \text{ lb.}$$

The heat equivalent of the work performed per pound of steam entering the turbine is

$$AW = h_A - h_B + (1 - M_1)(h_B - h_C) + (1 - M_1 - M_2)(h_C - h_D) \\ + (1 - M_1 - M_2 - M_3)(h_D - h_E) \\ = 1,307 - 1,177 + 0.9257(1,177 - 1,097) + 0.8544(1,097 - 1,014) \\ + 0.792(1,014 - 923) \\ = 347.2 \text{ B.t.u. per pound}$$

The heat added per pound in the boiler is

$$Q_1 = h_A - h_{fB} = 1,307 - 298.4 = 1008.6 \text{ B.t.u. per pound}$$

The efficiency of the cycle is

$$e = \frac{AW}{Q_1} = \frac{347.2}{1008.6} = 0.344$$

The steam consumption is 2,545/347.2, or 7.33 lb. per horsepower-hour. The efficiency of the equivalent Rankine cycle, pump work being neglected, is

$$e_R = \frac{1,307 - 923}{1,307 - 94} = 0.316$$

and its steam consumption is 2,545/(1,307 - 923), or 6.64 lb. per horsepower-hour.

An interesting interpretation of the true character of the regenerative vapor cycle may be obtained by representing it on  $TS$  coordinates as shown in Fig. 11:12. This chart has been constructed for the conditions of the foregoing example by the method outlined below.

The expansion of the steam through the turbine is represented by the adiabatic line  $AE$ . At  $E$  the steam enters the condenser where the condensation of the remaining portion  $(1 - M_1 - M_2 - M_3)$  takes place. The amount of heat rejected by the steam in the condenser is the product of its weight and the heat removed per pound, or

$$(1 - M_1 - M_2 - M_3)(h_E - h_{fE}) = 0.792(923 - 94.0) = 657 \text{ B.t.u.}$$

This amount of heat is rejected at a constant temperature of 126°F. or, approximately, 586°F. abs. and is represented by the area  $EFF_1E_1$ . The decrease of entropy for the process may be calculated by dividing the heat abstracted by the absolute temperature, or

$$s_E - s_F = \frac{657}{586} = 1.12$$

Since the entropy at E ( $=s_A$ ) is 1.589, the entropy at F is  $1.589 - 1.12$  or 0.469. The point F may now be located on the diagram.

The condensate is now heated to the temperature of the steam in the low-pressure heater or, approximately,  $193^\circ\text{F}$ . During the process the entropy is increased by an amount equal to the product of the weight (0.792 lb.) and the difference between the entropy of a pound of water

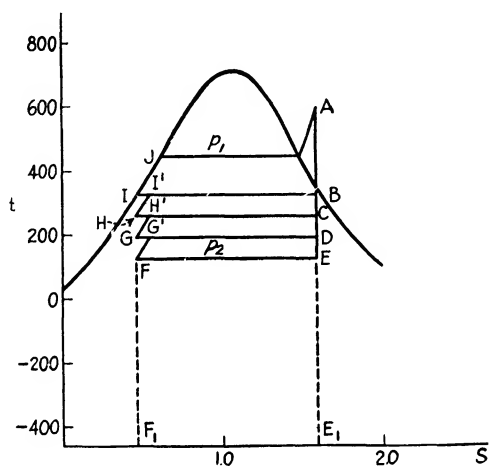


FIG. 11:12.—The regenerative cycle on  $TS$  coordinates.

at the saturation temperature corresponding to the pressure in the condenser ( $126^\circ\text{F}$ .) and at the saturation temperature equivalent to the pressure in No. 3 heater ( $193^\circ\text{F}$ .). The increase of entropy is therefore

$$s_{G'} - s_F = 0.792(0.2835 - 0.1749) = 0.086$$

Point  $G'$  can now be plotted.

The line  $G'G$  represents the rejection of heat due to the condensation of the steam bled to No. 3 heater. The amount of heat so rejected is

$$M_s(h_D - h_{fD}) = 0.0624(1,014 - 161.2) = 53.1 \text{ B.t.u.}$$

and the corresponding reduction of entropy is

$$s_{G'} - s_G = \frac{53.1}{193.2 + 459.6} = 0.081$$

The remaining points can be located by similar processes.

A study of the resulting diagram makes clear the fact that the regenerative vapor cycle is an approach from the Rankine toward the more

efficient Carnot cycle. If the initial condition of the steam (at  $A$ ) were in the saturated-vapor region and an infinite number of heaters were used, the Carnot cycle would, in theory, be exactly duplicated.

It has already been demonstrated that Rankine-cycle efficiency is less than that of the equivalent Carnot cycle because of the irreversible flow of heat across a finite temperature interval during the process of heating the feed water. If the average temperature interval can be reduced, it is only reasonable to conclude that the *degree* of irreversibility would also decrease and the efficiency would approach that of the Carnot cycle. In order to obtain a *minimum* average temperature interval, the *individual* temperature intervals between heater temperatures should be approximately uniform. Since the boiler also acts as a feed-water heater, this would also apply to the differential between its temperature and that maintained in the heater which operates at highest pressure. Thus the nearly ideal heater temperatures can be located by dividing the total temperature interval between  $t_1$  (the saturation temperature equivalent to  $p_1$ ) and  $t_2$  (the saturation temperature equivalent to  $p_2$ ) by one more than the number of heaters and subtracting this interval in succession from  $t_1$ , from the temperature of the highest pressure heater, etc. The corresponding heater pressures are, of course, the saturation pressures equivalent to these temperatures.

*Example B.* What would be the ideal heater pressures for Example A?

*Solution.* The saturation temperature equivalent to a pressure of 400 p.s.i. abs. is 444.59°F. The saturation temperature equivalent to 2 p.s.i. abs. is 126.08°F. The temperature interval per heater is

$$\frac{444.6 - 126.1}{4} = 79.6^\circ\text{F.}$$

The ideal temperature for the first heater is  $444.6 - 79.6 = 365^\circ\text{F.}$ , corresponding to a saturation pressure of 163 p.s.i. abs. in that heater. Similar calculations will indicate the ideal pressures for the second and third heaters to be 53 p.s.i. abs. and 13 p.s.i. abs., respectively.

In practice, certain other factors will also affect the choice of heater pressures. For instance, steam may be withdrawn from the turbine only between stages, and the design of the turbine will determine how closely the ideal heater pressures may be approached.

The regenerative cycle as outlined above would require the use of open feed-water heaters and a complicated feed-water pump arrangement. This is usually not considered desirable in the practical installation, and the condensate is drained from closed heaters to the steam space of the heater of next lower pressure or directly to a hot well, thus making it

possible for one pump to serve the entire system. The effect of changes of this character when introduced into the theoretical analysis of the cycle will be to decrease slightly the convertible energy per pound of steam entering the turbine, to increase slightly the amount of heat which must be supplied in the boiler, and thus to lower somewhat the efficiency of the cycle.

**11:9. The Binary Vapor Cycle.** Since the limiting efficiency of any means of producing power from heat is that of the Carnot cycle [ $e = (T_S - T_R)/T_S$ ], it is evident that the efficiency of a specific installation may be increased by either raising  $T_S$  or lowering  $T_R$ .  $T_R$  may not be lowered below the temperature of the surrounding atmosphere; but, aside from practical considerations,  $T_S$  may be increased without such limit. In vapor cycles, in order to approach the Carnot-cycle condition of isothermal addition of heat, it is desirable that the vapor begin its expansion from a saturated condition. The common medium in use is steam, but steam has a number of serious disadvantages. One of these is that the highest temperature at which it can exist in the saturated condition is only about 706°F. and that the saturation pressures are inconveniently high near this temperature level. A second is found in the large number of reheating stages that are necessary to avoid excessive blade erosion due to the large percentages of moisture which develop in the steam as the result of the adiabatic expansion that takes place in the turbine nozzles. A third results from the high specific volumes which are characteristic of steam at the lower temperature limits, making necessary the provision of large passages for the steam through the low-pressure stages of a turbine or of very large cylinders for engines operating at high vacuum.

No ideal fluid is known which has properties that are in all respects more desirable than those of steam but it has been suggested that, by combining two or more vapors each operating through a part of the temperature range, an advantage might be derived. Thus *mercury* has more satisfactory properties in the first respect since its critical temperature is much higher and the corresponding saturation pressures very much lower than those of steam. Its use at the lower range of temperature is entirely impractical, however. The accumulation of liquid particles in the vapor jet could be avoided by use of *diphenol oxide*. The  $TS$  diagram for this substance shows that the dry-vapor line slopes downward to the *left* instead of to the right as for most vapors, indicating that the vapor would become superheated as the result of an isentropic expansion. Even this is not entirely desirable, as the ideal vapor would have a *vertical* dry-vapor line: the use of diphenol oxide would be out of

the question for the lower range since it becomes a solid at ordinary atmospheric temperatures. *Sulphur dioxide* has certain advantages over steam at the lower range of temperatures owing to its low specific volume but has a very low critical temperature.

A number of practical installations combining the use of mercury at the higher range of temperature and of steam for the lower range have been constructed and have operated successfully. Two representative installations are located at Schenectady, N.Y. and Kearny, N.J. In a typical installation, the mercury is evaporated at a temperature of about 950°F., corresponding to a saturation pressure of about 140 p.s.i. abs., expanded in a mercury turbine to an absolute pressure of about 2.5 in. Hg, and exhausted to a mercury condenser, which also serves as a steam boiler, taking the place of the usual furnace in a steam-boiler unit. The temperature of the exhaust mercury is still high (above 450°F.) and the heat given up by the mercury vapor in condensing vaporizes the water into steam at a pressure of about 400 p.s.i. This steam is then expanded in a conventional steam turbine to the final temperature of the cycle. An interesting feature of some installations is the location of the mercury turbine and condenser on a platform above the mercury boiler; owing to the low saturation pressure of the mercury vapor and the high specific gravity of the mercury liquid, no feed pump is required for the mercury boiler. The heat remaining in the flue gases leaving the mercury boiler is utilized to superheat the saturated steam as it leaves the mercury-condenser steam-boiler unit. The weights of mercury and steam passing through the cycle in an equal time are proportioned in such a manner that the heat given up by the mercury vapor in the condenser can supply the heat necessary to evaporate the water into steam. This ratio of weights depends on the relative thermal-storage capacities of the two substances. The numerical ratio is about 9 lb. of mercury to 1 lb. of steam.

The use of sulphur dioxide for the low-temperature range and diphenol oxide for the higher temperatures has also been suggested but never applied in a major commercial installation.

**11:10. Methods of Governing.** When steam engines or turbines are to be operated at constant speed under varying loads, some type of governor must be employed. Thermodynamically, all such governors fall into two classes. As the load on the prime mover decreases and the speed in consequence tends to increase, (1) the steam may be admitted in smaller quantities (shorter cutoff in the engine or through a reduced number of nozzles to the turbine) but at full boiler pressure or (2) an automatic valve may be placed in the line carrying steam from the

boiler which will throttle the steam to a lower pressure before admitting it to the prime mover.

Governors of the first class do not affect the amount of energy theoretically convertible in each pound of steam, but those of the second class (called throttling governors) reduce the amount of thermal energy convertible into work by the Rankine cycle prime mover as seen in Fig. 11:13. The process consists of throttling the steam from  $p_1$  to some lower pressure  $p_i$ . By Eq. (10:17), it is evident that the change from the pressure in

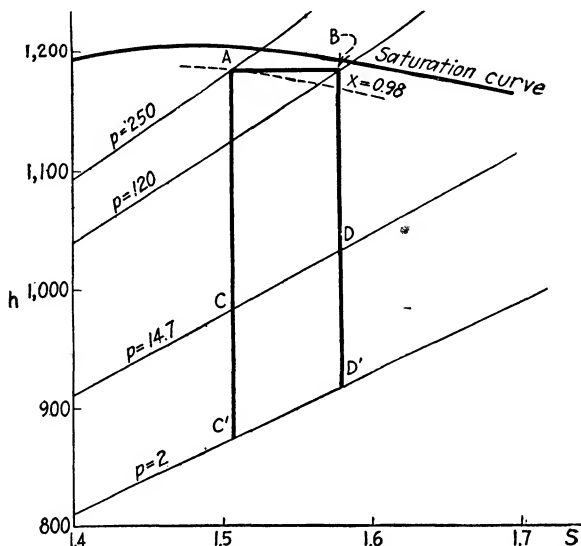


FIG. 11:13.—Effect of throttling on convertible energy of steam.

the steam line to that of the steam as it enters the prime mover takes place at constant enthalpy. This is represented in Fig. 11:13, which is a section of a Mollier diagram, by the line  $AB$ .

The convertible energy of the steam per pound in condition  $A$  based on the Rankine cycle has been shown to be  $h_A - h_C$ ; in condition  $B$ , the convertible energy per pound is  $h_B - h_D (= h_A - h_D)$ . The loss of convertible energy due to throttling the steam in the governor is

$$(h_A - h_C) - (h_A - h_D) = h_D - h_C$$

*Example.* Determine the percentage loss of convertible energy when steam at 250 p.s.i. abs., 98 per cent quality, is throttled to a pressure of 120 p.s.i. abs. before admission to a Rankine engine, (a) when the exhaust pressure is atmospheric and (b) when the engine operates condensing at an exhaust pressure of 2 p.s.i. abs.

*Solution.* (a) From the Mollier diagram,

$$h_A = 1185 \text{ B.t.u.}; \quad h_C = 983 \text{ B.t.u.}; \quad h_D = 1033 \text{ B.t.u.}$$

The actual loss of convertible energy per pound is

$$h_D - h_C = 1033 - 983 = 50 \text{ B.t.u. per pound}$$

The percentage loss is

$$\frac{h_D - h_C}{h_A - h_C} = \frac{50}{1185 - 983} = 0.247 \text{ or } 24.7 \text{ per cent}$$

$$(b) h_A = 1185 \text{ B.t.u.}; \quad h_C' = 874 \text{ B.t.u.}; \quad h_{D'} = 918 \text{ B.t.u.}$$

The actual loss is

$$h_{D'} - h_{C'} = 918 - 874 = 44 \text{ B.t.u. per pound}$$

The percentage loss is

$$\frac{h_{D'} - h_{C'}}{h_A - h_{C'}} = \frac{44}{1185 - 874} = 0.141 \text{ or } 14.1 \text{ per cent}$$

It may be noted that the decrease in convertible energy could, for this adiabatic process, as well have been calculated as the increase of unavailable energy (see Chap. V). In the example, the original entropy is 1.507, and the entropy after throttling the steam is 1.581. The temperature of the refrigerator corresponding to an exhaust pressure of 14.7 p.s.i. abs. is 212°F.; that corresponding to 2 p.s.i. abs. is 126°F. Thus the increase in unavailable energy is

$$(a) (1.581 - 1.507)(212 + 459.7) = 49.7 \text{ B.t.u. per pound}$$

$$(b) (1.581 - 1.507)(126 + 459.7) = 43.5 \text{ B.t.u. per pound}$$

These values should be compared with the corresponding values for the decrease of convertible energy as obtained above.

### Problems

1. What are the efficiency and the steam consumption of a Carnot-cycle steam engine that contains saturated water at the beginning and dry and saturated steam at 150 p.s.i. abs. at the end of the isothermal expansion if the exhaust pressure is 16 p.s.i. abs.? Illustrate the cycle on  $PV$ ,  $TS$ , and Mollier diagrams.
2. What are the efficiency and the steam consumption of a Carnot-cycle steam engine that contains saturated water at the beginning and dry and saturated steam at 250 p.s.i. abs. at the end of the isothermal expansion if the exhaust pressure is 16 p.s.i. abs.? Illustrate the cycle on  $PV$ ,  $TS$ , and Mollier diagrams.
3. What are the efficiency and steam consumption of a Carnot-cycle steam engine that contains saturated water at the beginning and dry and saturated steam at 150 p.s.i. abs. at the end of the isothermal expansion if the exhaust pressure is 4 p.s.i. abs.? Illustrate the cycle on  $PV$ ,  $TS$ , and Mollier diagrams.
4. What are the efficiency and steam consumption of a Carnot-cycle steam engine that contains saturated water at the beginning and superheated steam at a pressure of 150 p.s.i. abs., temperature of 400°F., at the end of the isothermal expansion if the exhaust pressure is 16 p.s.i. abs.? Illustrate the cycle on  $PV$ ,  $TS$ , and Mollier coordinates.
5. Making use of the temperature-entropy diagram, discuss the effect on the efficiency of a Carnot-cycle vapor engine of (a) raising the pressure at which evaporation takes place, (b) increasing the quality of the vapor as supplied the engine, and (c) raising the exhaust pressure.

6. What are the efficiency and steam consumption of a Rankine-cycle steam power plant if steam is supplied the prime mover dry and saturated at 150 p.s.i. abs. and the exhaust pressure is 16 p.s.i. abs.? What is the ratio of the efficiency to that of the equivalent Carnot cycle? What is the cutoff percentage if an ideal engine is used as the prime mover? Illustrate the Rankine cycle on  $PV$ ,  $TS$ , and Mollier diagrams.

7. What are the efficiency and steam consumption of a Rankine-cycle steam power plant if steam is supplied the prime mover dry and saturated at 250 p.s.i. abs. and the exhaust pressure is 16 p.s.i. abs.? What is the ratio of its efficiency to that of the equivalent Carnot cycle? Illustrate the Rankine cycle on  $PV$ ,  $TS$ , and Mollier coordinates.

8. What are the efficiency and steam consumption of a Rankine-cycle steam power plant if steam is supplied the prime mover dry and saturated at 150 p.s.i. abs. and the exhaust pressure is 4 p.s.i. abs.? What is the ratio of its efficiency to that of the equivalent Carnot cycle? Illustrate the cycle on  $PV$ ,  $TS$ , and Mollier diagrams.

9. Steam is supplied the prime mover of an ideal Rankine-cycle power plant at a pressure of 150 p.s.i. abs., quality of 0.97. Exhaust pressure is 16 p.s.i. abs. What is its efficiency? Explain why the efficiency of this Rankine cycle is slightly lower than that of the Rankine cycle described in Prob. 6.

10. What are the efficiency and steam consumption of a Rankine-cycle steam power plant if steam is supplied the prime mover at 150 p.s.i. abs., temperature of 400°F., and the exhaust pressure is 16 p.s.i. abs.? What is the ratio of its efficiency to that of the equivalent Carnot cycle (see Prob. 4)? Illustrate both the Rankine and equivalent Carnot cycles by superimposed diagrams on  $PV$ ,  $TS$ , and Mollier coordinates.

11. Making use of the temperature-entropy diagram, discuss the effect on the efficiency of a Rankine-cycle vapor power plant of (a) raising the pressure at which evaporation takes place, (b) increasing the quality of the steam supplied the prime mover (or using superheated vapor at the same pressure), and (c) raising the exhaust pressure. Compare your conclusions with those reached with respect to the effects on Carnot-cycle efficiency (see Prob. 5).

12. The prime mover of a pump cycle receives dry and saturated steam at 150 p.s.i. abs., and the exhaust pressure is 16 p.s.i. abs. What are the cycle efficiency and the steam consumption? Compare the cycle efficiency with that of the equivalent Carnot and Rankine cycles (Probs. 1 and 6). What is the quality of the steam as it leaves the prime mover?

13. What are the efficiency and the steam consumption of a pump cycle if steam is supplied the prime mover dry and saturated at 250 p.s.i. abs. and the exhaust pressure is 16 p.s.i. abs.? Compare the cycle efficiency with that of the equivalent Carnot and Rankine cycles (Probs. 2 and 7). What is the quality of the steam as it leaves the prime mover?

14. Plot the pump cycle of Prob. 12 on temperature-entropy coordinates so that the true condition of the steam as it leaves the prime mover will be represented. Compare with Fig. 11:7. How much heat is removed per pound of steam in condensing it to a saturated liquid? How is this heat flow represented on your diagram? Is it represented correctly on the  $TS$  diagram of Fig. 11:7? Does the enclosed area of the cycle represent the heat equivalent of the net work in either case?

15. In an incomplete-expansion Rankine cycle, steam is supplied the engine dry and saturated at a pressure of 150 p.s.i. abs. Adiabatic expansion ends at 40 p.s.i.



abs., exhaust pressure being 16 p.s.i. abs. What are the efficiency and the steam consumption of this cycle? What is the quality of the steam as it enters the condenser? What is the percentage of cutoff in the engine? (Note that the ideal cycle assumes no engine clearance). Compare the efficiency of the cycle with that of the equivalent Carnot, Rankine, and pump cycles (Probs. 1, 6, and 12). Compare the percentage of engine cutoff with that of the equivalent Rankine-cycle engine.

16. In an incomplete-expansion Rankine cycle, steam is supplied the engine dry and saturated at 250 p.s.i. abs. Adiabatic expansion ends at 80 p.s.i. abs., exhaust pressure being 16 p.s.i. abs. What are the efficiency and steam consumption of this cycle? What is the quality of the steam as it enters the condenser? What is the percentage of cutoff in the engine? Compare the efficiency of this cycle with that of the equivalent Carnot, Rankine, and pump cycles (Probs. 2, 7, and 13).

17. Steam is supplied the engine prime mover of an incomplete-expansion Rankine cycle dry and saturated at 150 p.s.i. abs. Adiabatic expansion ends at 50 p.s.i. abs., and the final exhaust pressure is 4 p.s.i. abs. What are the efficiency and steam consumption of this cycle? What is the quality of the steam as it enters the condenser? What is the percentage of cutoff in the engine? Compare the efficiency of this cycle with that of the equivalent Carnot and Rankine cycles (Probs. 3 and 8). Compare the percentage of engine cutoff with that of the equivalent Rankine-cycle engine.

18. The term *engine (turbine) efficiency* is customarily used in connection with vapor prime movers to express the ratio of the work performed by the actual prime mover to the work output of the ideal prime mover. An economy test on a certain power plant which is patterned after the Rankine cycle indicates that 4,800 lb. of steam per hour passes through the prime mover and that the average power delivery during the period of the test is 230 kw. Steam is supplied the turbine prime mover at a pressure of 200 p.s.i. abs., temperature of 500°F., and the exhaust pressure is 4 p.s.i. abs. What is the turbine efficiency of the turbine prime mover of the actual plant? Neglecting heat radiated directly to the surrounding atmosphere, what is the quality of the steam entering the condenser? Show on  $TS$  coordinates the curve representing approximately the expansion which actually takes place in the turbine.

19. A turbine receives steam at 400 p.s.i. abs. and 600°F. After expansion to 100 p.s.i. abs., the steam is extracted and reheated at constant pressure to a temperature of 500°F. and then completes its expansion in the lower pressure stages of the turbine to a final exhaust pressure of 2 p.s.i. abs. (a) Assuming the ideal cycle is followed, how much heat is added per pound of steam in the boiler and superheater? (b) How much in the reheater? (c) What is the quality of the steam as it enters the reheater? (d) What is its quality as it enters the condenser? (e) How much work is done per pound of steam in the high-pressure stages of the turbine? (f) How much in the low-pressure stages? (g) What is the efficiency of the cycle? (h) What is its steam consumption? (i) If the original expansion had been continued (without reheating) to the final pressure of 2 p.s.i. abs., what would have been the quality of the steam as it entered the condenser? (j) What would have been the efficiency of the Rankine cycle so formed? (k) Illustrate the cycle on  $TS$  and Mollier coordinates.

20. A reheat cycle supplies steam to the high-pressure turbine stages at 600 p.s.i. abs. and 600°F. When the quality of the steam has decreased to 98 per cent in its isentropic expansion through the high-pressure stages of the turbine, it is extracted and reheated at constant pressure to a temperature of 480°F. The final exhaust pressure is 28 in. Hg vacuum, referred to a 30-in. barometer. (a) What is the pressure at which the steam is reheated? (b) How much heat is added per pound of steam in

the boiler and superheater? (c) How much heat is added in the reheater? (d) How much work is done per pound of steam in the high-pressure stages of the turbine? (e) In the low-pressure stages? (f) What is the efficiency of the cycle? (g) What is the quality of the steam as it enters the condenser? (h) If the original expansion had been continued (without reheating) until the final exhaust pressure had been reached, what would have been the quality of the steam as it entered the condenser? (i) What would have been the efficiency of the Rankine cycle so formed? (j) Illustrate the reheat cycle on  $TS$  and Mollier coordinates.

21. In a regenerative cycle, steam is supplied the turbine at 600 p.s.i. abs. and 600°F. Steam is bled to heaters at pressures of 220, 60, and 10 p.s.i. abs. The condenser pressure is 2 in. Hg abs. Calculate the weights bled to each heater per pound of steam entering the turbine and the efficiency and steam consumption of the cycle. What are the efficiency and steam consumption of the equivalent Rankine cycle? Illustrate the regenerative cycle on  $TS$  and Mollier coordinates.

22. In Prob. 21, if two heaters (instead of three) were employed, what would be the ideal pressures at which they would operate?

23. In a regenerative vapor cycle, assume steam to be supplied the turbine dry and saturated at 450 p.s.i., abs., the condenser pressure being 2 p.s.i. abs. Steam is bled to feed-water heaters at pressures of 125 and 25 p.s.i. abs. Calculate the weights bled to each heater per pound of steam entering the turbine and the efficiency and steam consumption of the cycle. What is the efficiency of the equivalent Rankine cycle? What is the efficiency of the equivalent Carnot cycle? Illustrate this regenerative cycle on  $TS$  and Mollier coordinates.

24. (a) For the steam-supply and exhaust conditions as stated in Prob. 23, what would be the ideal bleeding pressures for a four-heater installation? (b) For a three-heater cycle? (c) If two heaters only are used? (d) If steam is bled to only one heater?

25. In a theoretical regenerative vapor cycle, steam is supplied to a 14-stage turbine at 500 p.s.i. abs. and 640°F. The condenser pressure is 1 in. Hg abs. Assuming that the expansion in the turbine is isentropic and that the decrease of enthalpy is the same in each stage, what are the pressures at the end of each stage? Select bleeding pressures for (a) a four-heater cycle and (b) a three-heater cycle to conform to the design of the turbine.

26. In a combination of the reheating and regenerative cycles, steam is supplied to the high-pressure stages of the turbine at 1,200 p.s.i. abs. and 900°F. After isentropic expansion to a pressure of 90 p.s.i. abs., all the steam is extracted; a part is bled to an open regenerative feed-water heater, and the balance is reheated at constant pressure to a temperature of 600°F. and then expands to condenser pressure of 1 p.s.i. abs. in the low-pressure stages of the turbine. The condensate is returned to the boiler through the regenerative heater by suitable pumps. Plot a flow diagram similar to Fig. 11:11 and a  $TS$  diagram of this cycle, lettering to correspond. Per pound of steam supplied to the high-pressure stages of the turbine, calculate (a) the weight of steam bled for feed-water heating, (b) weight of steam reheated, (c) heat added in the boiler-superheater unit, (d) heat added in reheater, (e) total heat added in the cycle from the fuel, (f) heat rejected in the condenser, and (g) work performed in the entire cycle. (h) What is the efficiency of this cycle? Neglect pump work throughout.

27. Find the actual loss of convertible energy per pound and the percentage loss when dry and saturated steam at 400 p.s.i. abs. is throttled to 300 p.s.i. abs. before entering an engine if the exhaust pressure is (a) 16 p.s.i. abs. and (b) 2 p.s.i. abs.

28. Find the actual loss of convertible energy per pound and the percentage loss when steam at 300 p.s.i. abs. and 540°F. is throttled to 240 p.s.i. abs. before entering a turbine if the exhaust pressure is (a) 20 p.s.i. abs. and (b) 2 p.s.i. abs.

## CHAPTER XII

### REFRIGERATION

#### 12:1. Comparison of Gases and Vapors as Refrigerating Mediums.

If the Carnot cycle is performed in the reverse direction, it has been shown that heat will be taken from the refrigerator and rejected to the source. The names "refrigerator" and "source" in the sense in which they were applied in direct operation of the cycle are now misleading, as the colder body has become the source of heat which is rejected to the body at higher temperature. The terms "hot body" and "cold body" will be used in this chapter to avoid confusion. The process of "pumping" heat from a low temperature to a higher temperature is called refrigeration and is always accompanied by the expenditure of energy on the system from some external source.

The purpose of refrigeration, which has wide uses in modern life, is the production and maintenance of a temperature below that of the surrounding atmosphere. The effectiveness of such a process is measured as the ratio of the rate at which heat is removed from the cold body to the rate at which energy must be supplied to the system from some external system. If this energy which enters from external sources is in the form of work, the ratio referred to above is called the *coefficient of performance*. The symbol  $\beta$  (beta) will be used to represent this ratio.

In Chap. V, two propositions were introduced with respect to the performance of a reversible engine as a prime mover. When a reversible cycle such as the Carnot is applied to the refrigeration process, two additional statements may be made. These are as follows:

1. *No irreversible refrigeration cycle may have a higher coefficient of performance than a reversible cycle which is operated between the same limiting temperatures of hot and cold bodies.*
2. *All reversible cycles, when operated between the same limiting temperatures, will have identical coefficients of performance.*

For the proof of the first of these propositions, the same general method will be used as for the demonstration of the truth of the Carnot principle. It will be assumed that an irreversible refrigeration cycle may be devised which will have a higher coefficient of performance than that of a reversible cycle which is operated as a "heat pump" between

the same limiting temperatures. By “irreversible refrigeration cycle” is meant a cycle which can be operated to draw heat from a cold body and discharge it to a body at higher temperature but which, because of the irreversibility of one or more of the processes which comprise it, cannot be operated as an “engine.” If it may be shown that this assumption leads to conclusions which are inconsistent with basic thermodynamic principles, the truth of the first statement will have been confirmed.

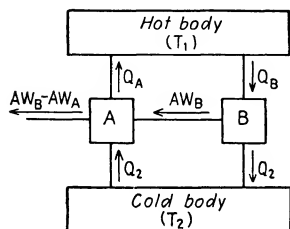


FIG. 12:1.—The Carnot principle as applied to the refrigerating machine.

For convenience, the irreversible machine will be designated as *A*, the reversible as *B*. Both will be placed between the same hot and cold bodies as illustrated in Fig. 12:1. Then if both were operated as refrigerating machines at a rate such that each would withdraw  $Q_2$  units of heat from the cold body in unit time, machine *A* would require a supply of  $W_A$  work units and *B* would absorb  $W_B$  work units in the same time interval; also, based on the first law, *A* would discharge  $Q_A (= Q_2 + AW_A)$  heat units to the hot body, and *B* would deposit  $Q_B (= Q_2 + AW_B)$  heat units in that body. Further, since it has been assumed that  $\beta (= Q_2/AW)$  is greater for *A* than for machine *B*,

$$\beta_A > \beta_B \quad \text{or} \quad Q_2/AW_A > Q_2/AW_B$$

from which  $AW_B > AW_A$  and  $Q_B > Q_A$ . However, *B*, being reversible, will be operated as an engine at an equivalent rate so that it will withdraw  $Q_B$  heat units from the hot body, deliver  $W_B$  units of work, and discharge  $Q_2$  units of heat to the cold body. Considering the system as a whole, we now find that the condition of the cold body will not have changed since it has given and received  $Q_2$  heat units, that  $W_B$  is greater than  $W_A$  and an excess of work,  $W_B - W_A$ , is available for other tasks, and that the hot body has supplied this energy. No violation of the first law is indicated since  $Q_B - Q_A = A(W_B - W_A)$ , but the assumed conditions are contrary to the second law since the energy taken from the hot body has been made *fully* available. The first proposition has therefore been confirmed.

A similar method of reasoning may be employed to prove the truth of the second statement. Based on the second proposition, we may substitute the coefficient of performance of *any* reversible cycle to represent that of *all* reversible cycles operating between the same temperature limits. The Carnot cycle is convenient for our purpose in calculating this limiting, or ideal, coefficient of performance. With reference to

Fig. 12:2, which shows the Carnot refrigeration cycle, the area  $EBCF$  represents the heat removed from the cold body (at temperature  $T_2$ ) and the area  $EADF$  the heat discharged to the hot body (at temperature  $T_1$ ), and the area  $ABCD$  is, by difference, the heat equivalent of the work expended. The ideal coefficient of performance is

$$\beta \text{ (ideal)} = \frac{\text{area } EBCF}{\text{area } ABCD} = \frac{T_2(S_2 - S_1)}{(T_1 - T_2)(S_2 - S_1)} = \frac{T_2}{T_1 - T_2} \quad (12:1)$$

The ratio of the actual coefficient of performance of a refrigerating machine to this ideal value may be termed the *refrigerating efficiency* of the machine. Since the value of the coefficient of performance will vary widely with changes in the limiting temperatures, the refrigerating efficiency is a much more accurate basis for evaluation of the performance of the practical refrigerating machine. It will be noted that refrigerating "efficiency" is not a ratio of output to input, as is the "efficiency" of a heat-engine cycle, but expresses the ratio of actual to ideal performance.

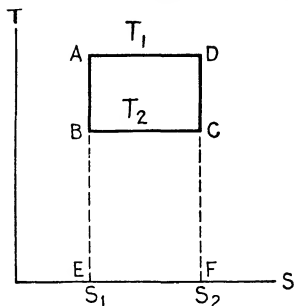


FIG. 12:2.—Carnot refrigeration cycle.

In practical applications, the heat is always absorbed and rejected at nearly constant pressures. When this feature is considered, the refrigeration cycle for gases is as illustrated in Fig. 12:3. The Carnot cycle for the same temperature limits is indicated by the dashed lines. It is evident that the heat extracted from the cold body is less and the work required greater than for the Carnot cycle and that therefore the coefficient of performance is smaller than for the ideal cycle.

On the other hand, if a vapor is employed, the constant-pressure line is also an isothermal if the vapor remains in the saturated state; consequently, the refrigeration cycle for a vapor may theoretically correspond exactly with the Carnot if the vapor is not allowed to become superheated. This comparison indicates that the use of a vapor for refrigeration in place of a gas will result in higher coefficients of performance, a fact which has been confirmed in practice. It is also clear that the coefficient of performance will, in general, increase as the ratio  $T_2/(T_1 - T_2)$  increases, and vice versa. This means, if it is assumed that  $T_1$  is fixed by ordinary atmospheric conditions and will therefore remain practically constant, that, as the temperature of the cold body is decreased and the range in temperature,  $T_1 - T_2$ , thereby increased, more energy will be required per unit of heat removed.

Formerly, air was sometimes used as a refrigerating medium although its effectiveness, as shown above, is necessarily lower than that of a suitable vapor. Its use was confined to installations, such as for theaters or on shipboard, where its advantages due to its lack of objectionable odor or poisonous effect in case of accidental escape compensated for its thermodynamic disadvantages. Although this type of system is now obsolete, the essentials of an air refrigerating system will be discussed in Art. 12:3 to illustrate more clearly the advantage of the vapor over the gas as the medium for refrigeration.

Water vapor is now being employed for cooling purposes in some air-conditioning systems although its saturation pressure becomes so low at ordinary refrigerator temperatures that its range of usefulness is limited to situations where the temperature to be maintained is only slightly below the temperature of the atmosphere. The usual cold-body temperature characteristic of water-vapor systems is seldom below 45°F., although lower temperatures have been attained. Because of the large volume of water vapor that must be handled and the relatively low

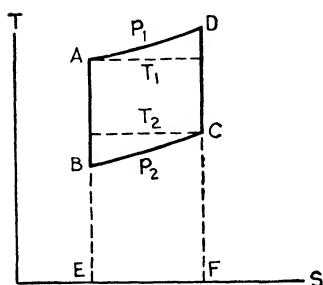


FIG. 12:3.—Refrigeration cycle for gases.

pressure range over which compression of this vapor need take place, the conventional compressor unit is customarily in the form of a steam ejector or centrifugal compressor.

Some of the vapors most commonly used for refrigeration are ammonia, sulphur dioxide, and carbon dioxide. Ammonia has good properties in most respects, but its penetrating odor is a disadvantage if it accidentally escapes from the system; it is the medium almost universally used in commercial refrigeration plants on account of its moderate pressures and volumes at refrigerator temperatures. Sulphur dioxide is often used in household refrigerators; it has the advantage of lower saturation pressures than ammonia but has rather high specific volumes and, if allowed to come into contact with the air through a leak in the system, will absorb moisture from it, forming a corrosive acid. It must, therefore, be carefully sealed. Carbon dioxide, at high pressures, assumes the characteristics of a vapor at atmospheric temperature and, since it is odorless and nonpoisonous, formerly was often used in theaters and auditoriums and on shipboard instead of the less efficient air system. A serious disadvantage of carbon dioxide is found in its extremely high saturation pressures (see Appendix I); the development of new vapors

which have all its advantages without its undesirable characteristics has made its use obsolete.

An excellent example of the way in which thermodynamics may guide the efforts of industrial research is found in the development of the Freon series of refrigerants. All the vapors mentioned in the preceding paragraphs were well known to the scientist before their use as refrigerating mediums was suggested; none had properties in all respects satisfactory for this use. The Freon series, compounds of carbon, chlorine, and fluorine, were developed to meet the indicated need. They have favorable pressure-volume-temperature relationships and, in addition, are nonflammable, nonpoisonous, noncorrosive, nonirritating, and practically odorless.

**12:2. Unit of Capacity in Refrigeration.** Just as the capacity of a heat engine is measured in horsepower or work performed in unit time, so also does the refrigerator have a capacity rating on the basis of the heat which it is capable of removing from the cold body in unit time. Although the capacity might very well be expressed simply as the number of B.t.u. removed per minute or per second, it is customary to express it in terms of *tons of ice per day*. A refrigerating machine with a capacity of 1 ton will equal in refrigerating effect the heat absorbed by the melting of 1 ton of ice per day; since the heat of fusion of ice is about 144 B.t.u. per pound, the *ton of refrigeration* is the equivalent of the removal of  $2,000 \times 144$  or 288,000 B.t.u. every 24 hr. or 200 B.t.u. per minute.

**12:3. Air Refrigeration.** Figure 12:4 illustrates diagrammatically the arrangement of an air refrigeration system. At *A* the air coming from the cooling coils is at a temperature only slightly above the temperature of the atmosphere but at a rather high pressure. It is allowed to perform work in the expansion cylinder and, as it expands adiabatically to a lower pressure, its temperature drops until at *B* it has fallen to a point considerably below the temperature in the cold room and is in a condition such that it can absorb heat from surrounding objects in that room. As it passes through the cold room, it absorbs heat and gradually increases in temperature at constant pressure until at *C* its temperature is nearly that of the cold room. It is now compressed adiabatically to its original pressure; the process of compression is accompanied by an increase in temperature so that the temperature at *D* is high. Since both expansion and compression take place over the same range of pressures and since the temperature at *C* is higher than at *B*, two effects are evident. First, the temperature at *D* is raised considerably above that at *A*; and, second, the power consumed by the compression process will exceed the power developed as the result of the expansion from *A*



to *B*. The latter effect will be understood when it is remembered that greater volumes must be handled in the compression cylinder owing to the higher temperature level. The power supplied by the expansion cylinder is used to furnish a part of the power needed for compression, and the difference must be supplied from external sources such as a booster cylinder. The air, now at high temperature and high pressure, is passed through cooling coils by means of which its temperature is reduced until it has again returned to its initial condition by the time it

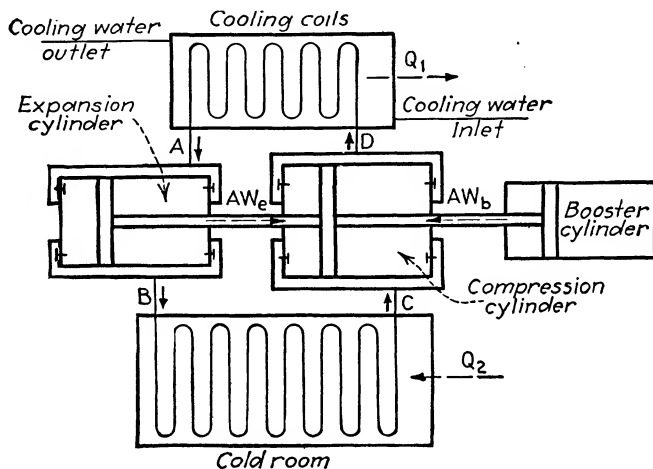


FIG. 12:4.—Diagram of air refrigeration system.

reaches point *A*. In practice, in order to reduce the size of the expanding and compressing cylinders and the necessary air passages, the air is often not allowed to fall to atmospheric pressure at any point in the cycle; this decreases the volume of air which must be handled. In such cases, the lower pressure is usually about 50 p.s.i. abs. and the upper about 200 p.s.i. abs. The pressure ratio is naturally dependent upon the temperature which is to be maintained in the cold room.

In the illustration (Fig. 12:4), the dashed arrows indicate the energy flowing into or out of the system at various points during the complete cycle.  $Q_1$  represents the heat carried away by the cooling water in the cooling coils and  $Q_2$  the heat absorbed from the cold room;  $AW_e$  indicates the heat equivalent of the mechanical energy developed in the expansion cylinder and restored to the system in the compression cylinder, and  $AW_b$  represents the heat equivalent of the additional work required for compression which is continuously supplied from an external system by means of the booster cylinder. If the energy absorbed in compression

is represented by  $AW_c$ , an energy balance for the entire cycle may be stated as follows:

$$Q_1 + AW_e = Q_2 + AW_c \quad (12:2)$$

and, since  $W_e + W_b = W_c$ ,

$$Q_1 = Q_2 + AW_b \quad (12:3)$$

The theoretical cycle has been illustrated on  $TS$  coordinates in Fig. 12:3. The area  $ABCD$  in this figure represents the heat equivalent of the work which must be supplied by the booster.

In the design of air refrigeration systems, the basis of the calculations is the desired capacity and the temperatures of cold room and cooling water. The temperature at  $C$  will usually be somewhat below the cold-room temperature in practice but may be assumed for the theoretical case at cold-room temperature. In the same way, the temperature at  $A$  may be established with reference to the temperature of the available cooling medium. If the pressure range in compression is not specified, it must be made large enough so that the temperature at  $D$  will be well above the temperature of the cooling medium.

The total heat which must be removed from the cold room per minute,  $Q_2$ , may be computed by multiplying the total desired capacity in tons of refrigeration by 200 (see Art. 12:2). The weight of air circulated per minute may then be calculated since

$$Q_2 = MC_p(T_C - T_B) \quad (12:4)$$

and

$$M = \frac{Q_2}{C_p(T_C - T_B)} \quad (12:5)$$

If it is assumed that both expansion and compression follow the theoretical adiabatic,  $T_B$  and  $T_D$  may be calculated since

$$T_B = T_A \left( \frac{p_B}{p_A} \right)^{\frac{k-1}{k}} \quad \text{and} \quad T_D = T_C \left( \frac{p_D}{p_C} \right)^{\frac{k-1}{k}}$$

Assuming complete expansion, the work performed in the expansion cylinder per minute is [see Eq. (7:4)]

$$W_e = \frac{k}{k-1} MR(T_A - T_B) = JMC_p(T_A - T_B) \quad (12:6)$$

and

$$W_c = JMC_p(T_D - T_C) \quad (12:7)$$

The net work required,

$$W_b = W_c - W_e = JMC_p(T_D - T_C - T_A + T_B) \quad (12:8)$$

and, substituting the value of  $M$  from Eq. (12:5),

$$\begin{aligned} W_b &= \frac{JQ_2C_p(T_D - T_C - T_A + T_B)}{C_p(T_C - T_B)} \\ &= JQ_2 \left( \frac{T_D - T_C - T_A + T_B}{T_C - T_B} \right) \end{aligned} \quad (12:9)$$

By proper water jacketing, the value of  $n$  during compression may be reduced to a value lower than  $k$ , and consequently a corresponding reduction may be made in the work of compression. In this case,

$$T_D = T_C \left( \frac{p_D}{p_C} \right)^{\frac{n-1}{n}}$$

and

$$W_c = \frac{n}{n-1} MR(T_D - T_C) \quad (12:10)$$

The work that must be supplied from an external source can be computed by applying Eq. (12:2). In practice, a pressure drop often accompanies the temperature drop in the cooling coils so that  $p_A$  is somewhat less than  $p_D$ ; the method of calculation will be unchanged. The size of the expansion and compression cylinders may be computed by methods discussed in Chap. VII.

*Example.* In a 10-ton air refrigerating machine, the pressure at the end of compression is 60 p.s.i. abs., at entrance to the expansion cylinder is 55 p.s.i. abs., and in the cold room is 15 p.s.i. abs. The temperature in the cold room is 30°F. and the air enters the expansion cylinder at 75°F. During compression,  $n$  is 1.35, and expansion is adiabatic. (a) What booster horsepower is theoretically required? (b) What is the theoretical coefficient of performance? (c) Assuming that the temperature of the cooling water is 65°F., what is the "efficiency" of the machine? (d) Calculate the cylinder volumes required if the machine makes 70 double strokes per minute.

*Solution.*

$$T_B = T_A \left( \frac{p_B}{p_A} \right)^{\frac{k-1}{k}} = 534.7 \left( \frac{15}{55} \right)^{0.288} = 368^\circ\text{F. abs.}$$

$$T_D = T_C \left( \frac{p_D}{p_C} \right)^{\frac{n-1}{n}} = 489.7 \left( \frac{60}{15} \right)^{0.259} = 702^\circ\text{F. abs.}$$

$$M = \frac{Q_2}{C_p(T_C - T_B)} = \frac{10 \times 200}{0.2375(489.7 - 368)} = 69.3 \text{ lb. per minute}$$

$$\begin{aligned} W_c &= \frac{n}{n-1} MR(T_D - T_C) = \frac{1.35}{0.35} \times 69.3 \times 53.3 \times (702 - 489.7) \\ &= 3,030,000 \text{ ft.-lb. per minute} \end{aligned}$$

$$W_c = JMC_p(T_A - T_B) = 778 \times 69.3 \times 0.2375 \times (534.7 - 368) \\ = 2,136,000 \text{ ft.-lb. per minute}$$

$$(a) \text{ Booster hp.} = \frac{W_c - W_e}{33,000} = \frac{3,030,000 - 2,136,000}{33,000} = 27.1 \text{ hp.}$$

$$(b) \text{ Coefficient of performance} = \beta = \frac{Q_2}{AW_b} = \frac{10 \times 200 \times 778}{27.1 \times 33,000} = 1.743$$

(c) The ideal coefficient of performance is

$$\frac{T_2}{T_1 - T_2} = \frac{489.7}{524.7 - 489.7} = 13.99$$

The efficiency of the machine is therefore

$$\frac{1.743}{13.99} \text{ or } 12.46 \text{ per cent}$$

(d) The volumes of air handled per stroke in the compressing and expanding cylinders are, respectively,

$$V_c = \frac{MRT_c}{2NP_c} = \frac{69.3 \times 53.3 \times 489.7}{2 \times 70 \times 144 \times 15} = 5.98 \text{ cu. ft. per stroke in compressor}$$

$$V_e = \frac{MRT_B}{2NP_B} = \frac{69.3 \times 53.3 \times 358}{2 \times 70 \times 144 \times 15} = 4.50 \text{ cu. ft. per stroke in the engine}$$

Assuming the clearance of the compressor to be 3 per cent and that of the expansion cylinder 4 per cent, the clearance factors are

$$1 + 0.03 - 0.03\left(\frac{6}{15}\right)^{1.35} = 0.947 \text{ for the compressor}$$

and

$$1 + 0.04 - 0.04\left(\frac{5}{15}\right)^{1.405} = 0.938 \text{ for the expansion cylinder}$$

The corrected piston displacements are therefore

$$\frac{5.98}{0.947} = 6.32 \text{ cu. ft. for the compressor}$$

and

$$\frac{4.50}{0.938} = 4.79 \text{ cu. ft. for the expansion cylinder}$$

**12:4. Vapor-compression Refrigeration.** The inherent advantages of a vapor over a gas as a medium for refrigeration have already been discussed. A vapor-compression system is diagramed in Fig. 12:5. At *A* the vapor has been condensed and is at a relatively high pressure and a temperature slightly above that of the cooling medium. As it passes through the expansion valve to *B*, the pressure drops instantaneously to that maintained on the suction side of the compressor and the temperature also decreases at once to the saturation temperature equivalent to this pressure for the vapor which is used in the cycle. This saturation temperature must be low enough so that heat may be absorbed from the

cold room. The change which takes place between  $A$  and  $B$  is a throttling process, and therefore  $h_A = h_B$  (see Chap. X). The enthalpy of the liquid at  $T_A$  is greater than at  $T_B$ , and a certain percentage of the liquid will evaporate. Since at  $A$  the enthalpy is that of the liquid at  $T_A$ , it may be readily obtained from a table of the properties of the vapor. From the same tables, the enthalpy of the liquid and the enthalpy of vaporization at the lower pressure and temperature may also be read.

Then

$$h_A = h_B = h_{f_B} + x_B h_{fg_B}$$

and

$$x_B = \frac{h_A - h_{f_B}}{h_{fg_B}} \quad (12.11)$$

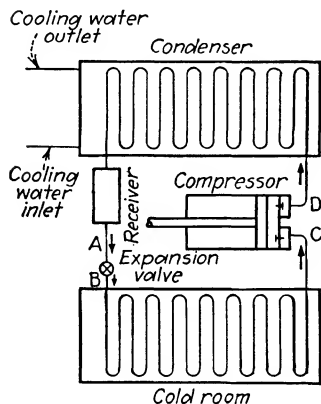


FIG. 12.5.—Diagram of vapor compression system.

As the vapor passes from  $B$  to  $C$  through the cold room, it absorbs heat at constant pressure, which, in this case, means also at constant temperature. If this absorption of heat is allowed to continue until the vapor is dry and saturated before it enters the compressor, it will, after compression, be in a superheated condition. It will now (at point  $D$ ) be at a relatively high pressure and temperature. As it passes

through the condenser, heat is removed at constant pressure. Since it was superheated as it left the compressor, the temperature will first be reduced to the saturation temperature during the first part of its passage through the condenser and thereafter will remain constant while the quality is decreased until the vapor is reconverted into a liquid and is in a condition to begin its cycle again. The cycle for these operations is represented on  $TS$  coordinates in Fig. 12.6. It will be noticed that it varies from the ideal Carnot in two respects, *viz.*, line  $AB$  is a line of constant enthalpy instead of constant entropy and line  $DD'A$ , representing the rejection of heat in the condenser, is an isothermal through only a part of its length.

It would be possible to allow the expansion from  $A$  to  $B$  to take place through some device such that external work might be developed and used to furnish a part of the power needed for compression; however, in practice, the advantages to be gained do not outweigh the disadvantages of greater complexity and first cost, and consequently the expansion valve is substituted. If process  $AB$  had been a reversible adiabatic, line  $AE$  would have been followed in the figure and the heat removed

from the cold room per pound of vapor would have been represented by area *FECH*. When an expansion valve is used, the heat removed per pound is represented by area *GBCH*. The *loss of refrigerating effect* per pound of vapor may therefore be measured as area *FEBG*, or

$$\begin{aligned}\text{Loss of refrigerating effect} &= \text{area } FEBG = T_B(S_B - S_E) \\ &= T_B(S_B - S_A)\end{aligned}\quad (12:12)$$

It will be observed that, in the air refrigerating system, the expansion cylinder could not have been replaced by an expansion valve since the throttling process for a gas would not produce the drop in temperature necessary at this point in the cycle (see Art. 4:10).

If the vapor, instead of continuing to absorb heat until it became dry and saturated, had been compressed when it reached the condition indicated by  $C'$  in Fig. 12:6, it would not have been in a superheated condition after compression and line  $D'A$ , an isothermal as well as a constant-pressure line, would have been traced by the action of the condenser. This is the *wet compression cycle*; and since it causes the cycle to conform more closely to the Carnot, its efficiency as a refrigerating cycle is higher than for dry compression. The gains which would result are not ordinarily considered large enough to balance the difficulties which would be encountered in design. When evaporation is allowed to proceed to point  $C$ , the absorption of further heat will result in increasing the temperature. Advantage may be taken of this effect to provide for automatic control of the operation of the compressor through a suitable thermostat.

For design purposes, the capacity in tons of refrigeration is, of course, known, and the values of  $p_D (= p_A)$  and  $p_B (= p_C)$  are selected on the basis of a consideration of the temperature of the available cooling medium, the temperature to be maintained in the cold room, and the properties of the vapor to be employed as a refrigerating medium. The enthalpy at  $A$  is that of the liquid at the upper pressure and may be read from the vapor tables, and the heat absorbed from the cold room per pound of vapor passing through the cycle is  $h_C - h_B$ , in which, for

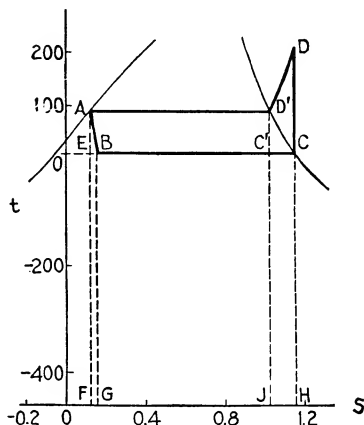


FIG. 12:6.--Vapor compression cycle.

dry compression,  $h_C$  is the total enthalpy of the dry and saturated vapor at suction pressure. The weight of vapor flowing is therefore

$$M = \frac{Q_2}{h_C - h_B} = \frac{Q_2}{h_C - h_A} \quad (12:13)$$

The heat equivalent of the work of compression per pound of vapor is equal to the difference of the enthalpies at the beginning and the end of compression, and the total work of compression per minute is equal to the difference between the mechanical equivalent of the heat rejected to the condenser and that absorbed from the cold room, or

$$\begin{aligned} W_C &= J(Q_1 - Q_2) = JM(h_D - h_A) - JM(h_C - h_A) \\ &= JM(h_D - h_C) \end{aligned} \quad (12:14)$$

In the vapor-compression cycle, as carried out in its practical application, it will be noted that a finite difference of temperature will be required for the removal of heat along line  $DA$  of Fig. 12:6. Thus, the saturated liquid at  $A$  is often subcooled along the liquid curve to a point below  $A$  before passing through the expansion valve. At first thought, this would seem to represent an improvement over the theoretical cycle, as the refrigerating effect per pound is thereby increased. However, if the necessary temperature difference could be reduced by more effective cooling in the condenser, it is evident that line  $DA$  could have been lowered and the coefficient of performance correspondingly increased. A degree of subcooling greater than that typical of good practice is therefore an indication of a loss which may be in part eliminated by more effective condensation or by regulation of the pressure at discharge from the compressor.

*Example.* A 10-ton dry-compression ammonia refrigerating machine has an ammonia temperature of  $10^\circ\text{F}$ . in the expansion coils and a pressure of 180 p.s.i. abs. in the condenser. It makes 100 effective strokes per minute. (a) Calculate the theoretical horsepower required for compression. (b) What is the coefficient of performance? (c) What is the efficiency of the machine if it is assumed that cooling water is available at  $75^\circ\text{F}$ .? (d) What size of compressor is required?

*Solution.* From the ammonia tables, the following enthalpies and entropies may be obtained:

$$h_A = h_f \text{ at } 180 \text{ p.s.i. abs.} = 143.3 \text{ B.t.u.}$$

$$h_B = h_A = 143.3 \text{ B.t.u.}$$

$$h_C = \text{total enthalpy of dry and saturated ammonia at } 10^\circ\text{F.} = 614.9 \text{ B.t.u.}$$

$$s_C = s_D = 1.3157$$

With reference to the superheat tables for ammonia, the enthalpy of the ammonia after adiabatic compression is found by locating the condition for which the entropy

is 1.3157 and the pressure 180 p.s.i. abs. The corresponding temperature is 209°F. for the superheated ammonia at point  $D$ , and the enthalpy  $h_D$  is 711.1 B.t.u.

$$M = \frac{Q_2}{h_C - h_A} = \frac{10 \times 200}{614.9 - 143.3} = 4.25 \text{ lb. per minute}$$

$$(a) W_c = JM(h_D - h_C) = 778 \times 4.25 \times (711.1 - 614.9) = 318,000 \text{ ft.-lb. per minute} = 9.65 \text{ hp.}$$

$$(b) \text{Coefficient of performance} = \beta = \frac{Q_2}{AW_c} = \frac{10 \times 200 \times 778}{318,000} = 4.9$$

(c) The ideal coefficient of performance is

$$\frac{T_2}{T_1 - T_2} = \frac{469.7}{534.7 - 469.7} = 7.22$$

The efficiency of the machine is therefore

$$\frac{4.9}{7.22} = 0.68 \text{ or } 68 \text{ per cent}$$

(d) The theoretical volume of the compressor must be sufficient to handle the given weight per minute at its specific volume at entrance to the compressor. The specific volume at entrance to the compressor is that of dry saturated ammonia at 10°F. (38.5 p.s.i. abs.) or 7.304 cu. ft. per pound. The total volume per minute is  $4.25 \times 7.304$ , and the necessary displacement volume of the compressor cylinder

$$V_C = \frac{Mv_C}{N} = \frac{4.25 \times 7.304}{100} = 0.311 \text{ cu. ft.}$$

A proper allowance must be added to this cylinder volume to take care of clearance and other effects.

An energy balance (usually termed *heat balance*) may be set up for this refrigerating plant. Thus, based on 1 lb. of ammonia circulated,

$$\text{Heat absorbed from cold room} = Q_2 = h_C - h_B = 614.9 - 143.3 = 471.6 \text{ B.t.u.}$$

$$\text{Heat equivalent of compressor work} = AW = h_D - h_C = 711.1 - 614.9 = 96.2 \text{ B.t.u.}$$

$$\text{Heat rejected to cooling medium} = Q_1 = h_D - h_A = 711.1 - 143.3 = 567.8 \text{ B.t.u.}$$

But the total energy flow inward must equal the total energy leaving the system during the cycle, or

$$Q_2 + AW = Q_1$$

Substituting values as determined above,

$$471.6 + 96.2 = 567.8 \text{ B.t.u.}$$

**12:5. The Ammonia-absorption System.** In this system the energy from the external system is largely supplied in the form of heat instead of work. The solubility of ammonia vapor in water is utilized, and the vapor compressor is replaced by an absorber and generator as shown in Fig. 12:7. The amount of ammonia which can be absorbed in water is dependent upon the temperature and the pressure. As the pressure



increases, the solubility increases; and as the temperature rises, the ammonia is progressively driven out of the solution.

When the vapor enters into solution, heat is generated from two sources. The first of these is due to the reduction of the vapor to the form of a liquid, and the amount of heat generated is equal to the difference between the enthalpy of the original vapor and the enthalpy of liquid ammonia at the temperature of the solution; the second is a chemical effect known as the heat of solution. In turn, if the solution

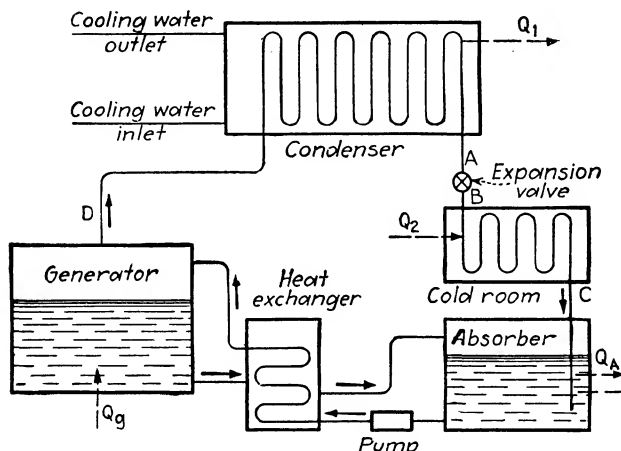


FIG. 12:7.—Diagram of ammonia-absorption system.

is heated, the amount of heat required to drive out of solution a unit weight of ammonia will be equal to the sum of these two requirements. With reference to Fig. 12:7, at *A* the system is composed of liquid ammonia (not a solution of ammonia in water) at a pressure and temperature which are comparable with the conditions at point *A* in Figs. 12:5 and 12:6. After passing through the expansion valve and the cold room, the vapor is in approximately the same condition as at point *C* in the same figures. In the vapor-compression system, it then enters the compressor; but, in the absorption system, it enters the absorber and is there dissolved in water. The pressure in the absorber is the same as that of the ammonia in the cooling coils of the cold room, or about 30 to 40 p.s.i. abs. As has been explained, heat is generated as the process of solution progresses, and this heat must be removed by cooling-water coils or some other method to prevent the temperature of the solution in the absorber from rising; if this temperature is too high, the required strength of solution cannot be maintained in the absorber.

The temperature of the solution in the absorber is considerably above the saturation temperature of ammonia at absorber pressure; but while this would prevent it from being directly condensed to liquid ammonia, it does not prevent it from changing into a liquid as it goes into solution. Absorber temperatures must always be higher than the temperature of the cooling water available and lower than the temperature at which the ammonia vapor will begin to leave the solution. This latter temperature is dependent upon the concentration of the solution, which is expressed in percentage by weight of the dissolved ammonia to the total weight of the solution, and the pressure, which will be the same as the pressure of the ammonia vapor in the expansion coils in the cold room. For instance, for a concentration of 32 per cent, which is a typical value for the solution in the absorber, and a pressure of 30 p.s.i. abs. corresponding to a temperature of about 0°F. in the expansion coils, the temperature at which the ammonia will begin to leave the solution is 111°F. If the cooling water is available at a temperature of 75°F., the temperature of the solution in the absorber may be anything between 75 and 111°F. As the ammonia vapor is dissolved and heat is liberated, the amount of heat designated as  $Q_A$  in Fig. 12:7 must be continuously removed by the cooling coils in the absorber to maintain the temperature at a level below the upper value.

As the solution becomes richer in ammonia, a portion is drawn from the absorber and pumped into the generator, where the pressure is approximately the same as at the discharge from the compressor in the ammonia-compressor system (point *D* in Figs. 12:5 and 12:6). Heat is applied to the generator, and ammonia vapor is driven off, passing first into a rectifier, or drier, which removes the water from the ammonia vapor, and then into the condenser, where heat is removed until it is reduced to liquid ammonia and returns to point *A*.

The concentration in the generator is maintained at a figure below that in the absorber. For example, to continue the illustration cited above, the concentration in the generator will probably be about 24 per cent. With cooling water at 75°F., the pressure maintained in the generator and condenser will probably be about 180 p.s.i. abs. For this concentration and pressure, the boiling temperature of the ammonia solution is 255°F., and heat is added to the solution by steam coils or other means to maintain it at about this temperature.

In order to maintain a balanced condition between generator and absorber, the weak solution is allowed to flow back to the absorber to receive a fresh charge of ammonia at a rate which will keep the liquid in the two units at a constant level. Thus there is a continual exchange

of solution between the absorber and the generator, strong ammonia solution passing from the absorber to the generator and weak solution in the reverse direction. The weight of strong solution flowing will be greater than the weight of weak solution by the weight of ammonia which passes through the condenser and expansion valve; the weight of flow in the system is therefore balanced.

A similar balance must exist in the heat flow. The amount of heat  $Q_2$  absorbed in the cold room will be similar in amount to that for the vapor-compression system. This heat together with the heat of solution of the ammonia is almost immediately removed by the cooling coils in the absorber; this is the amount of heat which has been designated as  $Q_A$ . The next essential heat process takes place in the generator, where an amount of heat  $Q_g$  enters the system; this amount is equal to the sum of the heat of solution and the heat of vaporization of the ammonia as it leaves the solution. Finally, the ammonia vapor is condensed to liquid ammonia in the condenser, representing the removal of  $Q_1$  heat units from the system. With all minor and incidental heat losses neglected, the heat balance may now be stated as

$$Q_2 + Q_g - Q_A - Q_1 = 0 \quad (12:15)$$

In the commercial application of this type of refrigeration, a pump is placed in the line carrying strong solution from absorber to generator; this is necessary since the pressure in the generator is much higher than in the absorber, but the amount of power required is small compared with that necessary to drive the compressor in the compressor system since the volumes are smaller. Also, a heat exchanger is used to remove heat from the weak solution and, at the same time, preheat the strong solution before it enters the generator.

An interesting application of this principle to household refrigeration eliminates the pump by utilizing the principle of partial pressures. The generator and condenser contain only ammonia; the expansion coils and absorber, a mixture of ammonia and insoluble hydrogen gas. The total pressures in the generator and absorber are nearly equal, but the partial pressure of the ammonia below the expansion valve is only 30 to 40 p.s.i. abs., the hydrogen accounting for the balance. Above the expansion valve, no hydrogen is present, and the pressure of the ammonia is the total pressure. Circulation is maintained by the "percolator" action in a tube-shaped generator. The head which can be created in this manner is, of course, quite small, and the pressures must be carefully balanced throughout the system.

An analysis of the ammonia-absorption system will show that, to

accomplish the same refrigeration effect, roughly twice the amount of heat must be removed from the system as would be necessary for the vapor-compression system. Therefore a greatly increased weight of cooling water is necessary. An additional disadvantage is its greater complexity. Its advantage lies in the saving in power; even when pumps are used, the volumes which must be handled are much smaller than those drawn into the vapor compressor.

### Problems

1. Prove that all reversible refrigerating cycles must have the same coefficient of performance if they operate between the same hot- and cold-body temperatures.

2. A reversible refrigeration cycle removes heat from a cold body at a temperature of  $10^{\circ}\text{F}$ . The temperature of the atmosphere is  $95^{\circ}\text{F}$ . (a) What is its (ideal) coefficient of performance? (b) What is its refrigerating efficiency? (c) If reversed (operated as a heat "engine") between the same temperatures, what would be its efficiency?

3. The temperature to be maintained in a refrigerator is  $0^{\circ}\text{F}$ . What is the ideal coefficient of performance if the temperature of the atmosphere (the hot body) is (a)  $80^{\circ}\text{F}$ . and (b)  $100^{\circ}\text{F}$ ?

4. The temperature of the atmosphere is  $90^{\circ}\text{F}$ . What is the ideal coefficient of performance if the temperature to be maintained in the cold body is (a)  $20^{\circ}\text{F}$ . and (b)  $0^{\circ}\text{F}$ ?

5. With reference to Fig. 12:3, let the temperature of the atmosphere,  $T_A$ , be  $95^{\circ}\text{F}$ . and that of the cold body,  $T_C$ ,  $5^{\circ}\text{F}$ . Assume the gas employed to be air, and let  $T_D = 130^{\circ}\text{F}$ . Assume the cycle to be frictionless. (a) What is the temperature at B? (b) How much heat is removed from the cold body per pound of air passing through the cycle? (c) How much heat is discharged to the atmosphere per pound of air? (d) What is the net work required per pound of air passing through the cycle? (e) What is the coefficient of performance of the cycle? (f) What is the ideal coefficient of performance of a reversed Carnot cycle operating between the same atmospheric and cold-body temperatures? (g) What is the refrigerating efficiency of the cycle ABCD? (h) Since the cycle ABCD has been assumed frictionless, why is its refrigerating efficiency not 100 per cent? (i) What general effect would have been produced on the answer to (g) if  $T_D$  had been  $110^{\circ}\text{F}$ ., all other data remaining the same? Explain. (j) Transfer the cycle to  $PV$  coordinates, calculate the net work based on its enclosed area, and compare with the answer to (d) above. [Let  $P_B = P_C = 20$  p.s.i. abs., and calculate  $P_D (= P_A)$ . Note that  $P_B$  could be assumed at any value without affecting the answer.]

6. A commercial cold room is to be maintained at a temperature of  $-10^{\circ}\text{F}$ . It receives 32,000 B.t.u. of heat per hour because of radiation from the surrounding atmosphere and 18,000 B.t.u. per hour through infiltration from the same source (due to leakage through cracks, because of the opening of doors, etc.), and 15,000 B.t.u. per hour is extracted from its contents in lowering them to cold-room temperature. (a) What is the capacity of the refrigerating plant which is required? (b) Assuming the temperature of the surrounding atmosphere is  $85^{\circ}\text{F}$ ., what *minimum* (ideal) horsepower would be required? (c) If the refrigerating efficiency of the actual plant is 67 per cent, what is the power requirement?

7. A refrigerating machine has a coefficient of performance of 4.6, and 40 horsepower is required to drive it. What is its capacity in tons of refrigeration?

8. (a) What *minimum* power is required to freeze (only) 1 ton of ice per day if the temperature of the atmosphere is  $90^{\circ}\text{F}$ .? (b) What is the corresponding coefficient of performance? (c) If the cooling of the water from a well temperature of  $60^{\circ}\text{F}$ . is included, what is the minimum power required?

9. A refrigerating machine with a coefficient of performance of 3.5 maintains a temperature of  $10^{\circ}\text{F}$ . in the cold room when the temperature of the atmosphere is  $90^{\circ}\text{F}$ . What is its efficiency?

10. If the temperature of a certain refrigerator is to remain constant at  $0^{\circ}\text{F}$ . when the temperature of the atmosphere is  $80^{\circ}\text{F}$ ., 25,000 B.t.u. per hour must be removed from it. If a reversed Carnot engine is employed for the purpose, what horsepower will be required?

11. A refrigerating machine removes heat from a cold room at  $-10^{\circ}\text{F}$ . to the atmosphere at  $95^{\circ}\text{F}$ . Its efficiency is 63 per cent. How many horsepower are required per ton of refrigerating capacity?

12. With the temperature of the atmosphere at  $100^{\circ}\text{F}$ ., a certain refrigerating machine operates with a coefficient of performance of 3.0 and a refrigerating efficiency of 60 per cent. What is the temperature of the cold room?

13. Twelve horsepower is required to operate a 10-ton refrigerating machine when the temperature of the cold room is  $15^{\circ}\text{F}$ . Its refrigerating efficiency is 65 per cent. What is the temperature of the atmosphere?

14. A 5-ton air-refrigerating machine operates between a cold-room temperature of  $25^{\circ}\text{F}$ . and an atmospheric (cooling-water) temperature of  $80^{\circ}\text{F}$ . The pressure in the cold room is 15 p.s.i. abs. and, at entrance to the expanding cylinder, 50 p.s.i. abs. Assume the pressure drop through the cold room and through the cooling coils to be negligible, that the air leaves the cold room  $5^{\circ}\text{F}$ . below cold-room temperature and enters the expanding cylinder at  $85^{\circ}\text{F}$ ., and that both the expansion and the compression are isentropic. Calculate (a) the booster horsepower required, (b) the coefficient of performance, (c) the refrigerating efficiency, and (d) the approximate cylinder volumes if both cylinders are double-acting and operate at 140 r.p.m.

15. Referring to Prob. 14, how would your answers be affected if the cold-room pressure had been 45 p.s.i. abs. and the pressure in the cooling coils 150 p.s.i. abs., all other data remaining unchanged?

16. Referring to the example of Art. 12:3, calculate the same quantities, with the following changes in data:  $P_B = P_C = 45$  p.s.i. abs.;  $P_D = 160$  p.s.i. abs.;  $P_A = 157$  p.s.i. abs. All other data are the same as in the example.

17. An ammonia dry-compression refrigerating cycle operates between a cold-room temperature of  $10^{\circ}\text{F}$ . and atmospheric temperature of  $90^{\circ}\text{F}$ . What is the coefficient of performance of the cycle? What is its refrigerating efficiency?

18. Repeat Prob. 17, using the same data but based on a wet-compression refrigeration cycle. Compare results with the answers to Prob. 17.

19. In a 30-ton dry-compression ammonia refrigeration machine, the temperature of the cold room is  $0^{\circ}\text{F}$ ., and that of the atmosphere is  $100^{\circ}\text{F}$ . (a) Between what pressure limits does compression take place? (b) What is the coefficient of performance for the cycle on which this plant operates? (c) What is its refrigerating efficiency? (d) How many pounds of ammonia must be circulated per hour? (e) What horsepower is required? (f) If the compressor is single-acting and makes 160 r.p.m., what approximate cylinder volume is required?

20. Repeat Prob. 19 for wet compression. Find the same quantities.

21. A sulphur dioxide dry-compression refrigerating cycle operates between a cold-room temperature of  $10^{\circ}\text{F.}$  and atmospheric temperature of  $90^{\circ}\text{F.}$  What is the coefficient of performance of the cycle? What is its refrigerating efficiency? Compare results with answers to Prob. 17. To what do you attribute the differences observed? Under what conditions would the coefficients of performance have been the same?

22. Repeat Prob. 21 for wet compression. Compare results with answers to Prob. 18.

23. For cycles using, respectively, ammonia, sulphur dioxide, carbon dioxide, and Freon as the refrigerating mediums in a wet-compression cycle operating between the temperatures of  $10$  and  $80^{\circ}\text{F.}$ , calculate the coefficients of performance and the refrigerating efficiencies. Between what pressures does the compressor operate in each case? What total volume of vapor (measured as it enters the compressor) must be handled per minute per ton of refrigeration in each case? What is the percentage loss of refrigerating effect in each case? Compare results for the various vapors.

24. (a) Calculate the coefficient of performance of a dry-compression refrigerating cycle using water vapor as the refrigerating medium. The lower temperature is  $45^{\circ}\text{F.}$ , and superheated vapor at a pressure of 1 p.s.i. abs. is delivered by the compressor to the condenser. (b) What is the refrigerating efficiency? (c) What is the pressure of the water vapor in the cold room? (d) What horsepower is required per ton of capacity? (e) What weight of water vapor must be compressed per minute per ton of capacity? (f) What volume, based on the condition at the beginning of compression, must be compressed per minute per ton of capacity?

## CHAPTER XIII

### MIXTURES OF AIR AND WATER VAPOR

**13:1. Atmospheric Moisture.** The air of the atmosphere is ordinarily a mixture of air and superheated water vapor. In the earlier chapters of this text, gases and vapors have been considered individually, but it is evident that the methods developed must be somewhat altered if they are to apply to such mixtures. A knowledge of the behavior of gas and vapor mixtures is important to an accurate analysis of the principles which govern the operation of internal-combustion engines, air compressors, air engines, and apparatus used in heating and ventilation, since the air which supplies all these is drawn from the atmosphere. Also, the same principles will influence the design of apparatus such as sprays and cooling towers, which have the purpose of cooling water by partly evaporating it in a stream of air.

**13:2. Mixtures of Air and Dry Saturated Water Vapor.** In Chap. III, attention has been given to mixtures of gases, and the methods of handling such mixtures have been outlined. Certain of the basic principles developed in that discussion will evidently also be valid in dealing with mixtures of a gas with a vapor; the volume of the mixture is also the volume of each constituent, its temperature is common to both gas and vapor, and Dalton's law will also apply, so that the total pressure of the mixture will be the sum of the individual partial pressures.

However, in dealing with gas mixtures, no interdependence of pressure and temperature, such as is set by the pressure-temperature relationships for steam discussed in Chap. VIII and outlined in the steam tables, need be considered. Atmospheric air is a mixture of dry air and either superheated or dry saturated water vapor; if, as when a drop in temperature causes the formation of *fog*, conditions bring about a partial *condensation* of the water-vapor content in the mixture, the suspended water particles that result from that condensation are no longer considered a part of the *thermodynamic* mixture, and a change in the ratio of vapor to gas content in the mixture is brought about.

Air is said to be *saturated* with moisture when the partial pressure of its water-vapor content is the saturation pressure equivalent to the temperature of the mixture. Thus, if the temperature of the saturated

air is 60°F., the partial pressure of the vapor portion is 0.2563 p.s.i. abs. as read from Table 1 of the steam tables. For a given mixture temperature, this represents the condition at which *maximum* water-vapor content can be associated with the mixture; the standard of measurement is usually the water-vapor content in pounds or grains per pound of dry air in the mixture. For instance, for atmospheric air having a total pressure of 14.7 p.s.i. abs. and a temperature of 60°F., the partial pressure of the dry-air portion is 14.4437 p.s.i. abs. (14.7 - 0.2563), and the volume of 1 lb. of dry air at this pressure and temperature (and therefore the volume of the mixture) is

$$V = \frac{MRT}{P} = \frac{1 \times 53.3 \times 519.7}{14.4437 \times 144} = 13.34 \text{ cu. ft.}$$

According to the steam tables, 1 lb. of dry saturated steam at 60°F. ( $p = 0.2563$  p.s.i. abs.) would require a space having a volume of 1,206.7 cu. ft. In the space available (13.34 cu. ft.), it is evident that 0.01106 lb. of water vapor (13.34/1,206.7) can be accommodated. Since 1 lb. weighs 7,000 gr., this is equivalent to about 77.5 gr. of water vapor. This is the *maximum* water-vapor content that can be associated with 1 lb. of dry air in a thermodynamic mixture having a temperature of 60°F. and a total pressure of 14.7 p.s.i. abs. At higher temperatures, it will be observed that the partial pressure of the water vapor will increase; that of the dry air will decrease; the volume of the mixture per pound of dry air will increase; and since the space required by 1 lb. of dry saturated water vapor at this higher partial pressure is sharply lower, the net result is to increase rapidly the maximum vapor content at increasing temperature. Thus, if the sun shines on a bank of fog, raising the temperature of the air in which it is suspended, the moisture particles of which the fog is composed will be evaporated and form a part of the vapor content of the air. When air contains the maximum amount of water-vapor content possible under given conditions of pressure and temperature as outlined above, it is said to be at its *dew point*; any further lowering of temperature of the air will result in the condensation of a part of its vapor content, as in the formation of *dew*.

**13:3. Mixtures of Air and Superheated Water Vapor.** If saturated air is heated at constant total mixture pressure in the absence of water that might be evaporated to increase the proportions of vapor to dry air in the mixture, it is evident that the partial pressures of both its gas and its vapor portions will also remain constant. Thus, the water vapor, which was present in the form of dry saturated steam in *saturated* air, now becomes *superheated*.



One of the distinguishing characteristics of a perfect gas, as developed from Joule's law in Chap. III, is that, if there is no change in temperature, there can be no change in internal energy. Similarly, since the difference between  $C_p$  and  $C_v$  is constant for a gas, a line of constant temperature can also be shown to be necessarily a line of constant enthalpy; indeed, this has already been implied in the statement of Eq. (4:28). Conversely, it may be said that any substance for which lines of constant temperature approximately coincide with lines of constant enthalpy is approaching the behavior of the perfect gas. The attention of the reader is directed to the Mollier diagram for steam; at the higher pressures, lines of constant temperature in the superheat region are curves which cross the horizontal lines representing constant enthalpies at an angle, but, near the right end of the saturation curve (corresponding to pressures as high as about 1.0 p.s.i. abs. and corresponding temperatures of 100°F. and below), lines of constant temperature leave the saturation curve and continue into the superheat region as almost exact horizontals. The possibility suggests itself that this indicates a degree of conformity to perfect-gas behavior such that mixtures of air with superheated water vapor may with negligible error be treated on much the same basis as mixtures of perfect gases if the temperature does not exceed ordinary atmospheric levels. The gas constant which applies to water vapor in this condition can be found from the molecular weight of water vapor, which is 18, and the universal gas constant, as  $1,544/18 = 85.7$ . This can be checked roughly by applying Eq. (3:6) to the properties of the dry saturated vapor at some convenient temperature within the specified range. Thus, at 60°F.,

$$R = \frac{Pv}{T} = \frac{(0.2563)(144)(1,206.7)}{519.7} = 85.7$$

It will be observed, however, that the requirement that the partial pressure of the water vapor may not exceed the saturation pressure of steam at the mixture temperature places a definite limit on the attainable proportion of water vapor in the mixture as outlined in Art. 13:2.

The condition of atmospheric air containing a maximum proportion of water vapor at a given temperature has been described above by speaking of it as *saturated* air. A similar method of designation for the condition of mixtures of air and superheated water vapor now becomes desirable. Based on our practice in describing the condition of steam up to the present chapter, it might appear that the number of degrees of superheat of the water-vapor portion could well be used as the method of designation; thus, for a mixture at a temperature of 80°F. but in which

the partial pressure of the water vapor was only 0.2563 p.s.i. abs., the saturation pressure equivalent to a temperature of 60°F., the mixture would be described as containing water vapor carrying 20° of superheat. A more descriptive term is desirable, however, one which will carry with it some reference as to the degree to which the mixture has satisfied its maximum capacity for mixing with water vapor at the given temperature. The term that meets this requirement and has become the standard basis for the description of such mixtures is *relative humidity*, de-

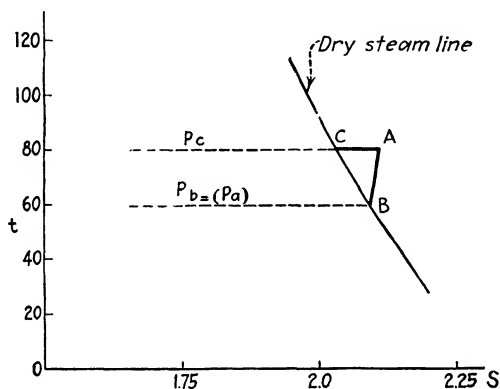


FIG. 13:1.—Relative humidity of moist air.

fined as the density of the water vapor in the mixture divided by its density in a saturated mixture having the same temperature.

Let a mixture containing  $M_1$  lb. of dry air and  $M_2$  lb. of superheated water vapor be considered; by Dalton's law, the total pressure of the mixture  $P$  is the sum of the partial pressures  $P_1 + P_2$ . In Fig. 13:1, the condition of the superheated water vapor in the mixture is represented by point  $A$ , and line  $AB$  is a constant-pressure line such that  $P_A = P_B = P_2$ ; the temperature at point  $B$  is therefore the *dew point* of the mixture. Referring to Fig. 13:1, the relative humidity of the mixture containing water vapor at condition  $A$  may be stated as the ratio of the density of the water vapor at point  $A$  divided by the density corresponding to point  $C$ , or  $d_A/d_C$ . Since density is the reciprocal of the specific volume, the relative humidity may also be stated as  $v_C/v_A$ . Because of the similarity in the behavior of superheated water vapor at temperatures such as are usually characteristic of atmospheric air to that of a perfect gas with a gas constant of 85.7, it is now possible to apply Eq. (3:6) to the condition of the water-vapor content at points  $A$  and  $C$  as follows:

$$v_A = \frac{RT_A}{P_A}; \quad v_C = \frac{RT_C}{P_C}$$

But

$$T_A = T_C \quad \text{and} \quad P_A = P_B = P_2$$

$$\text{Relative humidity} = \frac{v_C}{v_A} = \frac{P_A}{P_C} = \frac{P_B}{P_C} \quad (13:1)$$

Therefore, relative humidity may be expressed as the ratio of the saturation pressure equivalent to the dew point of the mixture to the saturation pressure of water vapor at the temperature of the mixture. For the mixture that was used as an example in the foregoing discussion, the temperature of which was 80°F. with dew point at 60°F., the relative humidity would be calculated as 0.2563/0.5069 or 50.5 per cent.

In addition to relative humidity, the terms "absolute humidity" and "specific humidity" also find application in describing the state of the atmospheric air mixture. *Absolute humidity* is defined as the weight of water vapor contained in 1 cu. ft. of the mixture. *Specific humidity* will be used in these pages as the weight of water vapor associated with 1 lb. of dry air; it is also defined by some authors as the weight of water vapor in 1 lb. of the mixture.

**13:4. The properties of atmospheric air** to which attention is given above (total and partial pressures, temperature, and relative humidity) are those which apply regardless of the mass of atmospheric air to which they refer. Certain others, such as enthalpy and total volume, must be stated with strict regard for the respective masses of dry air and of water vapor included in the mixture. It has become standard practice to express these properties as they apply to a constant weight of dry air (1 lb.) plus the (variable) weight of water vapor associated with it. Thus their values apply to a variable total mixture weight ranging from 1 lb. (for dry air without water-vapor content) upward.

The volume of this standard mixture may be developed by application of Eq. (3:6) to the (unit) weight of dry air, as has been done in Art. 13:2. Its enthalpy is the sum of the enthalpies of the dry air and water-vapor portions of the mixture. It has become conventional to measure the enthalpy of the dry-air portion above a base temperature of 0°F.; the enthalpy of the water vapor is based on steam-table values which begin the measurement of enthalpy at the condition of water at 32°F. Since it is *differences* of enthalpy with which we are concerned, this difference in bases from which measurement of enthalpy is made need not disturb us. For the standard mixture, the weight of dry air  $M_1$  is unity, and its enthalpy is

$$h_a = 1 \times 0.2375(t - 0) = 0.2375t$$

[see Eq. (4:28)]. For water vapor at these low pressures, it has already been noted that a line of constant temperature coincides with one of constant enthalpy; therefore, even if the vapor content of the mixture is in the form of the superheated vapor, its specific enthalpy may be recorded as the same as that of dry saturated water vapor at the mixture temperature. Thus,  $H_w = M_2 h_g$ , and the total enthalpy of the standard mixture is

$$H_m = 0.2375t + M_2 h_g \quad (13:2)$$

in which  $M_2$  is the specific humidity of the mixture in pounds per pound of dry air.

**13:5. The Adiabatic Saturation of Air.** When atmospheric air having a relative humidity less than 100 per cent is caused to come into intimate contact with water, as in passing through an air washer, a portion of the water will be vaporized and will increase the proportion of water vapor in the air. Even if the washer water is at the same temperature as the air as it enters the washer, it will be observed that the temperature of the air as it leaves the washer has been lowered.

This effect occurs because the air must supply heat sufficient to vaporize the additional weight of water evaporated. The limit of such a change is evidently attained when the air becomes saturated with water vapor, and the process is termed the *adiabatic saturation* of the air. The drop of temperature of the air during a process of this type is greater for air temperatures which are initially high and, of course, for relative humidities which are low.

Since the determination of the point at which condensation begins (the dew point) is not readily made experimentally, the ordinary method used for determining the relative humidity is based upon the principle of adiabatic saturation. Two thermometers are mounted beside each other, the bulb of one being dry and the bulb of the other surrounded by a wicking which is kept moistened with water. The dry-bulb temperature is read from the first and the wet-bulb temperature (the temperature of adiabatic saturation) from the second. These temperatures are then referred to standard tables or charts, and the relative humidity is read directly. A device of this type is called a psychrometer. The basis on which the charts and tables are prepared will be discussed in a later section of this chapter. A psychrometric chart will be found in Appendix III.

Certain compounds, such as silica gel and calcium chloride, have the property of absorbing water vapor from the air that passes over them. This water vapor joins the compound as a liquid, thus liberating its enthalpy of vaporization and causing the air stream to leave at increased

temperature. This is evidently the exact reverse of the process of adiabatic saturation and is called *adsorption*. Both processes are important in the conditioning of air.

Since, in both adiabatic saturation and adsorption, heat exchange takes place only between the parts of which the mixture is composed and since there is no change of total mixture pressure and but little of the partial pressures of the components, both processes may be considered as taking place at constant total enthalpy of the mixture. Also, since a condition of constant wet-bulb temperature is followed in both cases, it would appear that a change of condition during which the wet-bulb temperature is constant is one during which the total enthalpy of the mixture does not change. This is not exactly true since the water vapor which has entered or left the mixture during the process will have carried with it the enthalpy of the liquid, and this is equivalent to adding or removing heat. To account for this factor for the purposes of more accurate investigations, a new quantity has been introduced and given the name *sigma function*. The sigma function of the standard mixture has a value equal to the enthalpy of the mixture (per pound of dry air) less the product of the weight of the vapor portion of the mixture and the specific enthalpy of the liquid at wet-bulb temperature. Since this correction is relatively small and, for air at atmospheric temperatures, introduces only minor adjustments, it is mentioned here only to give the reader an understanding as to the meaning of the term.

**13:6. Constant-pressure Changes of Atmospheric Air.** The temperature changes which take place in the dry air contained in the atmospheric mixture occur at practically constant pressure since the partial pressure of the water vapor is always small relative to the partial pressure of the air. The heat absorbed or rejected by the dry air contained in the mixture may therefore be expressed as

$$Q_a = 0.2375M_1(t_2 - t_1)$$

During the heating or cooling process, the water-vapor content will have changed its condition and consequently will also have absorbed or rejected heat. This process may also be assumed to vary little from one of constant pressure for all practical considerations, and the heat absorbed or rejected by the water vapor during the change may be calculated as the difference between the initial and final enthalpies. Since the initial and final weights of water vapor may not be the same, the calculation involved is outlined as follows:

$$Q_w = M_b h_b - M_a h_a + (M_a - M_b) h_{f_b}$$

in which  $Q_w$  represents the heat absorbed or rejected by the water vapor,  $M_b$  the final weight of water vapor and  $M_a$  its initial weight,  $h_b$  and  $h_a$  the final and initial enthalpies of the water vapor per pound, and  $h_f$  the enthalpy of 1 lb. of water at the final temperature. The total heat flow during the process is the sum of the individual heat flows, or  $Q_a + Q_w$ . It will be observed that the total heat flow is not exactly equal to the change of enthalpy of the mixture because of the last term in the expression for  $Q_w$ ; thus a line of constant wet-bulb temperature is not precisely one of constant total enthalpy of the mixture. The relative magnitude of this term is so small, however, that the error involved may be ignored in most applications.

*Example.* Moist air at 80°F. with a relative humidity of 50 per cent is cooled to 50°F. at constant pressure. How much heat must be removed per pound of dry air? The original pressure of the mixture is 14.7 p.s.i. abs. What are the original and final partial pressures?

*Solution.* With reference to the steam tables, the saturation pressure corresponding to a temperature of 80°F. is 0.5069 p.s.i. abs. By Eq. (13:1), the saturation pressure  $p_B$  at the dew-point temperature is

$$0.50 \times 0.5069 = 0.2534 \text{ p.s.i. abs.}$$

The corresponding temperature as read from the steam tables is 59.7°F. Therefore, partial condensation takes place, and the final weight of water vapor is less than the initial weight. The original partial pressure of the water vapor is 0.2534 p.s.i. abs. The partial pressure of the dry air in the initial condition is

$$14.7 - 0.2534 = 14.4466 \text{ p.s.i. abs.}$$

In the final condition, the air is saturated with moisture at a temperature of 50°F. The saturation pressure for this temperature is 0.1781, which is the partial pressure of the water vapor in the final condition. The partial pressure of the dry air in the final condition is

$$14.7 - 0.1781 = 14.522 \text{ p.s.i. abs.}$$

The specific volume of the dry air at the dew point (59.7°F.) is

$$v_a = \frac{RT}{P_a} = \frac{53.3 \times (459.7 + 59.7)}{14.4466 \times 144} = 13.3 \text{ cu. ft. per pound}$$

The specific volume of the saturated water vapor at the dew point is found by interpolation from steam-table values to be 1,219.1 cu. ft. per pound. Therefore the original weight of water vapor per pound of dry air is  $13.3/1,219.1 = 0.0109$  lb.

The final specific volume of the dry air is

$$v_a = \frac{53.3 \times (459.7 + 50)}{14.522 \times 144} = 13.0 \text{ cu. ft. per pound}$$

The specific volume of the saturated water vapor at the final temperature of 50°F. is 1,703.2 cu. ft., and the final weight of water vapor per pound of dry air is  $13.0/1,703.2 = 0.0076$  lb.

The heat removed from the mixture per pound of dry air may now be calculated.

$$Q_a = 0.2375M_1(t_2 - t_1) = 0.2375 \times 1 \times (50 - 80) = -7.13 \text{ B.t.u.}$$

$$\begin{aligned} Q_w &= M_b h_b - M_a h_a + (M_a - M_b) h_{f_b} \\ &= 0.0076 \times 1083.7 - 0.0109 \times 1,096.6 + 0.0033 \times 18.07 \\ &= 8.24 - 11.95 + 0.06 \\ &= -3.65 \text{ B.t.u.} \end{aligned}$$

$$Q_{\text{total}} = Q_a + Q_w = -7.13 + (-3.65) = -10.78 \text{ B.t.u.}$$

The negative sign indicates that this amount of heat must be removed from the mixture. It will be noted that the value of  $h_a$  (the enthalpy of the superheated water vapor in its original condition) has been assumed equal to that of saturated water vapor at the original temperature of 80°F. This is possible for low vapor pressures, for reasons already advanced. The value of 1096.6 B.t.u. used above may be checked by a calculation based on a value of 0.45 for the specific heat of superheated steam at this pressure and temperature as read from Fig. 8.3.

$$h_a = h_g + 0.45(t_A - t_B) = 1087.9 + 0.45(80 - 59.7) = 1097.0 \text{ B.t.u. per pound}$$

in which  $h_g$  is the total enthalpy of saturated steam at the dew-point temperature (59.7°F.) and  $t_A - t_B$  represents the superheat of the water vapor in the original condition.

If the heat flow had been calculated as a simple difference of enthalpies, its amount would have been changed by the magnitude of the last term in the expression for  $Q_w$ , or 0.06 B.t.u.  $Q_w$  would have been -3.71 B.t.u. and the total heat flow -10.84 B.t.u. for this approximation. The error involved would, in this case, have been less than 1 per cent.

**13:7. Construction and Use of the Psychrometric Chart.** The solution of problems such as that of the preceding example is shortened and simplified by the use of psychrometric tables such as Table 10 of Appendix I or charts similar to that shown in Appendix III.

The psychrometric chart may be constructed on the basis of 1 lb. or 1 cu. ft. of dry air, the chart above referred to having been drawn on the basis of unit weight. The ordinates represent the specific humidity or vapor content per pound of dry air and the abscissas the dry-bulb temperatures. The saturation, or 100 per cent humidity, curve may be constructed on these coordinates by comparing the specific volume of the saturated water vapor at the dry-bulb temperature with the volume of 1 lb. of dry air at dry-bulb temperature and at the partial pressure of the dry air in the mixture, the latter being calculated by subtracting the saturation pressure of the water vapor from standard atmospheric pressure. An example of the method of calculation has already been presented in Art. 13:2.

The curves of constant relative humidity may be developed by the application of Eq. (13:1). For example, let it be required to establish the point on the chart which represents a dry-bulb temperature of 70°F. and a relative humidity of 40 per cent. The saturation pressure of

water at 70°F. is 0.3631 lb. per square inch absolute; the saturation pressure at the dew point (where the specific humidity is the same as for the given condition) is  $0.40 \times 0.3631 = 0.145$  p.s.i. abs., corresponding to a saturation temperature of about 44.5°F. Since the dew point represents a condition of 100 per cent relative humidity, the specific humidity can be read as about 44 gr. per pound of dry air at the intersection of the saturation curve previously developed and the vertical line corresponding to a dry-bulb temperature of 44.5°F. Both the specific humidity and the dry-bulb temperature now are known, and the required point may be located on the chart.

From the definition of the term "adiabatic saturation," lines of constant wet-bulb temperature are established as approximately the loci of mixture conditions having a constant total enthalpy. Let it be required to find the wet-bulb temperature corresponding to the given condition, *i.e.*, a dry-bulb temperature of 70°F. and relative humidity of 40 per cent. The total enthalpy of the dry air will be measured above a temperature of 0°F. for convenience; its value is

$$0.2375 \times 1 \times (70 - 0) = 16.6 \text{ B.t.u.}$$

The enthalpy of the water-vapor content is

$$\frac{44}{7,000} \times 1,092.3 = 6.9 \text{ B.t.u.}$$

and the total enthalpy of the mixture is therefore 23.5 B.t.u. It is evident that, at the saturation curve, dry- and wet-bulb temperatures will correspond. Let a wet-bulb temperature of 60°F. be tested; the vapor content per pound of dry air for saturated air at this temperature is 77 gr., and the total enthalpy of the mixture is

$$H_m = 0.2375 \times 1 \times (60 - 0) + \frac{77}{7000} \times 1,088.0 = 26.26 \text{ B.t.u.}$$

If a similar calculation is made based on a wet-bulb temperature of 55°F., the result is 23.1 B.t.u. By interpolation, the proper wet-bulb temperature may be established as about 55.6°F. That this method involves an approximation is brought out in Art. 13:6, but the amount of error introduced is small. For instance, if the wet-bulb-temperature calculation had been based on the sigma function instead of the total enthalpy as above, the wet-bulb temperature obtained would have been 55.8°F. The discrepancy will increase with the wet-bulb temperature and, for a given wet-bulb temperature, as the relative humidity decreases.

The curve labeled Total Heat is better described as a "total-enthalpy" curve and is plotted against points along the saturation, or 100 per cent



relative humidity, line. Thus, the total enthalpy of the standard saturated mixture at 60°F. is read as 26.2 B.t.u. However, since a line of constant wet-bulb temperature is also a line of constant total enthalpy, this value of the total enthalpy will also apply to any point on the chart having a wet-bulb temperature of 60°F.

Two curves are presented from which the volume may be read, the lower presenting the values for the dry air alone, the upper for the saturated mixture. These are based on the dry-bulb temperature; thus, a pound of dry air at 60°F. has a volume of 13.1 cu. ft., and the volume of a pound of dry air, saturated with moisture, at 60°F. is 13.35 cu. ft. The volume of air mixtures having intermediate relative humidities per pound of dry air in the mixture can be found by proportion. Thus, for a mixture at 60°F., 50 per cent relative humidity, the volume is  $13.1 + 0.5(13.35 - 13.1) = 13.23$  cu. ft.

The reader will observe that the chart as presented in the Appendix has been prepared on the basis of standard atmospheric pressure and would require some changes before it could be used to represent accurately atmospheric air conditions at any considerable elevation above sea level. Psychrometric charts are available in somewhat different form, however, from which values for any elevation can be read.

The psychrometric chart may be used to good advantage in the study of atmospheric air in heating, cooling, and ventilation processes and for the solution of cooling-tower design problems. In the cooling tower, which is often used when the cooling water used in the condensers or other heat-exchanging apparatus is to be recirculated, the warm water is brought into contact with a current of air at a relatively low temperature and a relative humidity less than 100 per cent. The water is cooled by the abstraction of the heat necessary to raise the temperature of the air to its final temperature and by the evaporation of a small portion of the water itself as the air approaches the saturated condition. If the total weight of water cooled per pound of dry air circulated is  $M$  lb. and the weight of water evaporated is denoted as  $M_3$ , the heat given up by the cooling water per pound of dry air circulated is

$$Mh_{f_1} - (M - M_3)h_{f_2}$$

in which  $h_{f_1}$  represents the enthalpy of the water per pound at entrance and  $h_{f_2}$  at exit from the tower. This must be equal to the increase of enthalpy of the moist air as it passes through the cooling tower, or

$$Mh_{f_1} - (M - M_3)h_{f_2} = H_{a_2} - H_{a_1} \quad (13:3)$$

in which  $H_{a_1}$  and  $H_{a_2}$  are, respectively, the total enthalpies, per pound

# APPENDIX II

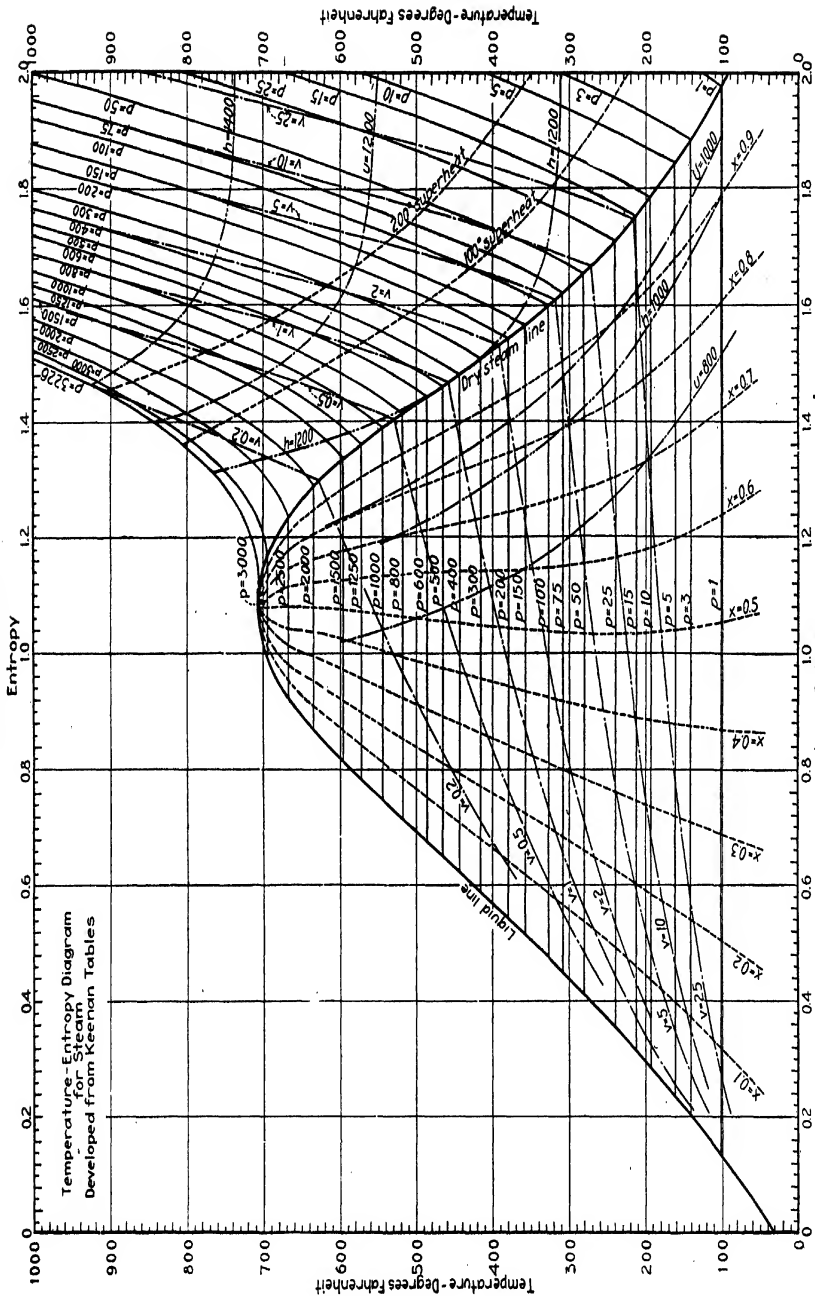
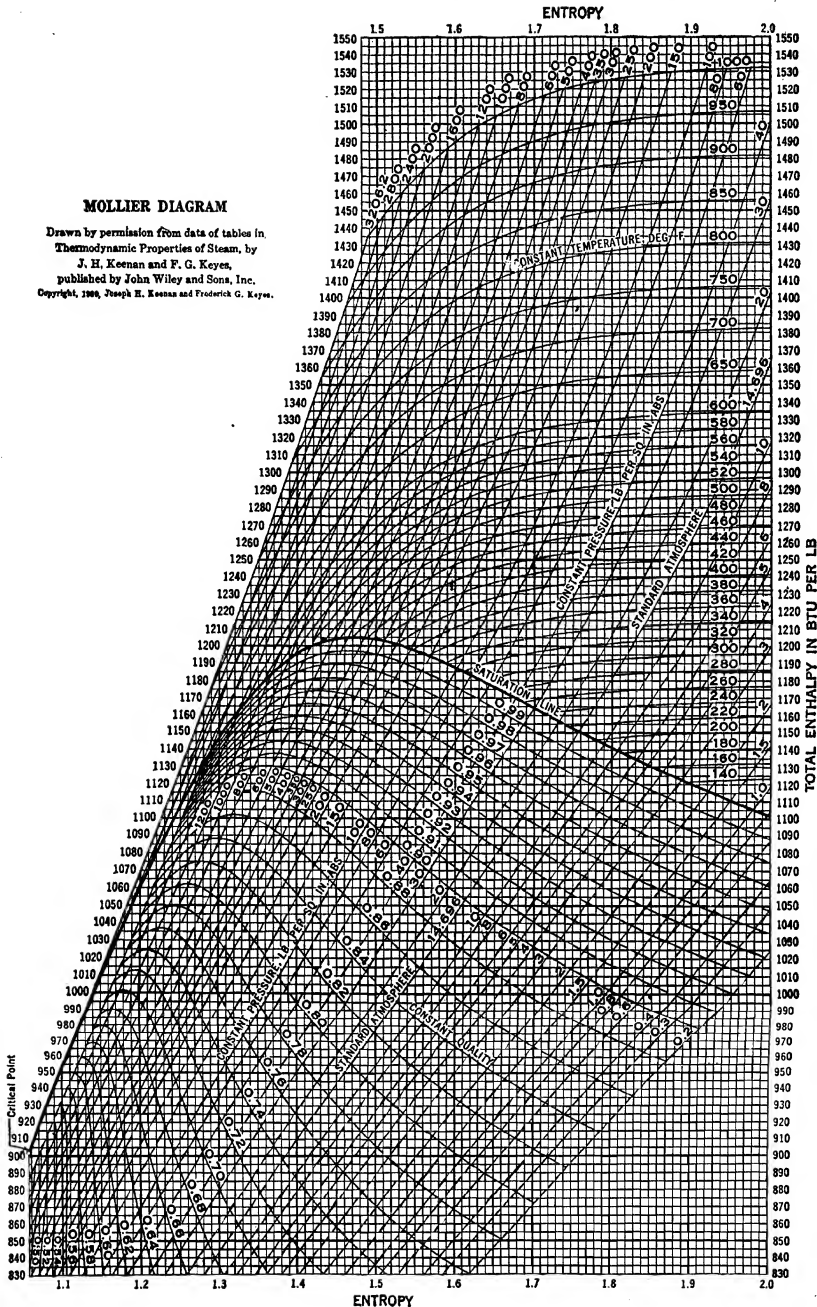


PLATE 1



# MOLLIER DIAGRAM

Drawn by permission from data of tables in  
Thermodynamic Properties of Steam, by  
J. H. Keenan and F. G. Keyes,  
published by John Wiley and Sons, Inc.  
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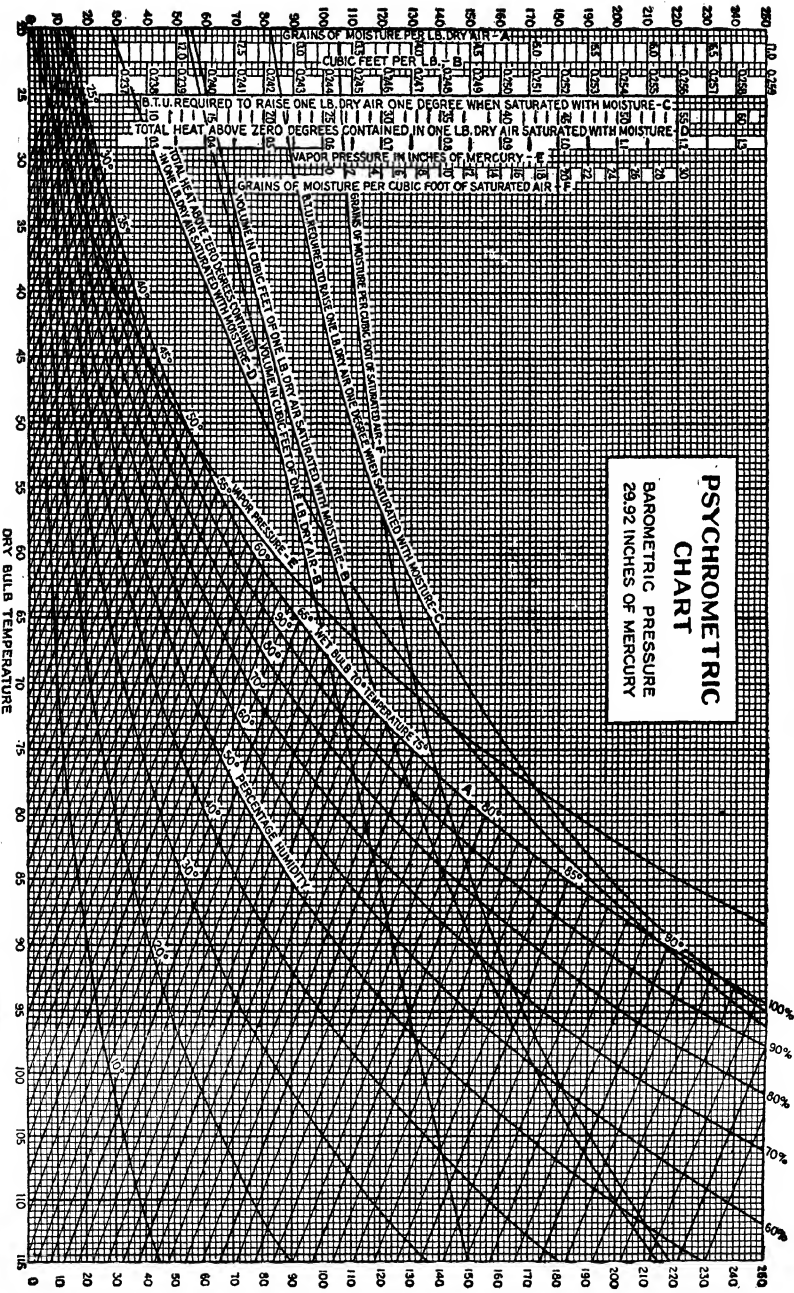
## APPENDIX II

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# APPENDIX III

## PSYCHROMETRIC CHART BAROMETRIC PRESSURE 29.92 INCHES OF MERCURY





of dry air supplied, of the moist air as it enters and as it leaves the cooling tower. If the initial and final temperatures and relative humidities of the air and the weight of water circulated are known, the drop in temperature of the water may be computed.

*Example.* Twenty thousand pounds of water per hour enters a cooling tower at 125°F.; 15,000 lb. of moist air at 70°F. and relative humidity of 50 per cent also is supplied. The air leaves at 110°F. saturated with moisture. What is the final temperature of the water?

*Solution.* From the psychrometric chart, the moisture content of the entering air is 55 gr. per pound of dry air. The weight of dry air circulated per hour is therefore

$$\frac{15,000}{1 + (55/7,000)} = 14,880 \text{ lb. per hour}$$

The weight of water vapor in the entering air is 55 gr. or 0.0078 lb. per pound of dry air. In the final condition the weight of water vapor is 415 gr. or 0.0593 lb. per pound of dry air. Therefore the weight of water evaporated per pound of dry air is

$$M_3 = 0.0593 - 0.0078 = 0.0515 \text{ lb.}$$

The weight of water entering the tower per pound of dry air circulated is

$$M = \frac{20,000}{14,880} = 1.343 \text{ lb.}$$

The original enthalpy of the moist air is

$$H_{a_1} = 0.2375(70 - 0) + 0.0078(1,092.3) = 25.1 \text{ B.t.u. per pound of dry air}$$

and, in the final condition,

$$H_{a_2} = 0.2375(110 - 0) + 0.0593(1,109.5) = 91.9 \text{ B.t.u. per pound of dry air}$$

Substituting in Eq. (13.3),

$$1.343(92.9) - (1.343 - 0.0515)h_{f_2} = 91.9 - 25.1$$

and

$$h_{f_2} = 45 \text{ B.t.u.}$$

This is the enthalpy of a pound of water at 77°F., the final temperature of the water as it leaves the tower.

**13.8. Other Processes for Moist Air.** The discussion in the preceding paragraphs has dealt with the changes which occur in moist air at or near atmospheric pressure. The same conclusions acceptable for this set of conditions will not apply for the higher range of pressure since, in the higher pressure range, water vapor can no longer be considered to have the properties of a gas; nor can they be applied to processes during which there is a considerable variation of pressure.



It is clear that when atmospheric air is drawn into the cylinder of an internal-combustion engine, for example, the nature of the compression, explosion, and expansion curves is somewhat affected by the moisture contained in the air. Since the weight of water vapor is relatively small, these effects are slight and, except in thermodynamic calculations of the highest refinement, may be neglected, just as in this text the effect of fuel mixtures on the behavior of the thermodynamic system has not been considered.

As the moist air passes through the successive processes in an internal-combustion engine, the amount of superheat possessed by the water-vapor content is greatly increased. As has been demonstrated, highly superheated vapors approach the behavior of a perfect gas and this is true of water vapor even at the higher range of pressures. Again, therefore, it is possible with sufficient accuracy to apply Avogadro's principle and Dalton's law exactly as if a mixture of gases existed.

**13:9. Mixtures of Steam and Air.** A mixture of air at high temperature and high pressure with high-pressure steam may be used as a medium for the generation of power. In general, the advantage is practical rather than theoretical as it reduces the conductivity of the steam and thus the initial condensation within the cylinder. In such cases as this, the partial pressure of the vapor content is no longer small as compared with the total pressure, and the methods previously discussed will not apply. However, the general methods of attack which have been discussed in previous chapters may be used individually for the gas and for the vapor content. A more complete discussion will be found in advanced texts on thermodynamics.

### Problems

1. Calculate the specific humidity of saturated air at a temperature of 70°F. and a total pressure of (a) 14.7 p.s.i. abs. and (b) 13.0 p.s.i. abs.
2. Calculate the specific humidity of saturated air at a temperature of 40°F. and a total pressure of (a) 14.7 p.s.i. abs. and (b) 12.0 p.s.i. abs.
3. What is the volume of a pound of steam if its temperature is 90°F. and it is superheated 20°F.?
4. What is the specific volume of superheated steam having a pressure of 0.20 p.s.i. abs. and a temperature of 80°F.?
5. The water vapor portion of a moist-air mixture has a partial pressure of 0.20 p.s.i. abs. The temperature of the mixture is 80°F. What is the relative humidity if the total pressure is (a) 14.7 p.s.i. abs. and (b) 13.0 p.s.i. abs.?
6. The temperature of atmospheric air is 90°F., and its dew point is 70°F. What is its relative humidity if the total pressure of the mixture is (a) 14.7 p.s.i. abs. and (b) 12.0 p.s.i. abs.?

7. What is the absolute humidity in grains per cubic foot of moist air in which the water-vapor portion has a partial pressure of 0.20 p.s.i. abs. if the temperature of the mixture is 80°F. and the total pressure of the mixture is (a) 14.7 p.s.i. abs. and (b) 13.0 p.s.i. abs.?

8. What is the absolute humidity in grains per cubic foot of atmospheric air with a temperature of 90°F. and a dew point of 70°F. and having a total mixture pressure of (a) 14.7 p.s.i. abs. and (b) 12.0 p.s.i. abs.?

9. Calculate the specific humidity of moist air in which the water-vapor portion has a partial pressure of 0.20 p.s.i. abs. if the temperature of the mixture is 80°F. and its total pressure is (a) 14.7 p.s.i. abs. and (b) 13.0 p.s.i. abs.

10. Calculate the specific humidity of atmospheric air with a temperature of 90°F. and a dew point of 70°F. if its total pressure is (a) 14.7 p.s.i. abs. and (b) 12.0 p.s.i. abs.

11. Calculate the specific humidity of moist air at 70°F. and relative humidity of 40 per cent if the total pressure is (a) 14.7 p.s.i. abs. and (b) 11.0 p.s.i. abs. What is its absolute humidity?

12. Calculate the enthalpy of the standard unit psychrometric mixture for saturated air at a temperature of 70°F. and a total pressure of (a) 14.7 p.s.i. abs. and (b) 13.0 p.s.i. abs.

13. Calculate the enthalpy of the standard unit psychrometric mixture for saturated air at a temperature of 40°F. and a total pressure of (a) 14.7 p.s.i. abs. and (b) 12.0 p.s.i. abs.

14. What is the specific enthalpy of superheated steam if (a) its temperature is 90°F. and it is superheated 20° and (b) its pressure is 0.20 p.s.i. abs. and its temperature is 80°F.?

15. Calculate the enthalpy of the standard unit psychrometric mixture when its specific humidity is 70 gr. per pound of dry air and its temperature 90°F. if the total pressure of the mixture is (a) 14.7 p.s.i. abs. and (b) 10.0 p.s.i. abs.

16. Calculate the enthalpy of the standard unit psychrometric mixture if its relative humidity is 40 per cent and its temperature 70°F. when the total pressure is (a) 14.7 p.s.i. abs. and (b) 11.0 p.s.i. abs.

17. What is the wet-bulb temperature of moist air having a temperature of 90°F. and a specific humidity of 70 gr. per pound of dry air if the total pressure of the mixture is (a) 14.7 p.s.i. abs. and (b) 10.0 p.s.i. abs.?

18. Calculate the wet-bulb temperature of moist air at a temperature of 70°F. and having a relative humidity of 40 per cent if the total pressure is (a) 14.7 p.s.i. abs. and (b) 11.0 p.s.i. abs. (NOTE: Assume the enthalpy to be constant along a line of constant wet-bulb temperature.)

19. A moist-air mixture has a total pressure of 12.0 p.s.i. abs., a temperature of 85°F., and relative humidity of 50 per cent. Calculate (a) dew-point temperature, (b) specific humidity, (c) absolute humidity, (d) volume of the standard unit mixture, (e) enthalpy of the standard unit mixture, and (f) wet-bulb temperature.

20. The total pressure of a moist-air mixture is 11.5 p.s.i. abs.; its dry-bulb temperature is 75°F. and wet-bulb temperature 65°F. Calculate (a) enthalpy of mixture per pound of dry air, (b) specific humidity, (c) dew-point temperature, (d) relative humidity, and (e) volume of mixture per pound of dry air.

21. The total pressure of a moist-air mixture is 12.5 p.s.i. abs.; its temperature is 65°F., and its dew point is 50°F. Calculate (a) relative humidity, (b) specific humidity, (c) volume of the standard unit mixture, (d) enthalpy of the standard unit mixture, and (e) wet-bulb temperature.

22. The dry-bulb temperature is  $70^{\circ}\text{F.}$  and the specific humidity is 60 gr. per pound of dry air for a moist-air mixture having a total pressure of 11.7 p.s.i. abs. Calculate (a) dew-point temperature, (b) relative humidity, (c) volume of the mixture per pound of dry air, (d) enthalpy of the standard unit mixture, and (e) wet-bulb temperature.

23. For air having a dry-bulb temperature of  $70^{\circ}\text{F.}$  and a wet-bulb temperature of  $60^{\circ}\text{F.}$ , read from the psychrometric chart (*i.e.*, for a total mixture pressure of 14.7 p.s.i. abs.) (a) the relative humidity, (b) the specific humidity, (c) the dew point-temperature, (d) the enthalpy, and (e) the volume.

24. Given air at a dry-bulb temperature of  $80^{\circ}\text{F.}$  and having a dew point at  $70^{\circ}\text{F.}$ , read the following values from the psychrometric chart: (a) relative humidity, (b) wet-bulb temperature, (c) specific humidity, (d) enthalpy, (e) volume.

25. Air at  $88^{\circ}\text{F.}$  and 40 per cent relative humidity is cooled adiabatically as it passes through a washer. What is its final temperature if it leaves the washer as saturated air? Use the psychrometric chart.

26. Air at  $88^{\circ}\text{F.}$  and 40 per cent relative humidity is cooled at constant specific humidity until it becomes saturated air. What is the final temperature? How much heat is removed per pound of dry air? Use the psychrometric chart.

27. Fourteen hundred cubic feet per minute of air at  $85^{\circ}\text{F.}$  dry-bulb temperature and 50 per cent relative humidity is mixed with 200 cu. ft. per minute of air at  $54^{\circ}\text{F.}$  dry-bulb temperature and 40 per cent relative humidity. Based on values from the psychrometric chart, find the resulting dry-bulb temperature, wet-bulb temperature, and relative humidity.

28. How many cubic feet per minute of air at  $70^{\circ}\text{F.}$  dry-bulb temperature and 50 per cent relative humidity must enter a cooling tower to cool 100 gal. per minute of water from  $85$  to  $70^{\circ}\text{F.}$ , assuming that the air is saturated at  $80^{\circ}\text{F.}$  as it leaves the top of the tower? Use the psychrometric chart.

29. An evaporative cooler is used to cool a house in the summer. The cooler draws 3,400 cu. ft. per minute of outside air with a dry-bulb temperature of  $92^{\circ}\text{F.}$  and a relative humidity of 20 per cent. While in the cooler the air is sprayed with water until when discharged into the house its relative humidity has been increased (adiabatically) to 90 per cent. (a) How many pounds of water are evaporated into the air per minute? (b) What is the final dry-bulb temperature of the air?

30. Ten thousand cubic feet per minute of air at  $80^{\circ}\text{F.}$  dry-bulb temperature and 40 per cent relative humidity is passed over a cooling coil the temperature of which is maintained at  $50^{\circ}\text{F.}$  Assuming that the air leaves the coil at  $55^{\circ}\text{F.}$  dry-bulb temperature, how many B.t.u. per hour are removed by the coil, and what is the equivalent capacity in tons of refrigeration?

31. (a) Find the weight of dry air that must be drawn hourly through a cooling tower to cool 60,000 lb. of water per hour from  $105$  to  $75^{\circ}\text{F.}$  if the air enters at  $70^{\circ}\text{F.}$  and with a relative humidity of 30 per cent and leaves at  $100^{\circ}\text{F.}$  and with a relative humidity of 95 per cent. (b) What percentage of the water is evaporated?

32. Construct a psychrometric chart on which the absolute humidity is plotted as the ordinate against dry-bulb temperatures as abscissas, plotting the following loci: 100 per cent relative humidity, 50 per cent relative humidity. Note that these loci are independent of any assumption as to the total pressure of the mixture. Assuming the total pressure to be 14.7 p.s.i. abs., plot the line which represents a constant wet-bulb temperature of  $70^{\circ}\text{F.}$

33. Construct a skeleton psychrometric chart similar to that of Appendix III but based on a total atmospheric pressure of 24.89 in. Hg (equivalent to an altitude of

5,000 ft. above sea level). Take the following steps in order: (a) Calculate data for saturation curve (100 per cent relative humidity) at  $10^{\circ}$  intervals, and plot. (b) Based on the dew-point temperature, calculate data for 50 per cent relative humidity line, and plot. (c) Calculate necessary data, and plot a line of  $70^{\circ}$  wet-bulb temperature. (d) Determine the locus of conditions for which the total volume of the standard unit mixture is 16 cu. ft., and plot this locus on your chart.



APPENDIX I  
TABLES OF PROPERTIES OF VAPORS

TABLE 1  
DRY SATURATED STEAM: TEMPERATURE TABLE \*

Temp., °F	Temp., °C	Abs Press., Lb Sq In.	Specific Volume			Enthalpy			Entropy			Temp., °F
			Sat. Liquid $v_f$	Evap. $v_{fg}$	Sat. Vapor $v_g$	Sat. Liquid $h_f$	Evap. $h_{fg}$	Sat. Vapor $h_g$	Sat. Liquid $s_f$	Evap. $s_{fg}$	Sat. Vapor $s_g$	
32	32	0.08554	0.01602	3306	3306	0.00	1075.8	1075.8	0.0000	2.1877	2.1877	32
35	35	0.09995	0.01602	2947	2947	3.02	1074.1	1077.1	0.0061	2.1709	2.1770	35
40	40	0.12170	0.01602	2444	2444	8.05	1071.3	1079.3	0.0162	2.1435	2.1597	40
45	45	0.14752	0.01602	2036.4	2036.4	13.06	1068.4	1081.5	0.0262	2.1167	2.1429	45
50	50	0.17811	0.01603	1703.2	1703.2	18.07	1065.6	1083.7	0.0361	2.0903	2.1264	50
60	60	0.2563	0.01604	1206.6	1206.6	28.06	1059.9	1088.0	0.0555	2.0393	2.0948	60
70	70	0.3631	0.01606	867.8	867.9	38.04	1054.3	1092.3	0.0745	1.9902	2.0647	70
80	80	0.5069	0.01608	633.1	633.1	48.02	1048.6	1096.6	0.0932	1.9428	2.0360	80
90	90	0.6982	0.01610	468.0	468.0	57.99	1042.9	1100.9	0.1115	1.8972	2.0087	90
100	100	0.9492	0.01613	350.3	350.4	67.97	1037.2	1105.2	0.1295	1.8531	1.9826	100
110	110	1.2748	0.01617	265.3	265.4	77.94	1031.6	1109.5	0.1471	1.8106	1.9577	110
120	120	1.6924	0.01620	203.25	203.27	87.92	1025.8	1113.7	0.1645	1.7684	1.9339	120
130	130	2.2226	0.01625	157.32	157.34	97.90	1020.0	1117.9	0.1816	1.7296	1.9112	130
140	140	2.8886	0.01629	122.99	123.01	107.89	1014.3	1122.0	0.1984	1.6940	1.8884	140
150	150	3.7115	0.01634	97.06	97.07	117.89	1008.5	1126.1	0.2149	1.6537	1.8683	150
160	160	4.741	0.01639	77.27	77.29	127.89	1002.3	1130.2	0.2311	1.6174	1.8485	160
170	170	5.992	0.01645	62.04	62.06	137.90	996.3	1134.2	0.2472	1.5822	1.8293	170
180	180	7.510	0.01651	50.21	50.23	147.92	990.2	1138.1	0.2630	1.5480	1.8109	180
190	190	9.339	0.01657	40.94	40.96	157.95	984.1	1142.0	0.2785	1.5147	1.7932	190
200	200	11.526	0.01663	33.62	33.64	167.99	977.9	1145.9	0.2938	1.4824	1.7762	200
210	210	14.123	0.01670	27.80	27.82	178.05	971.6	1149.7	0.3090	1.4508	1.7598	210
212	212	14.696	0.01672	26.78	26.80	180.07	970.3	1150.4	0.3120	1.4446	1.7566	212
220	220	17.186	0.01677	23.13	23.15	188.13	965.2	1153.4	0.3239	1.4201	1.7440	220
230	230	20.760	0.01684	19.365	19.385	198.23	958.8	1157.0	0.3387	1.3901	1.7288	230
240	240	24.969	0.01692	16.306	16.323	208.34	952.2	1160.5	0.3531	1.3609	1.7140	240
250	250	29.825	0.01700	13.804	13.821	218.48	945.5	1164.0	0.3675	1.3323	1.6998	250
260	260	35.459	0.01709	11.746	11.763	228.64	938.7	1167.3	0.3817	1.3043	1.6860	260
270	270	41.858	0.01717	10.044	10.061	238.84	931.8	1170.6	0.3958	1.2769	1.6727	270
280	280	49.203	0.01726	8.628	8.645	249.06	924.7	1173.8	0.4096	1.2501	1.6597	280
290	290	57.556	0.01735	7.444	7.461	259.31	917.5	1176.8	0.4234	1.2238	1.6472	290

300	67.013	0.01745	6.449	6.466	269.59	910.1	1179.7	0.4360	1.1980	1.6350	300
310	77.68	0.01755	5.609	5.626	279.92	902.6	1182.5	0.4637	1.1727	1.6321	310
320	89.66	0.01765	4.896	4.914	290.28	887.9	1187.2	0.4870	1.1533	1.6315	320
330	103.06	0.01776	4.289	4.307	300.18	870.0	1190.1	0.4900	1.1392	1.6002	330
340	118.01	0.01789	3.770	3.788	311.13	879.0				1.5891	340
350	134.63	0.01799	3.324	3.342	321.63	870.7	1192.3	0.5029	1.0754	1.5783	350
360	153.47	0.01811	2.930	2.957	332.18	862.2	1194.4	0.5158	1.0519	1.5677	360
370	173.37	0.01823	2.606	2.625	342.79	853.5	1196.3	0.5286	1.0287	1.5573	370
380	195.77	0.01836	2.317	2.335	353.45	844.6	1198.1	0.5413	1.0059	1.5471	380
390	220.37	0.01850	2.0651	2.0836	364.17	835.4	1199.6	0.5539	0.9832	1.5371	390
400	247.31	0.01864	1.8447	1.8633	374.97	826.0	1201.0	0.5664	0.9608	1.5272	400
410	276.75	0.01878	1.6512	1.6700	385.83	816.3	1202.1	0.5788	0.9386	1.5174	410
420	308.83	0.01894	1.4811	1.5000	396.77	806.3	1203.1	0.5912	0.9166	1.5078	420
430	343.72	0.01910	1.3308	1.3499	407.79	796.0	1203.8	0.6035	0.8947	1.4982	430
440	381.59	0.01926	1.1979	1.2171	418.90	785.4	1204.3	0.6158	0.8730	1.4887	440
450	422.6	0.0194	1.0799	1.0993	430.1	774.5	1204.6	0.6280	0.8513	1.4793	450
460	466.9	0.0196	0.9748	0.9944	441.4	763.2	1204.6	0.6402	0.8298	1.4700	460
470	514.7	0.0198	0.8811	0.9009	452.8	751.5	1204.3	0.6523	0.8083	1.4606	470
480	566.1	0.0200	0.7972	0.8172	464.4	739.4	1203.7	0.6645	0.7868	1.4513	480
490	621.4	0.0202	0.7221	0.7423	476.0	726.8	1202.8	0.6766	0.7653	1.4419	490
500	680.8	0.0204	0.6545	0.6749	487.8	713.9	1201.7	0.6887	0.7438	1.4325	500
520	812.4	0.0209	0.5385	0.5594	511.3	686.4	1198.2	0.7130	0.7008	1.4136	520
540	962.5	0.0215	0.4454	0.4668	539.2	656.6	1193.2	0.7374	0.6568	1.3942	540
560	1133.1	0.0221	0.3647	0.3868	567.2	626.2	1186.4	0.7621	0.6121	1.3742	560
580	1325.8	0.0228	0.2969	0.3217	598.9	598.4	1177.3	0.7872	0.5659	1.3532	580
600	1542.9	0.0236	0.2432	0.2668	617.0	548.5	1165.5	0.8131	0.5176	1.3307	600
620	1786.6	0.0247	0.1935	0.2201	646.7	503.6	1150.3	0.8398	0.4664	1.3062	620
640	2036.7	0.0260	0.1538	0.1798	678.6	452.0	1130.5	0.8679	0.4110	1.2789	640
660	2305.4	0.0278	0.1165	0.1442	714.2	390.2	1104.4	0.8987	0.3485	1.2472	660
680	2708.1	0.0305	0.0810	0.1115	757.3	309.9	1067.2	0.9351	0.2719	1.2071	680
700	3093.7	0.0369	0	0.0761	823.3	172.1	995.4	0.9905	0.1484	1.1389	700
705.4	3206.2	0.0503	0	0.0503	902.7	0	902.7	1.0580	0	1.0580	705.4

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TABLE 2  
DRY SATURATED STEAM: PRESSURE TABLE \*

Abs Press., Lb Sq In.	Temp., F	Specific Volume		Enthalpy			Entropy			Internal Energy		Abs Press., Lb Sq In.
		Sat. Liquid	Sat. Vapor	Sat. Liquid	Evap	Sat. Vapor	Sat. Liquid	Evap	Sat. Vapor	Sat. Liquid	Sat. Vapor	
$p$	$t$	$v_f$	$v_g$	$h_f$	$h_{fg}$	$h_g$	$s_f$	$s_{fg}$	$s_g$	$u_f$	$u_g$	$p$
0.491	79.03	0.01608	652.3	47.05	1049.2	1096.3	0.0914	1.9473	2.0387	47.05	1037.0	0.491
0.736	91.72	0.01611	444.9	59.71	1042.0	1101.7	0.1147	1.8894	2.0041	59.71	1041.1	0.736
0.983	101.14	0.01614	339.2	69.10	1036.6	1105.7	0.1316	1.8481	1.9797	69.10	1044.0	0.983
1.237	108.71	0.01616	274.9	76.65	1032.3	1108.9	0.1449	1.8160	1.9609	76.65	1046.4	1.237
1.473	115.06	0.01618	231.6	82.99	1028.6	1111.6	0.1560	1.7896	1.9456	82.99	1048.5	1.473
1.964	125.43	0.01622	176.7	93.34	1022.7	1116.0	0.1738	1.7476	1.9214	93.33	1051.8	1.964
2.455	133.76	0.01626	143.25	101.66	1017.7	1119.4	0.1879	1.7150	1.9028	101.65	1054.3	2.455
3.0	162.24	0.01640	73.52	130.13	1001.0	1131.1	0.2347	1.6094	1.8441	130.12	1063.1	3.0
4.0	193.21	0.01659	38.42	161.17	982.1	1143.3	0.2835	1.5041	1.7876	161.14	1072.2	4.0
5.0	212.0	0.01672	26.80	180.07	970.3	1150.4	0.3120	1.4446	1.7566	180.02	1077.5	5.0
15	213.03	0.01672	26.29	181.11	969.7	1150.8	0.3135	1.4415	1.7549	181.06	1077.8	15
16	216.32	0.01674	24.75	184.42	967.6	1152.0	0.3184	1.4313	1.7497	184.37	1078.7	16
18	222.41	0.01679	22.17	190.56	963.6	1154.2	0.3275	1.4128	1.7403	190.50	1080.4	18
20	227.96	0.01683	20.089	196.16	960.1	1156.3	0.3356	1.3962	1.7319	196.10	1081.9	20
25	240.07	0.01692	16.303	208.42	952.1	1160.6	0.3533	1.3606	1.7139	208.34	1085.1	25
30	250.33	0.01701	13.746	218.82	945.3	1164.1	0.3680	1.3313	1.6903	218.73	1087.8	30
35	259.53	0.01708	11.868	227.01	939.2	1167.1	0.3807	1.3063	1.6870	227.80	1090.1	35
40	267.53	0.01715	10.415	236.03	933.7	1169.7	0.3919	1.2844	1.6763	235.90	1092.0	40
45	274.44	0.01721	9.401	243.36	928.6	1172.1	0.4019	1.2650	1.6669	243.22	1093.7	45
50	281.01	0.01727	8.515	250.09	924.0	1174.1	0.4110	1.2474	1.6585	249.93	1095.3	50
55	287.07	0.01732	7.787	256.30	919.6	1175.9	0.4193	1.2316	1.6509	256.12	1096.7	55
60	292.71	0.01738	7.175	262.09	915.5	1177.6	0.4270	1.2168	1.6438	261.90	1097.9	60
65	297.97	0.01743	6.655	267.50	911.6	1179.1	0.4342	1.2032	1.6374	267.29	1099.1	65
70	302.92	0.01748	6.206	272.61	907.9	1180.6	0.4409	1.1906	1.6315	272.38	1100.2	70
75	307.60	0.01753	5.816	277.43	904.5	1181.9	0.4472	1.1787	1.6259	277.19	1101.2	75
80	312.03	0.01757	5.472	282.02	901.1	1183.1	0.4531	1.1676	1.6207	281.76	1102.1	80
85	316.25	0.01761	5.168	286.39	897.8	1184.2	0.4587	1.1571	1.6158	286.11	1103.7	85
90	320.27	0.01766	4.896	290.56	894.7	1185.3	0.4641	1.1471	1.6112	290.27	1105.2	90
100	327.81	0.01774	4.432	298.40	888.8	1187.2	0.4740	1.1286	1.6026	298.08	1106.5	100
110	334.77	0.01782	4.049	305.66	883.2	1188.9	0.4832	1.1117	1.5948	305.30	1108.2	110

130	341.25	0.01789	3.728	312.44	877.9	1190.4	0.4916	1.0062	1.5878	312.05	1107.6
130	347.32	0.01796	3.455	318.81	872.9	1191.7	0.4995	1.0817	1.5812	318.38	1108.6
140	353.02	0.01802	3.220	324.82	868.2	1193.0	0.5082	1.0682	1.5751	324.35	1109.6
150	358.42	0.01809	3.015	330.51	863.6	1194.1	0.5138	1.0536	1.5694	330.18	1110.6
160	363.53	0.01815	2.834	335.93	859.2	1195.1	0.5204	1.0436	1.5640	335.39	1111.2
170	368.41	0.01822	2.675	341.09	854.9	1196.0	0.5266	1.0324	1.5590	340.52	1111.9
180	373.06	0.01827	2.532	346.03	850.8	1196.9	0.5325	1.0217	1.5542	345.42	1112.5
190	377.49	0.01833	2.402	350.79	846.8	1197.6	0.5381	1.0116	1.5497	350.15	1113.1
200	381.72	0.01839	2.288	355.36	843.1	1198.4	0.5435	1.0013	1.5453	354.68	1113.7
250	400.35	0.01865	1.8438	376.00	825.1	1201.1	0.5675	0.9588	1.5263	375.14	1115.8
300	417.33	0.01890	1.5433	393.84	809.0	1202.8	0.5879	0.9225	1.5104	392.79	1117.1
350	431.72	0.01913	1.3260	409.69	794.2	1203.9	0.6056	0.8910	1.4966	408.45	1118.0
400	444.59	0.0193	1.1613	424.0	780.5	1204.6	0.6214	0.8630	1.4834	422.6	1118.5
450	456.28	0.0195	1.0320	437.2	767.4	1204.6	0.6356	0.8378	1.4734	435.5	1118.7
500	467.01	0.0197	0.9278	449.4	755.0	1204.4	0.6487	0.8147	1.4654	447.6	1118.6
550	476.94	0.0199	0.8424	460.8	743.1	1203.9	0.6608	0.7934	1.4542	458.8	1118.2
600	486.21	0.0201	0.7698	471.6	731.6	1203.2	0.6720	0.7734	1.4454	469.4	1117.7
650	494.90	0.0203	0.7083	481.8	720.5	1202.3	0.6826	0.7548	1.4374	479.4	1117.1
700	503.10	0.0205	0.6554	491.5	709.7	1201.2	0.6925	0.7371	1.4296	488.8	1116.3
750	510.86	0.0207	0.6092	500.8	699.2	1200.0	0.7019	0.7204	1.4223	498.0	1115.4
800	518.23	0.0209	0.5687	509.7	688.9	1198.6	0.7108	0.7045	1.4152	506.6	1114.4
850	525.26	0.0210	0.5327	518.3	678.8	1197.1	0.7194	0.6891	1.4082	515.1	1113.3
900	531.98	0.0212	0.5006	526.6	668.5	1195.4	0.7275	0.6743	1.4010	523.1	1112.1
950	538.43	0.0214	0.4717	534.6	659.1	1193.7	0.7355	0.6602	1.3957	530.9	1110.8
1000	544.61	0.0216	0.4456	542.4	649.1	1191.8	0.7430	0.6467	1.3897	538.4	1109.4
1100	556.31	0.0220	0.4001	557.4	630.4	1187.8	0.7575	0.6205	1.3780	552.9	1108.4
1200	567.22	0.0223	0.3619	571.7	611.7	1183.4	0.7711	0.5956	1.3667	566.7	1107.0
1300	577.46	0.0227	0.3293	585.4	593.2	1178.6	0.7840	0.5719	1.3559	580.0	1105.4
1400	587.10	0.0231	0.3012	598.7	574.7	1173.4	0.7963	0.5491	1.3454	592.7	1103.4
1500	596.23	0.0235	0.2765	611.6	556.3	1167.9	0.8082	0.5269	1.3351	605.1	1101.2
2000	635.82	0.0257	0.1878	671.7	463.4	1135.1	0.8619	0.4230	1.2849	662.2	1065.6
2500	668.13	0.0287	0.1307	730.6	360.5	1091.1	0.9126	0.3197	1.2322	717.3	1030.6
3000	695.36	0.0346	0.0858	802.7	217.8	1020.3	0.9731	0.1885	1.1615	783.4	972.7
3206.3	705.40	0.0503	0.0503	902.7	0	902.7	1.0580	0	1.0580	872.9	872.9

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TABLE 3  
PROPERTIES OF SUPERHEATED STEAM \*

Abs Press., Lb Sq In. (Sat. Temp.)		Temperature—Degrees Fahrenheit													
		200	220	300	350	400	450	500	550	600	700	800	900	1000	
1 (101.74)	<i>v</i>	392.6	404.5	452.3	482.2	512.0	541.8	571.6	601.4	631.2	690.8	750.4	809.9	869.5	
	<i>h</i>	1150.4	1159.5	1195.8	1218.7	1241.7	1264.9	1288.3	1312.0	1335.7	1385.8	1439.8	1492.7	1543.5	
5 (162.24)	<i>v</i>	78.16	80.59	90.25	96.26	102.26	108.24	114.22	120.19	126.16	138.10	150.03	161.95	173.87	
	<i>h</i>	1148.8	1158.1	1195.0	1218.1	1241.2	1264.5	1288.0	1311.7	1335.4	1383.6	1432.7	1482.6	1533.4	
10 (193.21)	<i>v</i>	38.85	40.09	45.00	48.03	51.04	54.05	57.05	60.04	63.03	69.01	74.98	80.95	86.92	
	<i>h</i>	1146.6	1156.2	1192.9	1217.2	1240.6	1264.0	1287.5	1311.3	1335.1	1383.4	1432.5	1482.4	1533.2	
14.696 (212.00)	<i>v</i>	27.15	30.53	32.62	34.68	36.73	38.78	40.82	42.86	46.94	51.00	55.07	59.13	63.19	
	<i>h</i>	1154.4	1192.8	1216.4	1239.9	1263.5	1287.1	1310.9	1334.8	1358.3	1383.2	1432.3	1482.3	1533.1	
20 (227.96)	<i>v</i>	22.36	23.91	25.43	26.95	28.46	29.97	31.47	32.97	34.47	37.46	40.45	43.44	46.43	
	<i>h</i>	1191.6	1215.6	1239.2	1262.9	1286.6	1310.5	1334.4	1358.2	1382.9	1432.1	1482.1	1532.4	1583.0	
40 (267.25)	<i>v</i>	11.040	11.843	12.628	13.401	14.168	14.93	15.688	16.461	17.218	18.702	20.20	21.70	23.20	
	<i>h</i>	1186.8	1211.9	1236.5	1260.7	1284.8	1308.9	1333.1	1357.3	1381.9	1431.3	1481.4	1531.9	1582.4	
60 (292.71)	<i>v</i>	7.259	7.818	8.357	8.884	9.403	9.916	10.427	10.937	11.441	12.449	13.452	14.454	15.456	
	<i>h</i>	1181.6	1208.2	1233.6	1258.5	1283.0	1307.4	1331.8	1356.9	1380.9	1430.5	1480.8	1531.8	1582.8	
80 (312.03)	<i>v</i>	5.803	6.22	6.624	7.020	7.416	7.812	8.208	8.604	9.000	9.922	10.977	10.830	11.833	
	<i>h</i>	1204.3	1230.7	1256.1	1281.1	1305.8	1330.5	1355.2	1379.9	1399.9	1429.7	1480.1	1531.3	1582.5	
100 (327.81)	<i>v</i>	4.592	4.937	5.268	5.589	5.905	6.218	6.528	6.836	7.141	7.446	8.052	8.656	9.260	
	<i>h</i>	1200.1	1227.6	1253.7	1279.1	1304.2	1329.1	1354.0	1378.9	1399.9	1429.9	1479.5	1529.8	1580.1	
130 (341.25)	<i>v</i>	3.783	4.081	4.363	4.636	4.902	5.165	5.428	5.691	5.954	6.195	6.702	7.207	7.712	
	<i>h</i>	1195.7	1224.4	1251.3	1277.2	1302.5	1327.7	1352.9	1377.8	1397.8	1428.1	1478.8	1529.5	1580.2	



TABLE 3—(Continued)  
PROPERTIES OF SUPERHEATED STEAM \*

Ab. Press. lb Sq. In. (Sat. Temp.)		Temperature—Degrees Fahrenheit—												
		500	550	600	650	700	800	900	1000	1200	1400	1600		
450 (456.28)	<i>t</i> .....	1.1231	1.2155	1.3005	1.3832	1.4678	1.5074	1.6074	1.7516	1.9228	2.170	2.443		
	<i>v</i> .....	1238.4	1272.0	1307.5	1344.6	1387.5	1443.3	1494.7	1621.0	1828.6	2178.7	2738.7		
	<i>s</i> .....	1.5095	1.5437	1.5758	1.5951	1.6153	1.6250	1.6699	1.7108	1.7486	1.8177	1.8803		
500 (467.01)	<i>t</i> .....	0.9927	1.0800	1.1591	1.2188	1.2763	1.3044	1.4405	1.5715	1.6996	1.9504	2.197		
	<i>v</i> .....	1231.3	1266.8	1298.6	1310.7	1322.6	1334.2	1412.1	1466.0	1519.6	1627.6	1737.9		
	<i>s</i> .....	1.4919	1.5280	1.5588	1.5701	1.5810	1.5915	1.6571	1.6982	1.7363	1.8056	1.8683		
550 (476.94)	<i>t</i> .....	0.8852	0.9686	1.0431	1.0989	1.1523	1.1783	1.3038	1.4241	1.5414	1.7706	2.219		
	<i>v</i> .....	1223.7	1261.2	1294.3	1306.8	1318.9	1324.5	1409.9	1464.3	1518.2	1626.6	1737.1		
	<i>s</i> .....	1.4751	1.5131	1.5451	1.5568	1.5680	1.5787	1.5991	1.6452	1.6868	1.7250	1.9155		
600 (486.21)	<i>t</i> .....	0.7947	0.8753	0.9463	0.9729	1.0241	1.0489	1.0732	1.1899	1.3013	1.4096	1.8279		
	<i>v</i> .....	1215.7	1255.5	1289.9	1302.7	1315.2	1327.4	1339.3	1407.7	1462.5	1516.7	1625.5		
	<i>s</i> .....	1.4586	1.4990	1.5323	1.5443	1.5558	1.5667	1.5875	1.6343	1.6762	1.7147	1.8476		
700 (503.10)	<i>t</i> .....	.....	0.7277	0.7934	0.8177	0.8411	0.8639	0.9077	1.0108	1.1082	1.3853	1.5641		
	<i>v</i> .....	.....	1243.2	1280.6	1294.3	1307.5	1320.3	1332.8	1403.2	1459.0	1513.9	1623.5		
	<i>s</i> .....	.....	1.4722	1.5084	1.5212	1.5333	1.5449	1.5665	1.6147	1.6573	1.6963	1.8299		
800 (518.23)	<i>t</i> .....	.....	0.6154	0.6779	0.7006	0.7223	0.7433	0.7833	0.8763	0.9633	1.0470	1.3662		
	<i>v</i> .....	.....	1229.8	1270.7	1285.4	1299.4	1312.9	1325.9	1395.6	1453.4	1511.0	1621.4		
	<i>s</i> .....	.....	1.4467	1.4863	1.5000	1.5129	1.5250	1.5366	1.5876	1.6301	1.6681	1.8146		
900 (531.98)	<i>t</i> .....	.....	0.5264	0.5873	0.6089	0.6294	0.6491	0.6880	0.8663	0.7716	0.8506	0.9262		
	<i>v</i> .....	.....	1215.0	1260.1	1275.9	1290.9	1305.1	1318.8	1393.9	1451.8	1508.1	1619.3		
	<i>s</i> .....	.....	1.4216	1.4653	1.4800	1.4938	1.5066	1.5187	1.5503	1.5814	1.6257	1.6656		
1000 (544.61)	<i>t</i> .....	.....	0.4533	0.5140	0.5350	0.5546	0.5733	0.5912	0.6084	0.6878	0.7604	0.8294		
	<i>v</i> .....	.....	1198.3	1248.8	1265.9	1281.9	1297.0	1311.4	1325.3	1389.2	1448.2	1505.1		
	<i>s</i> .....	.....	1.3961	1.4450	1.4610	1.4757	1.4893	1.5021	1.5141	1.5670	1.6121	1.6525		
1100 (556.31)	<i>t</i> .....	.....	0.4532	0.4738	0.4929	0.5110	0.5281	0.5445	0.6191	0.6866	0.7503	0.8716		
	<i>v</i> .....	.....	1236.7	1255.3	1272.4	1288.5	1303.7	1318.3	1398.3	1444.5	1502.2	1615.2		
	<i>s</i> .....	.....	1.4251	1.4425	1.4583	1.4728	1.4862	1.4989	1.5535	1.5995	1.6405	1.7130		
1300 (567.22)	<i>t</i> .....	.....	0.4016	0.4222	0.4410	0.4586	0.4752	0.4909	0.5617	0.6250	0.6843	0.7967		
	<i>v</i> .....	.....	1223.5	1243.9	1262.4	1279.6	1295.7	1311.0	1379.3	1440.7	1499.2	1613.1		
	<i>s</i> .....	.....	1.4052	1.4243	1.4413	1.4568	1.4710	1.4843	1.5409	1.5879	1.6293	1.7025		

<b>1400</b> (587.10)	$\bar{v}$ ..... $h$ ..... $s$ .....	0.3174 1193.0 1.3639	0.3390 1218.4 1.3877	0.3580 1240.4 1.4079	0.3753 1260.3 1.4258	0.3912 1278.5 1.4419	0.4062 1295.5 1.4567	0.4214 1311.1 1.4717	0.4381 1328.1 1.4866	0.4528 1343.1 1.5014	0.4678 1358.1 1.5161	0.4805 1373.1 1.5308	0.4927 1388.1 1.5455	0.5045 1403.1 1.5602	0.5158 1418.1 1.5749	0.5268 1433.1 1.5896	0.5375 1448.1 1.6043	0.5478 1463.1 1.6190	0.5578 1478.1 1.6337	0.5675 1493.1 1.6484	0.5768 1508.1 1.6631	0.5858 1523.1 1.6778	0.5945 1538.1 1.6925	0.6028 1553.1 1.7072	0.6108 1568.1 1.7219	0.6185 1583.1 1.7366	0.6258 1598.1 1.7513	0.6328 1613.1 1.7660	0.6395 1628.1 1.7807	0.6458 1643.1 1.7954	0.6518 1658.1 1.8101	0.6575 1673.1 1.8248	0.6628 1688.1 1.8395	0.6678 1703.1 1.8542	0.6725 1718.1 1.8689	0.6768 1733.1 1.8836	0.6808 1748.1 1.8983	0.6845 1763.1 1.9130	0.6878 1778.1 1.9277	0.6908 1793.1 1.9424	0.6935 1808.1 1.9571	0.6958 1823.1 1.9718	0.6978 1838.1 1.9865	0.6995 1853.1 2.0012	0.7008 1868.1 2.0159	0.7018 1883.1 2.0306	0.7025 1898.1 2.0453	0.7030 1913.1 2.0600	0.7035 1928.1 2.0747	0.7038 1943.1 2.0894	0.7040 1958.1 2.1041	0.7042 1973.1 2.1188	0.7045 1988.1 2.1335	0.7048 2003.1 2.1482	0.7050 2018.1 2.1629	0.7052 2033.1 2.1776	0.7055 2048.1 2.1923	0.7058 2063.1 2.2070	0.7060 2078.1 2.2217	0.7062 2093.1 2.2364	0.7065 2108.1 2.2511	0.7068 2123.1 2.2658	0.7070 2138.1 2.2805	0.7072 2153.1 2.2952	0.7075 2168.1 2.3099	0.7078 2183.1 2.3246	0.7080 2198.1 2.3393	0.7082 2213.1 2.3540	0.7085 2228.1 2.3687	0.7088 2243.1 2.3834	0.7090 2258.1 2.3981	0.7092 2273.1 2.4128	0.7095 2288.1 2.4275	0.7098 2303.1 2.4422	0.7100 2318.1 2.4569	0.7102 2333.1 2.4716	0.7105 2348.1 2.4863	0.7108 2363.1 2.5010	0.7110 2378.1 2.5157	0.7112 2393.1 2.5304	0.7115 2408.1 2.5451	0.7118 2423.1 2.5598	0.7120 2438.1 2.5745	0.7122 2453.1 2.5892	0.7125 2468.1 2.6039	0.7128 2483.1 2.6186	0.7130 2498.1 2.6333	0.7132 2513.1 2.6480	0.7135 2528.1 2.6627	0.7138 2543.1 2.6774	0.7140 2558.1 2.6921	0.7142 2573.1 2.7068	0.7145 2588.1 2.7215	0.7148 2603.1 2.7362	0.7150 2618.1 2.7509	0.7152 2633.1 2.7656	0.7155 2648.1 2.7803	0.7158 2663.1 2.7950	0.7160 2678.1 2.8097	0.7162 2693.1 2.8244	0.7165 2708.1 2.8391	0.7168 2723.1 2.8538	0.7170 2738.1 2.8685	0.7172 2753.1 2.8832	0.7175 2768.1 2.8979	0.7178 2783.1 2.9126	0.7180 2798.1 2.9273	0.7182 2813.1 2.9420	0.7185 2828.1 2.9567	0.7188 2843.1 2.9714	0.7190 2858.1 2.9861	0.7192 2873.1 3.0008	0.7195 2888.1 3.0155	0.7198 2903.1 3.0302	0.7200 2918.1 3.0449	0.7202 2933.1 3.0596	0.7205 2948.1 3.0743	0.7208 2963.1 3.0890	0.7210 2978.1 3.1037	0.7212 2993.1 3.1184	0.7215 3008.1 3.1331	0.7218 3023.1 3.1478	0.7220 3038.1 3.1625	0.7222 3053.1 3.1772	0.7225 3068.1 3.1919	0.7228 3083.1 3.2066	0.7230 3098.1 3.2213	0.7232 3113.1 3.2360	0.7235 3128.1 3.2507	0.7238 3143.1 3.2654	0.7240 3158.1 3.2801	0.7242 3173.1 3.2948	0.7245 3188.1 3.3095	0.7248 3203.1 3.3242	0.7250 3218.1 3.3389	0.7252 3233.1 3.3536	0.7255 3248.1 3.3683	0.7258 3263.1 3.3830	0.7260 3278.1 3.3977	0.7262 3293.1 3.4124	0.7265 3308.1 3.4271	0.7268 3323.1 3.4418	0.7270 3338.1 3.4565	0.7272 3353.1 3.4712	0.7275 3368.1 3.4859	0.7278 3383.1 3.5006	0.7280 3398.1 3.5153	0.7282 3413.1 3.5300	0.7285 3428.1 3.5447	0.7288 3443.1 3.5594	0.7290 3458.1 3.5741	0.7292 3473.1 3.5888	0.7295 3488.1 3.6035	0.7298 3503.1 3.6182	0.7300 3518.1 3.6329	0.7302 3533.1 3.6476	0.7305 3548.1 3.6623	0.7308 3563.1 3.6770	0.7310 3578.1 3.6917	0.7312 3593.1 3.7064	0.7315 3608.1 3.7211	0.7318 3623.1 3.7358	0.7320 3638.1 3.7505	0.7322 3653.1 3.7652	0.7325 3668.1 3.7799	0.7328 3683.1 3.7946	0.7330 3698.1 3.8093	0.7332 3713.1 3.8240	0.7335 3728.1 3.8387	0.7338 3743.1 3.8534	0.7340 3758.1 3.8681	0.7342 3773.1 3.8828	0.7345 3788.1 3.8975	0.7348 3803.1 3.9122	0.7350 3818.1 3.9269	0.7352 3833.1 3.9416	0.7355 3848.1 3.9563	0.7358 3863.1 3.9710	0.7360 3878.1 3.9857	0.7362 3893.1 4.0004	0.7365 3908.1 4.0151	0.7368 3923.1 4.0298	0.7370 3938.1 4.0445	0.7372 3953.1 4.0592	0.7375 3968.1 4.0739	0.7378 3983.1 4.0886	0.7380 3998.1 4.1033	0.7382 4013.1 4.1180	0.7385 4028.1 4.1327	0.7388 4043.1 4.1474	0.7390 4058.1 4.1621	0.7392 4073.1 4.1768	0.7395 4088.1 4.1915	0.7398 4103.1 4.2062	0.7400 4118.1 4.2209	0.7402 4133.1 4.2356	0.7405 4148.1 4.2503	0.7408 4163.1 4.2650	0.7410 4178.1 4.2797	0.7412 4193.1 4.2944	0.7415 4208.1 4.3091	0.7418 4223.1 4.3238	0.7420 4238.1 4.3385	0.7422 4253.1 4.3532	0.7425 4268.1 4.3679	0.7428 4283.1 4.3826	0.7430 4298.1 4.3973	0.7432 4313.1 4.4120	0.7435 4328.1 4.4267	0.7438 4343.1 4.4414	0.7440 4358.1 4.4561	0.7442 4373.1 4.4708	0.7445 4388.1 4.4855	0.7448 4403.1 4.5002	0.7450 4418.1 4.5149	0.7452 4433.1 4.5296	0.7455 4448.1 4.5443	0.7458 4463.1 4.5590	0.7460 4478.1 4.5737	0.7462 4493.1 4.5884	0.7465 4508.1 4.6031	0.7468 4523.1 4.6178	0.7470 4538.1 4.6325	0.7472 4553.1 4.6472	0.7475 4568.1 4.6619	0.7478 4583.1 4.6766	0.7480 4598.1 4.6913	0.7482 4613.1 4.7060	0.7485 4628.1 4.7207	0.7488 4643.1 4.7354	0.7490 4658.1 4.7501	0.7492 4673.1 4.7648	0.7495 4688.1 4.7795	0.7498 4703.1 4.7942	0.7500 4718.1 4.8089	0.7502 4733.1 4.8236	0.7505 4748.1 4.8383	0.7508 4763.1 4.8530	0.7510 4778.1 4.8677	0.7512 4793.1 4.8824	0.7515 4808.1 4.8971	0.7518 4823.1 4.9118	0.7520 4838.1 4.9265	0.7522 4853.1 4.9412	0.7525 4868.1 4.9559	0.7528 4883.1 4.9706	0.7530 4898.1 4.9853	0.7532 4913.1 5.0000	0.7535 4928.1 5.0147	0.7538 4943.1 5.0294	0.7540 4958.1 5.0441	0.7542 4973.1 5.0588	0.7545 4988.1 5.0735	0.7548 5003.1 5.0882	0.7550 5018.1 5.1029	0.7552 5033.1 5.1176	0.7555 5048.1 5.1323	0.7558 5063.1 5.1470	0.7560 5078.1 5.1617	0.7562 5093.1 5.1764	0.7565 5108.1 5.1911	0.7568 5123.1 5.2058	0.7570 5138.1 5.2205	0.7572 5153.1 5.2352	0.7575 5168.1 5.2499	0.7578 5183.1 5.2646	0.7580 5198.1 5.2793	0.7582 5213.1 5.2940	0.7585 5228.1 5.3087	0.7588 5243.1 5.3234	0.7590 5258.1 5.3381	0.7592 5273.1 5.3528	0.7595 5288.1 5.3675	0.7598 5303.1 5.3822	0.7600 5318.1 5.3969	0.7602 5333.1 5.4116	0.7605 5348.1 5.4263	0.7608 5363.1 5.4410	0.7610 5378.1 5.4557	0.7612 5393.1 5.4704	0.7615 5408.1 5.4851	0.7618 5423.1 5.4998	0.7620 5438.1 5.5145	0.7622 5453.1 5.5292	0.7625 5468.1 5.5439	0.7628 5483.1 5.5586	0.7630 5498.1 5.5733	0.7632 5513.1 5.5880	0.7635 5528.1 5.6027	0.7638 5543.1 5.6174	0.7640 5558.1 5.6321	0.7642 5573.1 5.6468	0.7645 5588.1 5.6615	0.7648 5603.1 5.6762	0.7650 5618.1 5.6909	0.7652 5633.1 5.7056	0.7655 5648.1 5.7203	0.7658 5663.1 5.7350	0.7660 5678.1 5.7497	0.7662 5693.1 5.7644	0.7665 5708.1 5.7791	0.7668 5723.1 5.7938	0.7670 5738.1 5.8085	0.7672 5753.1 5.8232	0.7675 5768.1 5.8379	0.7678 5783.1 5.8526	0.7680 5798.1 5.8673	0.7682 5813.1 5.8820	0.7685 5828.1 5.8967	0.7688 5843.1 5.9114	0.7690 5858.1 5.9261	0.7692 5873.1 5.9408	0.7695 5888.1 5.9555	0.7698 5903.1 5.9702	0.7700 5918.1 5.9849	0.7702 5933.1 5.9996	0.7705 5948.1 6.0143	0.7708 5963.1 6.0290	0.7710 5978.1 6.0437	0.7712 5993.1 6.0584	0.7715 6008.1 6.0731	0.7718 6023.1 6.0878	0.7720 6038.1 6.1025	0.7722 6053.1 6.1172	0.7725 6068.1 6.1319	0.7728 6083.1 6.1466	0.7730 6098.1 6.1613	0.7732 6113.1 6.1760	0.7735 6128.1 6.1907	0.7738 6143.1 6.2054	0.7740 6158.1 6.2201	0.7742 6173.1 6.2348	0.7745 6188.1 6.2495	0.7748 6203.1 6.2642	0.7750 6218.1 6.2789	0.7752 6233.1 6.2936	0.7755 6248.1 6.3083	0.7758 6263.1 6.3230	0.7760 6278.1 6.3377	0.7762 6293.1 6.3524	0.7765 6308.1 6.3671	0.7768 6323.1 6.3818	0.7770 6338.1 6.3965	0.7772 6353.1 6.4112	0.7775 6368.1 6.4259	0.7778 6383.1 6.4406	0.7780 6398.1 6.4553	0.7782 6413.1 6.4700	0.7785 6428.1 6.4847	0.7788 6443.1 6.4994	0.7790 6458.1 6.5141	0.7792 6473.1 6.5288	0.7795 6488.1 6.5435	0.7798 6503.1 6.5582	0.7800 6518.1 6.5729	0.7802 6533.1 6.5876	0.7805 6548.1 6.6023	0.7808 6563.1 6.6170	0.7810 6578.1 6.6317	0.7812 6593.1 6.6464	0.7815 6608.1 6.6611	0.7818 6623.1 6.6758	0.7820 6638.1 6.6905	0.7822 6653.1 6.7052	0.7825 6668.1 6.7199	0.7828 6683.1 6.7346	0.7830 6698.1 6.7493	0.7832 6713.1 6.7640	0.7835 6728.1 6.7787	0.7838 6743.1 6.7934	0.7840 6758.1 6.8081	0.7842 6773.1 6.8228	0.7845 6788.1 6.8375	0.7848 6803.1 6.8522	0.7850 6818.1 6.8669	0.7852 6833.1 6.8816	0.7855 6848.1 6.8963	0.7858 6863.1 6.9110	0.7860 6878.1 6.9257	0.7862 6893.1 6.9404	0.7865 6908.1 6.9551	0.7868 6923.1 6.9698	0.7870 6938.1 6.9845	0.7872 6953.1 6.9992	0.7875 6968.1 7.0139	0.7878 6983.1 7.0286	0.7880 6998.1 7.0433	0.7882 7013.1 7.0580	0.7885 7028.1 7.0727	0.7888 7043.1 7.0874	0.7890 7058.1 7.1021	0.7892 7073.1 7.1168	0.7895 7088.1 7.1315	0.7898 7103.1 7.1462	0.7900 7118.1 7.1609	0.7902 7133.1 7.1756	0.7905 7148.1 7.1903	0.7908 7163.1 7.2050	0.7910 7178.1 7.2197	0.7912 7193.1 7.2344	0.7915 7208.1 7.2491	0.7918 7223.1 7.2638	0.7920 7238.1 7.2785	0.7922 7253.1 7.2932	0.7925 7268.1 7.3079	0.7928 7283.1 7.3226	0.7930 7298.1 7.3373	0.7932 7313.1 7.3520	0.7935 7328.1 7.3667	0.7938 7343.1 7.3814	0.7940 7358.1 7.3961	0.7942 7373.1 7.4108	0.7945 7388.1 7.4255	0.7948 7403.1 7.4402	0.7950 7418.1 7.4549	0.7952 7433.1 7.4696	0.7955 7448.1 7.4843	0.7958 7463.1 7.4990	0.7960 7478.1 7.5137	0.7962 7493.1 7.5284	0.7965 7508.1 7.5431	0.7968 7523.1 7.5578	0.7970 7538.1 7.5725	0.7972 7553.1 7.5872	0.7975 7568.1 7.6019	0.7978 7583.1 7.6166	0.7980 7598.1 7.6313	0.7982 7613.1 7.6460	0.7985 7628.1 7.6607	0.7988 7643.1 7.6754	0.7990 7658.1 7.6901	0.7992 7673.1 7.7048	0.7995 7688.1 7.7195	0.7998 7703.1 7.7342	0.8000 7718.1 7.7489	0.8002 7733.1 7.7636	0.8005 7748.1 7.7783	0.8008 7763.1 7.7930	0.8010 7778.1 7.8077	0.8012 7793.1 7.8224	0.8015 7808.1 7.8371	0.8018 7823.1 7.8518	0.8020 7838.1 7.8665	0.8022 7853.1 7.8812	0.8025 7868.1 7.8959	0.8028 7883.1 7.9106	0.8030 7898.1 7.9253	0.8032 7913.1 7.9400	0.8035 7928.1 7.9547	0.8038 7943.1 7.9694	0.8040 7958.1 7.9841	0.8042 7973.1 7.9988	0.8045 7988.1 8.0135	0.8048 8003.1 8.0282</
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TABLE 4.—PROPERTIES OF SATURATED AMMONIA<sup>1</sup>  
(Entropy and enthalpy measurements are from -40°F.)

Temp., °F. <i>t</i>	Pres- sure, lb. per sq. in. abs. <i>p</i>	Specific volume, cu. ft. per lb.		Enthalpy of liquid <i>h<sub>f</sub></i>	Enthalpy of vaporiza- tion <i>h<sub>fg</sub></i>	Enthalpy of vapor <i>h<sub>g</sub></i>	Entropy		
		Liquid <i>v<sub>f</sub></i>	Vapor <i>v<sub>g</sub></i>				of liquid <i>s<sub>f</sub></i>	of evap. <i>s<sub>fg</sub></i>	of vapor <i>s<sub>g</sub></i>
-40	10.41	0.02322	24.86	0.0	597.6	597.6	0.000	1.4242	1.4242
-38	11.04	0.02326	23.53	2.1	596.2	598.3	0.0051	1.4142	1.4193
-36	11.71	0.02331	22.27	4.3	594.8	599.1	0.0101	1.4048	1.4144
-34	12.41	0.02335	21.10	6.4	593.5	599.9	0.0151	1.3945	1.4096
-32	13.14	0.02340	20.00	8.5	592.1	600.6	0.0201	1.3847	1.4043
-30	13.90	0.02345	18.97	10.7	590.7	601.4	0.0250	1.3751	1.4001
-28	14.71	0.02349	18.00	12.8	589.3	602.1	0.0300	1.3655	1.3955
-26	15.55	0.02354	17.09	14.9	587.9	602.8	0.0350	1.3559	1.3909
-24	16.42	0.02359	16.24	17.1	586.5	603.6	0.0399	1.3464	1.3864
-22	17.34	0.02364	15.43	19.2	585.1	604.3	0.0448	1.3370	1.3818
-20	18.30	0.02369	14.68	21.4	583.6	605.0	0.0497	1.3277	1.3774
-18	19.30	0.02374	13.97	23.5	582.2	605.7	0.0545	1.3184	1.3729
-16	20.34	0.02378	13.29	25.6	580.8	606.4	0.0594	1.3092	1.3686
-14	21.43	0.02383	12.66	27.8	579.3	607.1	0.0642	1.3001	1.3643
-12	22.56	0.02384	12.06	30.0	577.8	607.8	0.0690	1.2910	1.3600
-10	23.74	0.02393	11.50	32.1	576.4	608.5	0.0738	1.2820	1.3558
-8	24.97	0.02399	10.97	34.3	574.9	609.2	0.0786	1.2730	1.3516
-6	26.26	0.02404	10.47	36.4	573.4	609.8	0.0833	1.2641	1.3474
-4	27.59	0.02409	9.991	38.6	571.9	610.5	0.0880	1.2553	1.3433
-2	28.98	0.02414	9.541	40.7	570.4	611.1	0.0928	1.2465	1.3393
0	30.42	0.02419	9.116	42.9	568.9	611.8	0.0975	1.2377	1.3352
2	31.92	0.02424	8.714	45.1	567.3	612.4	0.1022	1.2290	1.3312
4	33.47	0.02430	8.333	47.2	565.8	613.0	0.1069	1.2204	1.3273
6	35.09	0.02435	7.971	49.4	564.2	613.6	0.1115	1.2119	1.3234
8	36.77	0.02440	7.629	51.6	562.7	614.3	0.1162	1.2033	1.3195
10	38.51	0.02446	7.304	53.8	561.1	614.9	0.1208	1.1949	1.3157
12	40.31	0.02451	6.996	56.0	559.5	615.5	0.1254	1.1864	1.3118
14	42.18	0.02457	6.703	58.2	557.9	616.1	0.1300	1.1781	1.3081
16	44.12	0.02462	6.425	60.3	556.3	616.6	0.1346	1.1697	1.3043
18	46.13	0.02468	6.161	62.5	554.7	617.2	0.1392	1.1614	1.3006
20	48.21	0.02474	5.910	64.7	553.1	617.8	0.1437	1.1532	1.2969
22	50.36	0.02479	5.671	66.9	551.4	618.3	0.1483	1.1450	1.2933
24	52.59	0.02485	5.443	69.1	549.8	618.9	0.1528	1.1369	1.2897
26	54.90	0.02491	5.227	71.3	548.1	619.4	0.1573	1.1288	1.2861
28	57.28	0.02497	5.021	73.5	546.4	619.9	0.1618	1.1207	1.2825
30	59.74	0.02503	4.825	75.7	544.8	620.5	0.1663	1.1127	1.2790
32	62.29	0.02508	4.637	77.9	543.1	621.0	0.1708	1.1047	1.2755
34	64.91	0.02514	4.459	80.1	541.4	621.5	0.1753	1.0968	1.2721
36	67.63	0.02521	4.289	82.3	539.7	622.0	0.1797	1.0889	1.2686
38	70.43	0.02527	4.126	84.6	537.9	622.5	0.1841	1.0811	1.2652
40	73.32	0.02533	3.971	86.8	536.2	623.0	0.1885	1.0733	1.2618
42	76.31	0.02539	3.823	89.0	534.4	623.4	0.1930	1.0655	1.2585
44	79.38	0.02545	3.682	91.2	532.7	623.9	0.1974	1.0578	1.2552
46	82.55	0.02551	3.547	93.5	530.9	624.4	0.2018	1.0501	1.2519
48	85.82	0.02558	3.418	95.7	529.1	624.8	0.2062	1.0424	1.2486
50	89.19	0.02564	3.294	97.9	527.3	625.2	0.2105	1.0348	1.2453
52	92.66	0.02571	3.176	100.2	525.5	625.7	0.2149	1.0272	1.2421
54	96.23	0.02577	3.063	102.4	523.7	626.1	0.2192	1.0197	1.2389
56	99.91	0.02584	2.954	104.7	521.8	626.5	0.2236	1.0121	1.2357
58	103.7	0.02590	2.851	106.9	520.0	626.9	0.2279	1.0046	1.2325
60	107.6	0.02597	2.751	109.2	518.1	627.3	0.2322	0.9972	1.2294
62	111.6	0.02604	2.656	111.5	516.2	627.7	0.2365	0.9897	1.2262
64	115.7	0.02611	2.565	113.7	514.3	628.0	0.2408	0.9823	1.2231
66	120.0	0.02618	2.477	116.0	512.4	628.4	0.2451	0.9750	1.2201
68	124.3	0.02625	2.393	118.3	510.5	628.8	0.2494	0.9676	1.2170
70	128.8	0.02632	2.312	120.5	508.6	629.1	0.2537	0.9603	1.2140
72	133.4	0.02639	2.235	122.8	506.6	629.4	0.2579	0.9531	1.2110
74	138.1	0.02646	2.161	125.1	504.7	629.8	0.2622	0.9458	1.2080
76	143.0	0.02653	2.089	127.4	502.7	630.1	0.2664	0.9386	1.2050
78	147.9	0.02661	2.021	129.7	500.7	630.4	0.2706	0.9314	1.2020
80	153.0	0.02668	1.955	132.0	498.7	630.7	0.2749	0.9242	1.1991
82	158.3	0.02675	1.892	134.3	496.7	631.0	0.2791	0.9171	1.1962
84	163.7	0.02684	1.831	136.6	494.7	631.3	0.2833	0.9100	1.1933
86	169.2	0.02691	1.772	138.9	492.6	631.5	0.2875	0.9029	1.1904
88	174.8	0.02699	1.716	141.2	490.6	631.8	0.2917	0.8958	1.1875
90	180.6	0.02707	1.661	143.5	488.5	632.0	0.2958	0.8888	1.1846
92	186.6	0.02715	1.609	145.8	486.4	632.2	0.3000	0.8818	1.1818
94	192.7	0.02723	1.559	148.2	484.3	632.5	0.3041	0.8748	1.1789
96	198.9	0.02731	1.510	150.5	482.1	632.6	0.3083	0.8678	1.1761
98	205.3	0.02739	1.464	152.9	480.0	632.9	0.3125	0.8608	1.1733
100	211.9	0.02747	1.419	155.2	477.8	633.0	0.3166	0.8539	1.1705
105	228.9	0.02769	1.313	161.1	472.3	633.4	0.3269	0.8366	1.1635
110	247.0	0.02790	1.217	167.0	466.7	633.7	0.3372	0.8194	1.1566
115	266.2	0.02813	1.128	173.0	460.9	633.9	0.3474	0.8023	1.1497
120	286.4	0.02836	1.047	179.0	455.0	634.0	0.3576	0.7851	1.1427

<sup>1</sup> Marks, "Mechanical Engineers' Handbook."

TABLE 5.—PROPERTIES OF SUPERHEATED AMMONIA<sup>1</sup>  
(Condensed from *Circular No. 142* of the U. S. Bureau of Standards, 1923)

$v$  = specific volume in cu. ft. per lb.;  $h$  = enthalpy in B.t.u. per lb.;  
 $s$  = entropy.

( $h$  and  $s$  are measured from  $-40^{\circ}\text{F.}$ )

Pressure lb. per sq. in. abs.	Temp. of satu- rated vapor, $^{\circ}\text{F.}$	Temperature of superheated vapor, $^{\circ}\text{F.}$								
		-30	-20	-10	0	10	20	30	40	50
10	-41.34	$v$ 26.58	27.26	27.92	28.58	29.24	29.90	30.55	31.20	31.85
		$h$ 603.2	608.5	613.7	618.9	624.0	629.1	634.2	639.3	644.4
		$s$ 1.4420	1.4542	1.4659	1.4773	1.4884	1.4992	1.5097	1.5200	1.5301
20	-16.64	$v$ .....	.....	13.74	14.09	14.44	14.78	15.11	15.45	15.78
		$h$ .....	.....	610.0	615.5	621.0	626.4	631.7	637.0	642.3
		$s$ .....	.....	1.3784	1.3907	1.4025	1.4138	1.4240	1.4356	1.4460
30	- 0.57	$v$ .....	.....	.....	9.257	9.492	9.731	9.966	10.20	10.43
		$h$ .....	.....	.....	611.9	617.8	623.5	629.1	634.6	640.1
		$s$ .....	.....	.....	1.3371	1.3497	1.3618	1.3733	1.3845	1.3953
40	11.66	$v$ .....	.....	.....	.....	.....	7.203	7.387	7.568	7.746
		$h$ .....	.....	.....	.....	.....	620.4	626.3	632.1	637.8
		$s$ .....	.....	.....	.....	.....	1.3231	1.3353	1.3470	1.3583
50	21.67	$v$ .....	.....	.....	.....	.....	.....	5.838	5.988	6.135
		$h$ .....	.....	.....	.....	.....	.....	623.4	629.5	635.4
		$s$ .....	.....	.....	.....	.....	.....	1.3046	1.3169	1.3286
80	44.40	$v$ .....	.....	.....	.....	.....	.....	.....	.....	.....
		$h$ .....	.....	.....	.....	.....	.....	.....	.....	.....
		$s$ .....	.....	.....	.....	.....	.....	.....	.....	.....
100	56.05	$v$ .....	.....	.....	.....	.....	.....	.....	.....	.....
		$h$ .....	.....	.....	.....	.....	.....	.....	.....	.....
		$s$ .....	.....	.....	.....	.....	.....	.....	.....	.....
120	66.02	$v$ .....	.....	.....	.....	.....	.....	.....	.....	.....
		$h$ .....	.....	.....	.....	.....	.....	.....	.....	.....
		$s$ .....	.....	.....	.....	.....	.....	.....	.....	.....
140	74.79	$v$ .....	.....	.....	.....	.....	.....	.....	.....	.....
		$h$ .....	.....	.....	.....	.....	.....	.....	.....	.....
		$s$ .....	.....	.....	.....	.....	.....	.....	.....	.....
160	82.64	$v$ .....	.....	.....	.....	.....	.....	.....	.....	.....
		$h$ .....	.....	.....	.....	.....	.....	.....	.....	.....
		$s$ .....	.....	.....	.....	.....	.....	.....	.....	.....
180	89.78	$v$ .....	.....	.....	.....	.....	.....	.....	.....	.....
		$h$ .....	.....	.....	.....	.....	.....	.....	.....	.....
		$s$ .....	.....	.....	.....	.....	.....	.....	.....	.....
200	96.34	$v$ .....	.....	.....	.....	.....	.....	.....	.....	.....
		$h$ .....	.....	.....	.....	.....	.....	.....	.....	.....
		$s$ .....	.....	.....	.....	.....	.....	.....	.....	.....
220	102.42	$v$ .....	.....	.....	.....	.....	.....	.....	.....	.....
		$h$ .....	.....	.....	.....	.....	.....	.....	.....	.....
		$s$ .....	.....	.....	.....	.....	.....	.....	.....	.....
240	108.09	$v$ .....	.....	.....	.....	.....	.....	.....	.....	.....
		$h$ .....	.....	.....	.....	.....	.....	.....	.....	.....
		$s$ .....	.....	.....	.....	.....	.....	.....	.....	.....
260	113.42	$v$ .....	.....	.....	.....	.....	.....	.....	.....	.....
		$h$ .....	.....	.....	.....	.....	.....	.....	.....	.....
		$s$ .....	.....	.....	.....	.....	.....	.....	.....	.....

<sup>1</sup> Marks, "Mechanical Engineers' Handbook."



TABLE 6.—PROPERTIES OF SATURATED SULPHUR DIOXIDE<sup>1</sup>  
(*h* and *s* are measured from  $-40^{\circ}\text{F.}$ )

Temp., °F.	Pres- sure, lb. per sq. in. abs.	Specific volume, cu. ft. per lb.		En- thalpy of liquid, B.t.u. per lb.	En- thalpy of vapor- ization, B.t.u.	En- thalpy of vapor, B.t.u.	Entropy		
		Liquid	Vapor				Of liquid	Of vapor- ization	Of vapor
<i>t</i>	<i>p</i>	<i>v<sub>f</sub></i>	<i>v<sub>g</sub></i>	<i>h<sub>f</sub></i>	<i>h<sub>fg</sub></i>	<i>h<sub>g</sub></i>	<i>s<sub>f</sub></i>	<i>s<sub>fg</sub></i>	<i>s<sub>g</sub></i>
-40	3.136	0.01044	22.42	0.00	178.6	178.6	0.0000	0.4256	0.4256
-30	4.331	0.01053	16.56	2.93	177.0	179.9	0.00674	0.4119	0.4186
-20	5.883	0.01062	12.42	5.98	175.1	181.1	0.01366	0.3983	0.4119
-10	7.863	0.01072	9.44	9.16	173.0	182.1	0.02075	0.3847	0.4054
0	10.35	0.01082	7.28	12.44	170.6	183.1	0.02795	0.3712	0.3992
10	13.42	0.01092	5.682	15.80	168.1	183.9	0.03519	0.3570	0.3931
20	17.18	0.01103	4.487	19.20	165.3	184.5	0.04241	0.3447	0.3871
30	21.70	0.01114	3.581	22.64	162.4	185.0	0.04956	0.3316	0.3812
40	27.10	0.01126	2.887	26.12	159.3	185.4	0.05668	0.3187	0.3754
50	33.45	0.01138	2.348	29.61	156.0	185.6	0.06370	0.3060	0.3697
60	40.93	0.01150	1.926	33.10	152.5	185.6	0.07060	0.2935	0.3641
70	49.62	0.01163	1.590	36.58	148.9	185.5	0.07736	0.2811	0.3585
80	59.68	0.01176	1.321	40.05	145.1	185.2	0.08399	0.2690	0.3529
90	71.25	0.01190	1.104	43.50	141.2	184.7	0.09038	0.2569	0.3473
100	84.52	0.01204	0.9262	46.90	137.2	184.1	0.09657	0.2452	0.3417
110	99.76	0.01219	0.7804	50.26	133.1	183.3	0.1025	0.2336	0.3361
120	120.9	0.01236	0.6598	53.58	128.8	182.4	0.1083	0.2222	0.3305
130	136.5	0.01253	0.5595	56.85	124.4	181.2	0.1138	0.2110	0.3247
140	158.6	0.01272	0.4758	60.04	119.9	179.9	0.1189	0.1999	0.3189

<sup>1</sup> Marks, "Mechanical Engineers' Handbook."

TABLE 7.—PROPERTIES OF SUPERHEATED SULPHUR DIOXIDE<sup>1</sup>  
 ( $v$  = specific volume in cu. ft. per lb.;  $h$  = enthalpy in B.t.u. per lb.;  
 $s$  = entropy)  
 ( $h$  and  $s$  are measured from  $-40^{\circ}\text{F.}$ )

Pressure lb. per sq. in. abs.	Temp. of satu- rated vapor, $^{\circ}\text{F.}$	Temperature of superheated vapor, $^{\circ}\text{F.}$								
		0	20	40	60	80	100	120	140	160
6	-19.37	$v$ 12.75	13.34	13.93	14.52	15.11	15.69	16.26	16.82	17.35
		$h$ 184.3	187.5	190.7	193.9	197.2	200.5	203.8	207.1	210.4
		$s$ 0.4185	0.4254	0.4320	0.4383	0.4444	0.4504	0.4561	0.4618	0.4672
10	-1.34	$v$ 7.545	7.939	8.316	8.681	9.038	9.389	9.736	10.08	10.42
		$h$ 183.2	186.7	190.1	193.5	196.9	200.3	203.7	207.1	210.5
		$s$ 0.4005	0.4080	0.4151	0.4216	0.4280	0.4341	0.4400	0.4457	0.4512
15	14.43	$v$ .....	5.192	5.470	5.734	5.988	6.233	6.471	6.705	6.937
		$h$ .....	185.4	189.2	192.8	196.4	199.9	203.3	206.7	210.1
		$s$ .....	0.3927	0.4005	0.4078	0.4144	0.4208	0.4268	0.4326	0.4383
20	26.44	$v$ .....	.....	4.035	4.251	4.454	4.648	4.834	5.015	5.193
		$h$ .....	.....	187.8	191.8	195.6	199.3	202.9	206.5	209.9
		$s$ .....	.....	0.3896	0.3972	0.4043	0.4109	0.4173	0.4232	0.4290
25	36.33	$v$ .....	.....	3.181	3.363	3.536	3.696	3.848	3.998	4.145
		$h$ .....	.....	186.1	190.6	194.7	198.6	202.4	206.0	209.6
		$s$ .....	.....	0.3793	0.3880	0.3958	0.4029	0.4095	0.4157	0.4216
30	44.76	$v$ .....	.....	.....	.....	.....	.....	.....	.....	.....
		$h$ .....	.....	.....	.....	.....	.....	.....	.....	.....
		$s$ .....	.....	.....	.....	.....	.....	.....	.....	.....
40	58.83	$v$ .....	.....	.....	.....	.....	.....	.....	.....	.....
		$h$ .....	.....	.....	.....	.....	.....	.....	.....	.....
		$s$ .....	.....	.....	.....	.....	.....	.....	.....	.....
60	80.29	$v$ .....	.....	.....	.....	.....	.....	.....	.....	.....
		$h$ .....	.....	.....	.....	.....	.....	.....	.....	.....
		$s$ .....	.....	.....	.....	.....	.....	.....	.....	.....
80	96.88	$v$ .....	.....	.....	.....	.....	.....	.....	.....	.....
		$h$ .....	.....	.....	.....	.....	.....	.....	.....	.....
		$s$ .....	.....	.....	.....	.....	.....	.....	.....	.....

<sup>1</sup> Marks, "Mechanical Engineers' Handbook."

TABLE 8.—PROPERTIES OF CARBON DIOXIDE<sup>1</sup>  
(Enthalpy measurements are from 32°F.)

Temperature, °F.	Pressure, lb. per sq. in. abs.	Density, lb. per cu. ft.		B.t.u. per lb.			Entropy	
		of the liquid	of the vapor	Enthalpy of the liquid	Enthalpy of vapor- ization	Enthalpy of satu- rated vapor	of the liquid	of the vapor
<i>t</i>	<i>p</i>	$\frac{1}{v_f}$	$\frac{1}{v_g}$	<i>h<sub>f</sub></i>	<i>h<sub>fg</sub></i>	<i>h<sub>g</sub></i>	<i>s<sub>f</sub></i>	<i>s<sub>g</sub></i>
-40	145.87	69.8	1.64	-38.5	136.5	98.0	-0.0850	0.2400
-35	161.33	69.1	1.83	-35.8	134.3	98.5	-0.0793	0.2367
-30	177.97	68.3	2.02	-33.1	132.1	99.0	-0.0735	0.2336
-25	195.85	67.6	2.23	-30.4	129.8	99.4	-0.0676	0.2306
-20	215.02	66.9	2.44	-27.7	127.5	99.8	-0.0619	0.2277
-15	235.53	66.1	2.66	-24.9	125.0	100.1	-0.0560	0.2250
-10	257.46	65.3	2.91	-22.1	122.4	100.3	-0.0500	0.2220
-5	280.85	64.5	3.17	-19.4	120.0	100.6	-0.0440	0.2198
0	305.76	63.6	3.46	-16.7	117.5	100.8	-0.0381	0.2173
5	332.2	62.8	3.77	-14.0	115.0	101.0	-0.0322	0.2151
10	360.4	61.9	4.12	-11.2	112.2	101.0	-0.0264	0.2124
15	390.2	61.0	4.49	-8.4	109.4	101.0	-0.0204	0.2100
20	421.8	60.0	4.89	-5.5	106.3	100.8	-0.0144	0.2071
25	455.3	59.0	5.33	-2.5	103.1	100.6	-0.0083	0.2043
30	490.6	58.0	5.81	+ 0.4	99.7	100.1	-0.0021	0.2012
35	528.0	57.0	6.35	3.5	95.8	99.3	+0.0039	0.1975
40	567.3	55.9	6.91	6.6	91.8	98.4	0.0099	0.1934
45	608.9	54.7	7.60	9.8	87.5	97.3	0.0160	0.1892
50	652.7	53.4	8.37	12.9	83.2	96.1	0.0220	0.1852
55	698.8	52.1	9.27	16.1	78.7	94.8	0.0282	0.1809
60	747.4	50.7	10.2	19.4	74.0	93.4	0.0345	0.1767
65	798.6	49.1	11.3	22.9	68.9	91.8	0.0412	0.1724
70	852.4	47.3	12.6	26.6	62.7	89.3	0.0482	0.1665
75	909.3	45.1	14.2	30.9	54.8	85.7	0.0562	0.1587
80	969.3	42.4	16.2	35.6	44.0	79.6	0.0649	0.1464
85	1032.7	38.2	19.1	41.7	27.5	69.2	0.0761	0.1265
88	1072.1	32.9	25.4	Critical Point				

<sup>1</sup> Marks, "Mechanical Engineers' Handbook."

TABLE 9.—PROPERTIES OF DICHLORODIFLUOROMETHANE (F-12-FREON)<sup>1</sup>  
(Entropy and enthalpy measurements are from -40°F.)

Temp., °F. <i>t</i>	Pressure, p.s.i. abs. <i>p</i>	Specific volume, cu. ft. per lb.		Enthalpy		Entropy	
		Liquid <i>v<sub>f</sub></i>	Vapor <i>v<sub>g</sub></i>	Liquid <i>h<sub>f</sub></i>	Vapor <i>h<sub>g</sub></i>	Liquid <i>s<sub>f</sub></i>	Vapor <i>s<sub>g</sub></i>
-40	9.33	0.0106	3.911	0	73.50	0	0.17517
-30	12.02	0.0107	3.088	2.03	74.70	0.00471	0.17387
-20	15.28	0.0108	2.474	4.07	75.87	0.00940	0.17275
-10	19.20	0.0109	2.003	6.14	77.05	0.01403	0.17175
0	23.87	0.0110	1.637	8.25	78.21	0.01869	0.17091
2	24.89	0.0111	1.574	8.67	78.44	0.01961	0.17075
4	25.96	0.0111	1.514	9.10	78.67	0.02052	0.17060
6	27.05	0.0111	1.457	9.53	78.90	0.02143	0.17045
8	28.18	0.0111	1.403	9.96	79.13	0.02235	0.17030
10	29.35	0.0112	1.351	10.39	79.36	0.02328	0.17015
12	30.56	0.0112	1.301	10.82	79.59	0.02419	0.17001
14	31.80	0.0112	1.253	11.26	79.82	0.02510	0.16987
16	33.08	0.0113	1.207	11.70	80.05	0.02601	0.16974
18	34.40	0.0113	1.163	12.12	80.27	0.02692	0.16961
20	35.75	0.0113	1.121	12.55	80.49	0.02783	0.16949
22	37.15	0.0113	1.081	13.00	80.72	0.02873	0.16938
24	38.58	0.0114	1.043	13.44	80.95	0.02963	0.16926
26	40.07	0.0114	1.007	13.88	81.17	0.03053	0.16913
28	41.59	0.0114	0.973	14.32	81.39	0.03143	0.16900
30	43.16	0.0115	0.939	14.76	81.61	0.03233	0.16887
32	44.77	0.0115	0.908	15.21	81.83	0.03323	0.16876
34	46.42	0.0115	0.877	15.65	82.05	0.03413	0.16865
36	48.13	0.0115	0.848	16.10	82.27	0.03502	0.16854
38	49.88	0.0116	0.819	16.55	82.49	0.03591	0.16843
40	51.68	0.0116	0.792	17.00	82.71	0.03680	0.16833
42	53.51	0.0116	0.767	17.46	82.93	0.03770	0.16823
44	55.40	0.0117	0.742	17.91	83.15	0.03859	0.16813
46	57.35	0.0117	0.718	18.36	83.36	0.03948	0.16803
48	59.35	0.0117	0.695	18.82	83.57	0.04037	0.16794
50	61.39	0.0118	0.673	19.27	83.78	0.04126	0.16785
52	63.49	0.0118	0.652	19.72	83.99	0.04215	0.16776
54	65.63	0.0118	0.632	20.18	84.20	0.04304	0.16767
56	67.84	0.0119	0.612	20.64	84.41	0.04392	0.16758
58	70.10	0.0119	0.593	21.11	84.62	0.04480	0.16749
60	72.41	0.0119	0.575	21.57	84.82	0.04568	0.16741
62	74.77	0.0120	0.557	22.03	85.02	0.04657	0.16733
64	77.20	0.0120	0.540	22.49	85.22	0.04745	0.16725
66	79.67	0.0120	0.524	22.95	85.42	0.04833	0.16717
68	82.24	0.0121	0.508	23.42	85.62	0.04921	0.16709
70	84.82	0.0121	0.493	23.90	85.82	0.05009	0.16701
72	87.50	0.0121	0.479	24.37	86.02	0.05097	0.16693
74	90.20	0.0122	0.464	24.84	86.22	0.05185	0.16685
76	93.00	0.0122	0.451	25.32	86.42	0.05272	0.16677
78	95.85	0.0122	0.438	25.80	86.61	0.05359	0.16669
80	98.76	0.0123	0.425	26.28	86.80	0.05446	0.16662
82	101.7	0.0123	0.413	26.76	86.99	0.05534	0.16655
84	104.8	0.0124	0.401	27.24	87.18	0.05621	0.16648
86	107.9	0.0124	0.389	27.72	87.37	0.05708	0.16640
88	111.1	0.0124	0.378	28.21	87.56	0.05795	0.16632
90	114.3	0.0125	0.368	28.79	87.74	0.05882	0.16624
92	117.7	0.0125	0.357	29.19	87.92	0.05969	0.16616
94	121.0	0.0125	0.347	29.68	88.10	0.06056	0.16608
96	124.5	0.0126	0.338	30.18	88.28	0.06143	0.16600
98	128.0	0.0126	0.328	30.67	88.45	0.06230	0.16592
100	131.6	0.0127	0.319	31.16	88.62	0.06316	0.16584
110	150.7	0.0129	0.277	33.65	89.43	0.06749	0.16542
120	171.8	0.0131	0.240	36.16	90.15	0.07180	0.16495

<sup>1</sup> Adapted from "Refrigerating Data Book."

TABLE 10.—DATA FOR MIXTURES OF AIR AND SATURATED WATER VAPOR

Temperature, °F	Weight of saturated vapor		Volume, cu. ft.	
	Lb. per cu. ft. of dry air	Lb. per lb. of dry air	Of 1 lb. of dry air	Of 1 lb. of dry air plus vapor to saturate it
0	0.0000674	0.000781	11.58	11.59
6	0.0000909	0.001067	11.73	11.75
10	0.0001103	0.001309	11.83	11.86
16	0.000147	0.001764	11.99	12.02
20	0.000177	0.002144	12.09	12.13
26	0.000232	0.002854	12.24	12.30
30	0.000278	0.003444	12.34	12.41
35	0.000340	0.004268	12.47	12.55
40	0.000410	0.005202	12.59	12.70
45	0.000492	0.00632	12.72	12.85
50	0.000588	0.00764	12.84	13.00
55	0.000699	0.00920	12.97	13.16
60	0.000829	0.01105	13.10	13.33
65	0.000979	0.01323	13.22	13.50
70	0.001153	0.01578	13.35	13.69
75	0.001352	0.01877	13.48	13.88
80	0.001580	0.02226	13.60	14.09
85	0.001841	0.02634	13.73	14.31
90	0.002137	0.03109	13.86	14.55
95	0.002474	0.03662	13.98	14.80
100	0.002855	0.04305	14.11	15.08
105	0.003285	0.0505	14.24	15.39
110	0.003769	0.0593	14.36	15.73
115	0.004312	0.0694	14.49	16.10
120	0.004920	0.0813	14.62	16.52
125	0.005599	0.0953	14.75	16.99
130	0.006356	0.1114	14.88	17.53
135	0.007197	0.1305	15.00	18.13
140	0.008130	0.1532	15.13	18.84
145	0.00916	0.1800	15.26	19.64
150	0.01030	0.2122	15.39	20.60
155	0.01156	0.2511	15.52	21.73
160	0.01294	0.2987	15.64	23.09
165	0.01445	0.3577	15.77	24.75
170	0.01611	0.4324	15.90	26.84
175	0.01793	0.5290	16.03	29.51
180	0.01991	0.6577	16.16	33.04
185	0.02206	0.8359	16.28	37.89
190	0.02441	1.0985	16.41	45.00
200	0.02972	2.2953	16.67	77.24

TABLE 11.—MANTISSAS OF COMMON LOGARITHMS

	0	1	2	3	4	5	6	7	8	9
100	0000	0004	0009	0013	0017	0022	0026	0030	0035	0039
101	0043	0048	0052	0056	0060	0065	0069	0073	0077	0082
102	0086	0090	0095	0099	0103	0107	0111	0116	0120	0124
103	0128	0133	0137	0141	0145	0149	0154	0158	0162	0166
104	0170	0175	0179	0183	0187	0191	0195	0199	0204	0208
105	0212	0216	0220	0224	0228	0233	0237	0241	0245	0249
106	0253	0257	0261	0. 65	0269	0273	0278	0282	0286	0290
107	0294	0298	0302	0306	0310	0314	0318	0322	0326	0330
108	0334	0338	0342	0346	0350	0354	0358	0362	0366	0370
109	0374	0378	0382	0386	0390	0394	0398	0402	0406	0410
110	0414	0418	0422	0426	0430	0434	0438	0441	0445	0449
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981

The Napierian logarithm equals 2.3026 times the common logarithm  
 $\log_e n = 2.3026 \times \log_{10} n$

TABLE 11.—MANTISSAS OF COMMON LOGARITHMS (Continued)

	0	1	2	3	4	5	6	7	8	9
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802
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95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818
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97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996

The Napierian logarithm equals 2.3026 times the common logarithm  
 $\log_e n = 2.3026 \times \log_{10} n$

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